Research on Load Sharing Characteristics of Planetary Transmission System

Liu Zhiyu
School of Mechatronic Engineering
Xi'an Technological University
Xi'an, China
E-mail: Lzy9302@foxmail.com

Liu Bo
School of Mechatronic Engineering
Xi'an Technological University
Xi'an, China
E-mail: 976361823@qq.com

Dong Hao
School of Mechatronic Engineering
Xi'an Technological University
Xi'an, China
E-mail: 283414672@qq.com

***Abstract*—In order to study the average load sharing characteristics of planetary transmission, based on the lumped parameter theory, the dynamic model is established by considering the factors such as error, support clearance and support stiffness. The Runge-Kutta numerical method is used to solve the model, and the relationship between parameters and load sharing characteristics is obtained. The results show that with the increase of the support gap, the load factor tends to be flat after the rapid decline; with the change of manufacturing error, the load factor and the error approximate a linear relationship; with the increase of the support stiffness, the system load factor After a rapid increase, it tends to be gentle.**

Keywords-Planetary Transmission; Load Sharing; Clearance; Error

I. INTRODUCTION

Planetary transmission has the characteristics of small size, light weight and high transmission efficiency. It has been widely used in the aviation defense industry, attracting many domestic and foreign scholars to conduct a lot of research on the dynamic load sharing characteristics of planetary transmission systems. The characteristics have been extensively studied. Kahraman[1-2] analyzed the uniform load problem of planetary gear trains from the perspective of statics and dynamics, and proposed three parameters: dynamic load sharing coefficient, static average load coefficient and dynamic load coefficient to characterize planetary gear transmission. The uniform load effect; Montestruc[3-4] calculated the load sharing model of the planetary gear transmission system by numerical method, and analyzed the influence of various error factors on the load sharing characteristics; Zhou Wei[5] established the dynamic load sharing analysis model. The influence of error on the uniform load characteristics of the planetary transmission system is studied. Zhu Zengbao[6] analyzed the influence of support stiffness on the dynamic load sharing characteristics of the planetary transmission system. Lu Junhua[7] established the calculation model of the 2K-H planetary transmission system from the dynamic point of view. The load sharing characteristics of the system are studied. Bao Heyun[8] analyzes the influence of the support stiffness of the sun gear on the dynamic load sharing characteristics of the planetary transmission system by establishing the dynamic load model of the planetary gear transmission system; Guo Fang[9] The generalized dynamic model of star-shaped transmission is established, and the influence of eccentricity error of different positions and different numbers on the dynamic load-carrying characteristics of powertrain is studied. Wu Shijing and Peng Zeming et al[10-11] The composite wheel train is the research object, and the influence of meshing error, rotation speed and load on the dynamic load sharing characteristics of the system is analyzed. Xu Xiangyang et al.[12] analyzed the support stiffness of the planet carrier by establishing a dynamic load sharing model of the planetary transmission considering multi-component floating. , planetary wheel error variation and sun gear floating, planetary carrier floating, and external load changes, the planetary wheel load change trend; Fang Zongde, Shen Yunwen et al[13] using the Fourier series to solve the dynamic equation, to obtain the system time The domain and frequency domain solutions are used to analyze the self-excited vibration characteristics of the system. The uniformity of load distribution on each planet gear and the influence of gear eccentricity error on gear load uniformity are calculated.

Based on the above research, it can be seen that there are few studies on the influence of the number of planetary wheels on the dynamic load sharing characteristics of the system. In this paper, considering the error, support stiffness and flank clearance, the 2K-H single-stage spur gear planetary gear train is taken as the research object, and the bending-torsional coupled vibration model is established. The numerical method is used to solve the problem. Under the influence of factors such as error, support stiffness and flank clearance, the difference of the number of planetary wheels affects the load sharing characteristics of the system.

II. System Dynamics Model

Figure 1 shows the planetary gear transmission dynamics model based on the lumped mass method. The model includes four components: the sun gear, the planetary gear, the inner ring gear and the planet carrier, which are denoted by *S, P, R* and *C* respectively.

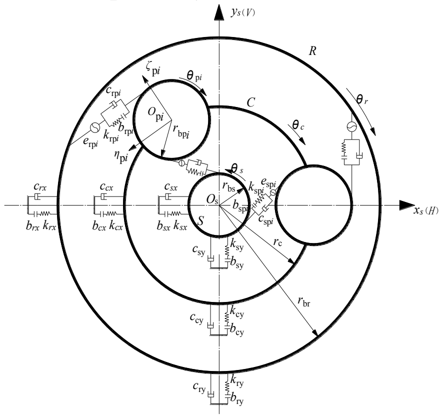

Figure 1. Multi-gap bending-torsional coupled dynamic vibration model

In Fig. 1, the micro-displacement of the sun gear, the planetary gear, the ring gear and the planet carrier along the meshing line are represented by θ_s, θ_p, θ_I and θ_c, respectively; the lateral displacements of the sun gear, the ring gear and the planet carrier are respectively X_s, H_I And H_c indicate that the longitudinal displacement is represented by Y_s, V_I, and V_c, respectively; the tangential displacement and the normal displacement of the planetary gear are represented by η_{pi} and ζ_{pi}, respectively. The meshing rigidity, the half-tooth side clearance and the meshing damping of the *i*-th sun gear and the planetary gear are respectively k_{spi}, b_{spi} and c_{spi}; the inner meshing pair of the *i*-th planetary gear and the inner ring gear meshes the rigidity and the half-tooth side clearance And meshing damping with $k_{rp}i$, $b_{rp}i$ and c_{rpi}, respectively. The stiffness, semi-support clearance and damping of the sun gear support bearing are denoted by k_s, b_s and c_s respectively; the stiffness, semi-support clearance and damping of the support bearing of the planetary gear are denoted by k_p, b_p and c_p respectively; the support bearing of the inner ring gear The stiffness, semi-support clearance and damping are denoted by k_I, b_I and c_I, respectively; the stiffness, semi-support clearance and damping of the support bearings of the planet carrier are denoted by k_c, b_c and c_c, respectively.

The errors affecting the load distribution between the planetary wheels mainly include the sun wheel eccentricity error E_s, the planetary wheel eccentricity error E_{pi}, the inner ring gear eccentricity error E_I, the carrier eccentricity error E_c.

The equivalent cumulative meshing error generated on the inner and outer meshing lines of each error can be expressed as a sinusoidal form:

$$\begin{aligned} e_{spi}(t) = & -E_s \sin\left(\left(\omega_s - \omega_c\right)t + \beta_s + \alpha_w - \varphi_i\right) \\ & - E_{pi} \sin\left(\left(\omega_p - \omega_c\right)t + \beta_{pi} + \alpha_w\right) \\ & + E_c \sin\left(\beta_c - \varphi_i\right)\cos\alpha_w \end{aligned} \tag{1}$$

$$\begin{aligned} e_{piI}(t) = & +E_I \sin\left(-\omega_c + \beta_I - \alpha_n - \varphi_i\right) \\ & + E_{pi} \sin\left(\left(\omega_p - \omega_c\right)t + \beta_{pi} - \alpha_n\right) \end{aligned} \tag{2}$$

Where ω_s, ω_p, and ω_c are the angular velocities of the sun gear, planet gear, and planet carrier; β_s, β_{pi}, β_I, and β_c are the phase angles of the geometric eccentricity errors of the sun gear, the planet gear, the ring gear, and the planet carrier, respectively; γ_s、γ_{pi} and γ_I are the phase angles of the installation error of the sun gear, the planetary gear and the ring gear, respectively; α_w and α_n are the meshing angles of the external gear transmission and the internal gear transmission respectively; φ_i is the i-th (i=1, 2, ..., n) the position angle of the planet wheels relative to the first planet gear.

The piecewise linear displacement function of the gear system the support clearance can be expressed as:

$$f(X,b) = \begin{cases} X-b & X>b \\ 0 & |X| \le b \\ X+b & X<-b \end{cases} \tag{3}$$

Where X—lateral (longitudinal) displacement of each wheel

b—half semi-support clearance

In the gear transmission system, the dynamic meshing force is composed of elastic restoring force and damping force.

The elastic restoring force can be expressed as:

$$\begin{cases} F_{spi} = k_{spi}(t)\big(\theta_s - \theta_{pi} - \theta_c - \xi_{pi}\sin\alpha_w \\ \quad -\eta_{pi}\cos\alpha_w - X_s\sin\left(\varphi_i - \alpha_w\right) \\ \quad + Y_s\cos\left(\varphi_i - \alpha_w\right) + H_c\sin\left(\varphi_i + \alpha_w\right) \\ \quad - V_c\cos\left(\varphi_i + \alpha_w\right) - e_{spi}(t)\big) \\ F_{piI} = k_{piI}(t)\big(\theta_{pi} - \theta_I - \theta_c + \xi_{pi}\sin\alpha_n \\ \quad + \eta_{pi}\cos\alpha_n + H_c\sin\left(\varphi_i + \alpha_n\right) \\ \quad - V_c\cos\left(\varphi_i + \alpha_n\right) + H_I\sin\left(\varphi_i + \alpha_n\right) \\ \quad - V_I\cos\left(\varphi_i + \alpha_n\right) - e_{piI}(t)\big) \end{cases} \tag{4}$$

The damping force can be expressed as:

$$\begin{cases} D_{\mathrm{sp}i} = c_{\mathrm{sp}i}(t)\Big[\dot{\iota}_{\mathrm{s}} \quad \dot{}_{\mathrm{p}i} \quad \dot{}_{\mathrm{c}} \quad \dot{}_{\mathrm{p}i} \quad {}_{\mathrm{w}} \\ \quad -\dot{i} \quad\quad -\dot{}_{\mathrm{s}} \quad (\quad -\alpha_{\mathrm{w}}) \\ \quad +\dot{1}_{\mathrm{s}} \quad (\quad {}_{\mathrm{w}}) \quad \dot{}_{\mathrm{c}} \quad (\quad {}_{\mathrm{w}}) \\ \quad -\dot{I}_{c} \quad (\quad -\alpha_{\mathrm{w}}) - \dot{(} \quad \Big] \\ D_{\mathrm{p}i\mathrm{I}} = c_{\mathrm{p}i\mathrm{I}}(t)\Big[\dot{\iota}_{\mathrm{p}i} \quad \dot{}_{\mathrm{I}} \quad \dot{}_{\mathrm{c}} \quad \dot{}_{\mathrm{p}i} \quad {}_{\mathrm{n}} \\ \quad +\dot{i} \quad\quad +\dot{I}_{\mathrm{c}} \quad (\quad +\alpha_{\mathrm{n}}) \\ \quad -\dot{I}_{c} \quad (\quad -\alpha_{\mathrm{n}}) + \dot{I}_{\mathrm{I}} \quad (\quad -\alpha_{\mathrm{n}}) \\ \quad -\dot{I}_{\mathrm{I}} \quad (\quad -\alpha_{\mathrm{n}}) - \dot{(} \quad \Big] \end{cases} \tag{5}$$

System differential equations:

$$\begin{cases} m_{\mathrm{seq}}\ddot{\iota}_{\mathrm{s}} \cdot \sum_{i=1}^{N}(F_{\mathrm{sp}i} + D_{\mathrm{sp}i}) = F_{\mathrm{in}} \\ m_{\mathrm{s}}\ddot{}_{\mathrm{s}} \quad \sum_{i=1}^{N}(F_{\mathrm{sp}i} + D_{\mathrm{sp}i})\sin(\varphi_i - \alpha_{\mathrm{w}}) \\ \quad + k_{\mathrm{s}} f_{\mathrm{sx}}(X_{\mathrm{s}}, b_{\mathrm{s}}) + c_{\mathrm{s}}\dot{}_{\mathrm{s}} \\ m_{\mathrm{s}}\ddot{}_{\mathrm{s}} \cdot \sum_{i=1}^{N}(F_{\mathrm{sp}i} + D_{\mathrm{sp}i})\cos(\varphi_i - \alpha_{\mathrm{w}}) \\ \quad + k_{\mathrm{s}} f_{\mathrm{sy}}(Y_{\mathrm{s}}, b_{\mathrm{s}})_{\mathrm{s}} + c_{\mathrm{s}}\dot{1}_{\mathrm{s}} \\ m_{\mathrm{peq}}\ddot{\iota}_{\mathrm{s}} \quad (\;_{\mathrm{sp}i} + D_{\mathrm{sp}i}) + (F_{\mathrm{p}i\mathrm{I}} + D_{\mathrm{p}i\mathrm{I}}) = 0 \\ m_{\mathrm{p}}\ddot{\varsigma}_{\mathrm{p}i} \quad (\;_{\mathrm{sp}i} + D_{\mathrm{sp}i})\sin\alpha_{\mathrm{w}} + (F_{\mathrm{p}i\mathrm{I}} + D_{\mathrm{p}i\mathrm{I}})\sin\alpha_{\mathrm{n}} \\ \quad + k_{\mathrm{p}} f_{\mathrm{px}}(\xi_{\mathrm{p}}, b_{\mathrm{p}}) + c_{\mathrm{p}}\dot{\varsigma}_{\mathrm{p}i} \\ m_{\mathrm{p}}\ddot{} \quad (\;_{\mathrm{sp}i} + D_{\mathrm{sp}i})\cos\alpha_{\mathrm{w}} + (F_{\mathrm{p}i\mathrm{I}} + D_{\mathrm{p}i\mathrm{I}})\cos\alpha_{\mathrm{n}} \\ \quad + k_{\mathrm{p}}\left(f_{\mathrm{py}}(\eta_{\mathrm{p}}, b_{\mathrm{p}}) - \theta_{\mathrm{c}}\right) + c_{\mathrm{p}}\dot{i} \\ m_{\mathrm{ceq}}\ddot{\iota}_{\mathrm{c}} \cdot \sum_{i=1}^{N}\;_{\mathrm{p}}(\theta_{\mathrm{c}} - \eta_{\mathrm{p}i}) + k_{\mathrm{o}}\theta_{\mathrm{c}} = -F_{\mathrm{out}} \\ m_{\mathrm{I}}\ddot{I}_{\mathrm{I}} \cdot \sum_{i=1}^{N}(F_{\mathrm{p}i\mathrm{I}} + D_{\mathrm{p}i\mathrm{I}})\sin(\varphi_i + \alpha_{\mathrm{n}}) \\ \quad + k_{\mathrm{I}} f_{\mathrm{Ix}}(H_{\mathrm{I}}, b_{\mathrm{I}}) + c_{\mathrm{I}}\dot{I}_{\mathrm{I}} \\ m_{\mathrm{I}}\ddot{I}_{\mathrm{I}} + \sum_{i=1}^{N}(F_{\mathrm{p}i\mathrm{I}} + D_{\mathrm{p}i\mathrm{I}})\cos(\varphi_i + \alpha_{\mathrm{n}}) \\ \quad + k_{\mathrm{I}} f_{\mathrm{Iy}}(V_{\mathrm{I}}, b_{\mathrm{I}}) + c_{\mathrm{I}}\dot{I}_{\mathrm{I}} \end{cases} \tag{6}$$

In equation (6), m_{seq}, m_{peq}, m_{Ieq}, and m_{ceq} are the equivalent masses of the sun gear, planet gear, ring gear, and planet carrier, respectively; m_{s}, m_{p}, m_{I}, and m_{c} are the sun gear, the planet gear, the ring gear, and The mass of the planet carrier; F_{in} and F_{out} are the input force and output force respectively; k_{o} is the torsional stiffness of the planet carrier.

III. SYSTEM LOAD CHARACTERISTICS

The 4-5-order Runge-Kutta numerical method is used to solve the problem, and the load-carrying characteristics under different parameters of the flank clearance, support clearance, error and support stiffness are obtained.

The input power of the given system is 400P/kW, the input speed is 1500r/min, the modulus is 3.5m/mm, the internal and external pressure angles are 20$^{\mathrm{o}}$, and the number of teeth of the sun gear, planetary gear and inner ring gear are 40, 20 and 80 respectively.

A. Load sharing characteristics of the system as a function of support clearance

Figure 2 is a graph showing the variation of the load sharing coefficient of the system with the support gap when the manufacturing installation error of each wheel is 10μm, the support stiffness is 1x10^{9} N·mm^{-1}, and the number of planetary wheels is 3~6.

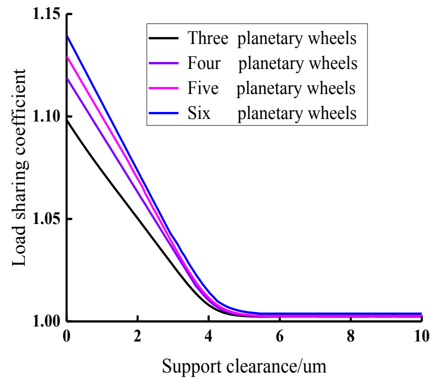

Figure 2. The variation of load-sharing coefficient with support clearance under different number of wheels

It can be seen from Fig. 2 that the uniform load characteristic of the system is highly sensitive to the number of planetary wheels in the small support gap. When the support gap is increased to a certain extent, the average load coefficient will no longer change with the increase of the gap; When the support gap or the support gap is small, the load-carrying characteristics of the system will decrease with the increase of the number of planet wheels. As the support gap increases, the gap will compensate the load-carrying characteristics, so that the number of planetary wheels is equal to the system. The impact is small. Therefore, in order to eliminate the influence of the number of planetary wheels on the load sharing characteristics and obtain better load sharing characteristics, it is very important to select a suitable support gap.

B. The load sharing characteristics of the system as a function of manufacturing error

The manufacturing installation error of each wheel in the system is 10μm, the support stiffness is 1x10^{9} N·mm-1 (the other parameters remain unchanged when one parameter changes), and the system increases with the manufacturing error when the number of planetary wheels is 3~6. The change of the average load factor of the large time system is shown in Fig. 3.

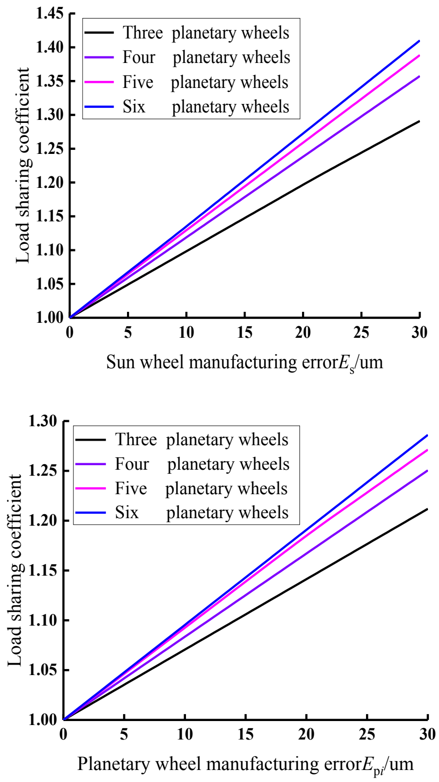

Figure 3. Variation of load-sharing coefficient with manufacturing errors under different number of wheels

It can be seen from Fig. 3(a)~3(b) that the manufacturing error of each wheel in the system has a great influence on the load sharing characteristics of the system, and the load factor and the manufacturing error show a linear upward trend, while the center wheel has a greater influence, and the number of rounds increases. The speed at which the system load characteristics deteriorates is accelerated, but the influence of the number of rounds on the rate is gradually weakened.

C. Load sharing characteristics of the system as a function of support stiffness

Figure 4 shows that the manufacturing installation error of each wheel in the system is 10μm, and the support stiffness is $1\text{x}10^9$ $\text{N}\cdot\text{mm}^{-1}$, when the number of planetary wheels is 3~6 The system's load factor changes as the manufacturing error increases.

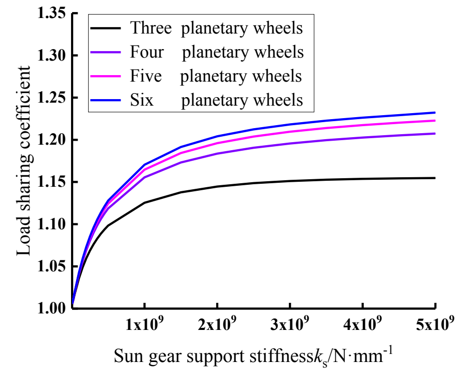

Figure 4. Variation of load-sharing coefficient with support stiffness under different number of wheels

It can be seen from Fig. 4 that during the process from the floating state to the rigid support of the sun gear, the support stiffness has a great influence on the load sharing characteristics of the system; and with the increase of the number of planet wheels, the more uniform load characteristics of the system The difference is that the final peak of the average load factor is also larger, but the influence of the number of rounds on the final stable peak of the system is gradually reduced.

IV. CONCLUSION

The differential equations of motion of 2K-H planetary transmission are established. The numerical method is used to analyze the load-carrying characteristics of the parameters of different planetary wheels, with the flank clearance, support clearance, error and support stiffness. The conclusions are as follows:

- As the support gap increases, the load sharing coefficient of the system decreases rapidly after a rapid decrease at an almost constant rate, and the more the number of planetary wheels, the worse the load carrying characteristics.
- As the manufacturing error increases, the load sharing coefficient of the system and the manufacturing error increase linearly, and the more the number of planetary wheels, the faster the load-carrying characteristics deteriorate.
- As the support stiffness increases, the support stiffness has a great influence on the load sharing characteristics of the system during the process from the floating state to the rigid support. The more the number of planetary wheels, the larger the stable peak.

ACKNOWLEDGMENT

Thanks to the National Natural Science Foundation of China (51705390); Shaanxi Natural Science Foundation Project (2018JQ5029); Xi'an University of Technology Research Innovation Team Construction Project Funding support the publication of articles

REFERENCES

[1] Kahraman.A.Load Sharing Characteristics of Planetary Transmissions [J]. Mech. Mach. Theory, 1994,29(8): 1151~1165

[2] Kahraman A. Static Load Sharing Characteristics of Transmission Planetary Gear Sets Model and Experiment[J]. Society of Automotive Engineers, 1999: 1954~1963

[3] Montestruc A N. Influence of Planet Pin Stiffness on Load-sharing in Planetary Gear Drives[J]. Journal of Mechanical Design, 2011, 133(1):014501.

[4] Montestruc A N. A Nμmerical Approach to Calculation of Load-sharing in Planetary Gear Drives[J]. Journal of Mechanical Design, 2010, 132(1):014503.

[5] Zhou Lu, Wu Shijing, Li Jing. Analysis of the Influence of Error on Load-sharing Characteristic of Planetary Transmission System[J]. Journal of Mechanical Transmission, 2017(6): 1-8. (in Chinese)

[6] Zhu zengbao, Jiang zhixiang, Yin min. Impact of support stiffness on dynamic load-sharing characteristics of planetary train system[J]. Journal of Aerospace Power,2016,31(4): 986- 992. (in Chinese)

[7] Lu Junhua, Zhu Rupeng, Jin Guanghu. Analysis of Dynamic Load-sharing Be-havior in Planetary Gearing[J]. Journal of Mechanical Engineering, 2009, 45(5):85-90. (in Chinese)

[8] Bao Heyun, Zhan Xiaoqing. The Influence of the Sun-Gear Support Stiffness on the Load Sharing Behavior of Pl-anetary Gear System[J]. Journal of Me-chanical Engineering, 2012(10):72-74.

[9] Guo Fang, Fang Zongde, Zhang Yongzhen. Effect of star gears' eccentric errors on dynamic performances of a star gear transmission system with clearance fl-oating[J]. Journal of Vibration and Shock, 2018, 37(3):98-104.

[10] Wu Shijing, Peng Zeming, Wng Xiaosun. Impact of Mesh Errors on Dynamic Load Sharing Characteristics of Compound Pl-anetary Gear Sets[J]. Journal of Mech-anical Engineering, 2015,51(3):29-36.

[11] Peng Zeming, Wu Shijing, Wang Xiaosun. Virtual prototype simulation of load shar-ing characteristics of compound planetary gear sets[J]. Journal of Mechanical & Ele-ctrical Engineering, 2015, 32(2):157-162.

[12] Xu Xiangyang, YangWang, Diao Peng. Research of the Dynamic Load Sharing of Heavy Load Planetary Gear System with Multi-floating Component[J]. Jou-rnal of Mechanical Transmission, 2016,12:6-11.

[13] Fang Zongde, Shen Yunwen, huang zhen-dong.Analysis of dynamic behave-orur of 2K—H planetary gearing [J].Journal of North-western Polytechni-cal University, 1990,8(4)：361—371.