

# A Fuzzy Clustering Mathematical Model for Sub-assemblies Based on Game Theory

Xiaoxi Kou\*

School of Mechatronic Engineering  
Xi'an Technological University  
Shaanxi, China  
E-mail: 878014795@qq.com

Hu Qiao

School of Mechatronic Engineering  
Xi'an Technological University  
Shaanxi, China  
E-mail: 418141845@qq.com

Yan Cao

School of Mechatronic Engineering  
Xi'an Technological University  
Shaanxi, China  
E-mail: 670533578@qq.com

Yujia Wu

School of Mechatronic Engineering  
Xi'an Technological University  
Shaanxi, China  
E-mail: 286356270@qq.com

**Abstract**—Taking sub-assembly recognition problem as the research object and combining with the mathematical model of sub-assembly recognition algorithm, this paper analyzes the similarity and corresponding mapping between the objective function problem of sub-assembly recognition and the game problem. The object function of sub-assembly recognition is transformed into the benefit of game participants, and the parameter variable is transformed into game strategy space. The steps of game analysis for sub-assembly recognition algorithm are systematically planned, and the fuzzy clustering mathematical model of sub-assembly based on game theory is given. It is beneficial to the determination of each parameter of the target function of sub-assembly recognition algorithm, and can reduce the influence of parameter weight on the result of the algorithm. It has practical engineering application significance.

**Keywords**—*Mathematical Model; Objective Function; Game Analysis*

## I. INTRODUCTION

In the manufacturing industry, more and more intelligent algorithms are applied to solve complex problems in the engineering field. Sub-assembly recognition algorithm is an intelligent algorithm which applies the concepts of set clustering and fuzzy set to assembly field. Sub-assembly recognition algorithm is a multi-objective problem in theory. It needs to meet the requirements of different parameters in the application process. The difficulty of solving multi-objective problem is to adjust the value of each parameter and reconcile the contradictions among conflicting objects under the condition of satisfying multiple constraints. The change of one parameter may lead to the weakening of other objectives. The tendency of change of one parameter leads to the lack of uniform criteria for solving multi-objective problems. The common solution of multi-objective intelligent algorithm is to change the multi-objective problem into a single-objective problem in some way, so as to obtain a solution set with pareto optimal solution, or a

non-dominated set[1]. Game theory, as a well-known optimization method for studying contradictions, reconciling or resolving conflicts, can change the decision under given information to maximize the benefits of each participant in the game[2]. The multi-objective intelligent algorithm combined with the thought of game analysis is to transform the multi-objective problem into a game optimization problem, which can accelerate the iterative convergence rate, overcome the local optimization, and obtain the non-dominated set or pareto optimal solution[3].

In recent years, game theory has been used to solve multi-objective problems. R. Spallino et al. combined fuzzy clustering method and game optimization idea, and considered some conflicting objectives in the multi-objective optimization design of composite laminated boards[4]. Periaux et al studied the multi-objective optimization of aerodynamic shape of airfoil under transonic flow on the basis of fuzzy clustering and game optimization[5]. According to the basic game solution model, Nengang Xie, Songlin Sun, Xingwen Guo et al. obtained the strategy subset of game division by fuzzy clustering, established the technical route and solution model of multi-objective game optimization design, and successfully applied it to the multi-objective optimization design of variable amplitude mechanism of compensation pulley block[6].

Engineering practice has proved that the combination of intelligent algorithm and game analysis method can give overall consideration to multiple objectives and solve multi-objective problems more effectively[7]. Combining game theory with sub-assembly recognition algorithm can optimize the solving process of a complex assembly problem.

## II. DESCRIPTION OF SUB-ASSEMBLY RECOGNITION PROBLEM

To realize the process of sub-assembly recognition, firstly, the number of sub-assemblies and the central parts of sub-assemblies should be preset. Secondly, the actual assembly relation of the assembly is transformed into the

correlation relation matrix between each part. Then the membership degree of each part to the preset sub-assembly center will be obtained through algorithm analysis. Finally, according to the size of membership, the parts are classified into sub-assemblies with the highest membership. The sub-assembly recognition algorithm divides the assembly connection relation into several data matrix subsets. These data matrix subsets are transformed into recognized sub-assemblies according to their corresponding model relationships.

In mathematical theory, the data of assembly incidence matrix to be processed can be regarded as an n-dimensional data space. The data information of each part is a point in the data space. The task of sub-assembly recognition is to search the region presenting the densest possible points in the n-dimensional data space, most of which are based on "distance".

According to the above principles, the concept of fuzzy set combined with clustering analysis can be applied to the recognition of sub-assemblies in the assembly process, and the sub-assembly recognition algorithm based on the objective function can be generated.

Given  $X = \{x_1, x_2, \dots, x_n\}$  represents the assembly part set, n is the quantity of parts,  $c(1 < c < n)$  is the quantity of pre-identified sub-assemblies. The mathematical model of the recognition algorithm is as follows:

$$\text{Min } J_m(U, V) = \sum_{i=1}^c \sum_{j=1}^n u_{ij}^{m^*} d_{ij}^2 \quad (1)$$

Meeting:

$$\begin{aligned} \sum_{i=1}^c u_{ij} &= 1, \quad 1 \leq j \leq n \\ \sum_{j=1}^n u_{ij} &> 0, \quad 1 \leq i \leq c \\ u_{ij} &\geq 0, \quad 1 \leq i \leq c, \quad 1 \leq j \leq n \end{aligned}$$

In the formula,  $m^*$  is the fuzzy index, also called weighting index, and the range is  $(1, +\infty)$ .  $U = u_{ij}$  is a  $c \times n$  sub-assembly partition matrix,  $u_{ij}$  is the j-th part  $p_j$  belongs to the i-class sub-assembly membership value.  $V = [v_1, v_2, \dots, v_n]$  is composed of c sub-assembly centers.  $d_{ij} = \|x_j - v_i\|$  is the distance from the part to the sub-assembly center.

### III. MATHEMATICAL MODELING OF SUB-ASSEMBLY RECOGNITION BASED ON GAME THEORY

#### A. Theory and basic steps of game analysis

The basic seven elements of the game include players, behavior, strategy, information, benefits, results and equilibrium:

1) Game participants, also known as "players", refer to the individuals who make independent decisions and benefits

in the game process, and the decision makes them gain the most.

2) Behavior refers to all decision variables and action sets of participants.

3) Strategy is the collection of all behaviors or strategies that game players can choose.

4) Game information refers to information that will affect the decision-making of players in game behavior.

5) Benefits refers to the decision-making benefit obtained by game participants from the game process.

6) The result is a collection of the execution behaviors of all game players.

7) Game equilibrium is an optimal combination of balanced actions achieved by all game players[8].

The game mathematical model for n game players is as follows:

$$G = (P, S, F) \quad (2)$$

In the above equation, P is a finite set, representing the game participants.  $S_i = \{s_1, s_2, \dots, s_m\}$  represents the strategy set of game participant i.  $S = \{s_1, s_2, \dots, s_n\}$  represents the strategy space of all game players.  $F: \sum \rightarrow R$  is the revenue function.

The equilibrium strategy of a game is that multiple players reach equilibrium simultaneously. The relative income combination of equilibrium strategy is considered as equilibrium income. It can be concluded that game equilibrium needs to meet the following characteristics[9]:

1) Given the preferences of all participants.

2) The game players search their respective maximum return coefficients under various strategies.

3) The equilibrium strategy corresponds to the highest return coefficient.

4) The strategy with the highest profit selected by the participant.

5) In the case of equilibrium, the income coefficients of all participants are high, and the income is also in an equilibrium stable state.

The basic steps of game theory analysis are shown in Figure 1.

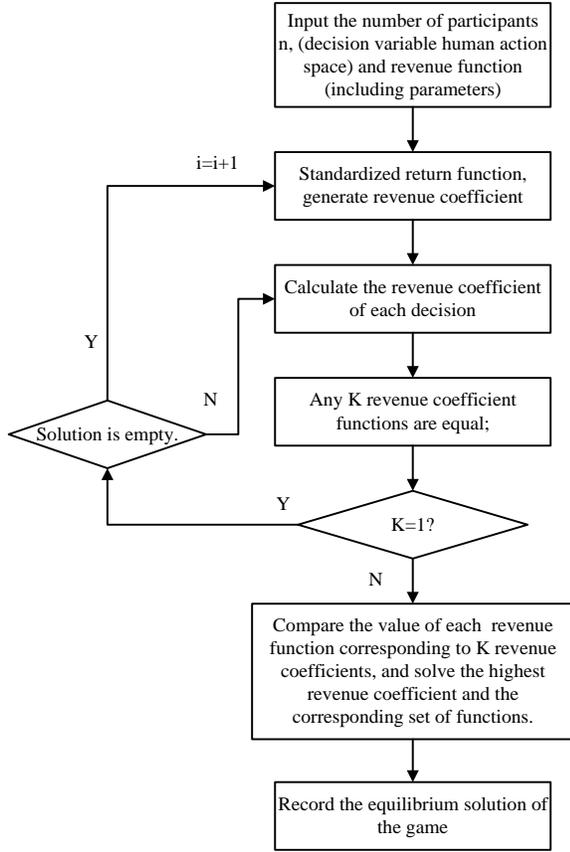


Figure 1. Basic steps of game theory analysis

To sum up, game analysis needs to solve the equilibrium solution. In the state of equilibrium solution, each participant will choose the strategy corresponding to the highest return coefficient within their own constraints, and finally reach a stable equilibrium of cooperation and competition among all participants[10].

### B. Game theory mapping of sub-assembly recognition

The objective function of the sub-assembly recognition algorithm is described by game theory. The basic steps to map the algorithm's objective function problem into a game problem are as follows:

- 1) Convert each objective into game participants.
- 2) Convert each parameter variable into game strategy set.
- 3) The value of each target under the same condition corresponds to the benefit of game participants under the same strategy.
- 4) The objective function problem has the same constraint conditions as the game.
- 5) Each game player takes its own profit function as the goal to obtain an independent optimal single-objective optimization strategy relative to other game players. The single objective optimization strategy of each game player is the initial strategy of the next iteration. According to this

idea, the optimal equilibrium solution of the objective function can be obtained after several iterations.

### C. Mathematical modeling of sub-assembly recognition based on game theory

According to game theory[11],  $G = \{S_1, S_2, \dots, S_m; f_1, f_2, \dots, f_m\}$  represents a game. There is  $m$  game players in  $G$ , and the strategy set is represented by  $S_1, S_2, \dots, S_m$ .  $S_{ij} \in S_i$  represents the  $j$ -th strategy of game participant  $i$ , and  $f_1, f_2, \dots, f_m$  is the benefit function of each game player.

The steps of game analysis for sub-assembly recognition algorithm are as follows:

1) The objective function  $J(U, V)$  of the sub-assembly recognition algorithm can be define. Variable parameters are the number of sub-assemblies  $C$  and fuzzy index  $m^*$ , and constraint conditions is  $U$ . The purpose of transforming the objective function  $J(U, V)$  into the game problem  $G(S)$  is to obtain the combination of strategy sets.

2) Initialization of game analysis.  $S^{(0)} = \{S_1^{(0)}, S_2^{(0)}, \dots, S_m^{(0)}\}$  represents the initial policy set of  $G(S)$ .

3) Income models of game participants can be determined. Let  $\bar{S}^{(0)} = \{\bar{S}_1^{(0)}, \bar{S}_2^{(0)}, \dots, \bar{S}_m^{(0)}\}$  be the complement of  $S_1^{(0)}, S_2^{(0)}, \dots, S_m^{(0)}$  in  $S^{(0)}$ . The benefits of all game participants are as follows:

$$\begin{aligned}
 F_i &= w_{ii} \bar{f}_i + \sum_{j=1(j \neq i)} w_{ij} \hat{f}_{ij} \\
 &= w_{ij} \frac{f_i(s_i^*, \bar{S}_1^{(0)})}{f_i(s_i^0, \bar{S}_1^{(0)})} + \sum_{j=1(j \neq i)} w_{ij} \frac{f_j(s_i^*, \bar{S}_1^{(0)})}{f_j(s_i^0, \bar{S}_1^{(0)})}
 \end{aligned} \quad (3)$$

Where,  $\bar{f}_i$  is the benefit of game participant  $i$  when  $i$  takes action, and it is also the dimensionless value normalized by the objective function  $f_i$ .  $\hat{f}_{ij}$  ( $j = 1, 2, \dots, i-1, i+1, \dots, m$ ) is the benefit of game participant  $j$  when  $i$  takes action, and it is also the dimensionless value normalized by the objective function  $f_j$ .  $w_{ij}$  ( $i, j = 1, 2, \dots, m$ ) is the weight coefficient, reflecting the degree of cooperation and competition between  $i$  and  $j$  game players. When  $w_{ij} = 0$ , it means completely uncooperative, while when  $w_{ij} = 1$ , it means completely cooperative. The value of  $w_{ij}$  is inversely proportional to the degree of cooperation, while the degree of competition is proportional.

4) Solve the game player  $i$  optimal strategy  $S_i^*$ . Firstly, single objective optimization is carried out for each game player, and the game player's income  $f_1(s), f_2(s), \dots, f_m(s)$  is the optimization objective.

Secondly, the initial value complement  $\bar{S}_1^{(0)}, \bar{S}_2^{(0)}, \dots, \bar{S}_m^{(0)}$  of each game participant can be fixed, and the corresponding optimization design can be carried out in the strategy set  $S_1, S_2, \dots, S_m$  to solve the optimal strategy  $S_i^*$  of any game participant i, so that the game income  $f_i(s_i^*, \bar{S}_1^{(0)}) = w_{ii}\bar{f}_i + \sum_{j=1(j \neq i)}^m w_{ij}\hat{f}_{ij} \rightarrow \min$  can meet the constraint condition  $h_k(s_i^*, \bar{S}_1^{(0)}) (k=1,2,\dots,q)$ .

5) Game convergence cycle calculation.  $S^* = S_1^* \cup S_2^* \cup \dots \cup S_m^*$ , the distance between the two strategies of the upper and lower rounds can satisfy the convergence criterion  $\|S^* - S^{(0)}\| \leq \varepsilon$ . If the convergence criterion is satisfied, the game ends. If not, replace  $S^{(0)}$  with  $S^*$ , turn 3), and repeat the cyclic calculation until the convergence rule is satisfied. The above game analysis steps are shown in Figure 2.

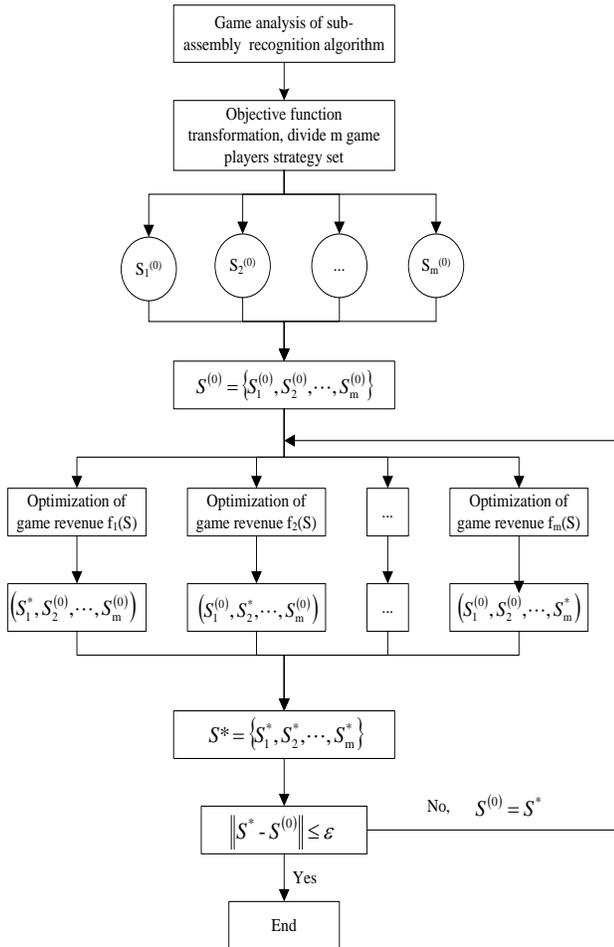


Figure 2. Game analysis steps of sub-assembly recognition algorithm

In conclusion, the mathematical model of sub-assembly recognition based on game theory can be obtained as follows: Game revenue objective function

$$F_i = w_{ii}\bar{f}_i + \sum_{j=1(j \neq i)} w_{ij}\hat{f}_{ij} \quad (4)$$

$$= w_{ij} \frac{f_i(s_i^*, \bar{S}_1^{(0)})}{f_i(s_i^0, \bar{S}_1^{(0)})} + \sum_{j=1(j \neq i)} w_{ij} \frac{f_j(s_i^*, \bar{S}_1^{(0)})}{f_j(s_i^0, \bar{S}_1^{(0)})}$$

meet

$$h_k(s_i^*, \bar{S}_1^{(0)}) (k=1,2,\dots,q)$$

Where,  $\bar{f}_i$  is the benefit of game participant i when i takes action, and it is also the dimensionless value normalized by the objective function  $f_i$ ,  $\hat{f}_{ij} (j=1,2,\dots,i-1,i+1,\dots,m)$  is the benefit of game participant j when i takes action, and it is also the dimensionless value normalized by the objective function  $f_j$ ,  $w_{ij} (i,j=1,2,\dots,m)$  is the weight coefficient, reflecting the degree of cooperation and competition between i and j game players. When  $w_{ij} = 0$ , it means completely uncooperative, while when  $w_{ij} = 1$ , it means completely cooperative. The value of  $w_{ij}$  is inversely proportional to the degree of cooperation, while the degree of competition is proportional. Constraint  $h_k$  is the constraint of sub-assembly recognition algorithm.

The programming design based on Matlab can realize the transformation and operation of the mathematical model of sub-assembly recognition based on game theory, and the game analysis of sub-assembly recognition can be carried out through examples.

#### IV. CONCLUSION

Taking sub-assembly recognition problem as the research object and combining with the mathematical model of sub-assembly recognition algorithm, this paper analyzes the similarity and corresponding mapping between the objective function problem of sub-assembly recognition and the game problem. The object function of sub-assembly recognition is transformed into the benefit of game participants, and the parameter variable is transformed into game strategy space. The steps of game analysis for sub-assembly recognition algorithm are systematically planned, and the fuzzy clustering mathematical model of sub-assembly based on game theory is given. It is beneficial to the determination of each parameter of the target function of sub-assembly recognition algorithm, and can reduce the influence of parameter weight on the result of the algorithm. It has practical engineering application significance.

**ACKNOWLEDGMENT**

Xiaoxi Kou gratefully acknowledge the Professor Yan Cao for guiding and reviewing the dissertation. Thanks to Hu Qiao and Yujia Wu for their opinions and suggestions in the thesis writing.

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