

## Analysis of the Influence of Eccentricity on the Performance of Eccentric Involute Flexible Spring

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**Abstract**—Flexible plate spring plays an important role in gap seal and reliability of free piston Stirling heat pump. Eccentric flexible plate spring is widely used in Stirling machine because of its better stress distribution and larger radial stiffness ratio. In this paper, the influence factor of three-groove eccentric involute flexible plate spring eccentricity is analyzed in detail, and the value range of eccentricity in the design process of eccentric flexible spring is obtained by establishing a mathematical model. Through finite element analysis, the variation of radial and axial stiffness of eccentric flexible spring with different eccentricity is summarized. This study provides a basis for the design of later eccentric flexible spring.

**Keywords**—Eccentricity; Flexible Spring; Finite Element; Radial And Axial Stiffness

### I. INTRODUCTION

Flexible plate spring (later referred to as plate spring) is on the circular elastic metal sheet through laser cutting and other methods to process a variety of lines. Through the action of axial force, the elastic sheet can have a certain axial displacement and ensure the plate spring has a great stiffness in the radial direction[1].The overall structure of the free piston Stirling heat pump for pure electric vehicle air conditioning heating research shown in this paper is shown in Figure 1. The blue part is a plate spring, and the plate spring is used to support the power piston and the gas distribution piston. The sealing gap between the piston and the cylinder during the movement, the performance of the plate spring is directly related to the smoothness and output efficiency of the Stirling heat pump. Therefore, the study on the properties of plate spring has important theoretical and engineering value.

In 1981, Dave[2] of Oxford University first applied the plate spring to the cryocooler, which made the gap sealing and oil-free lubrication technology of the refrigerator possible. Since then, the plate spring has been widely concerned and applied in the field of Stirling machine. T. E.

Wong et al[3-4]. Optimized the design of flexible spring to meet the work needs of linear compression system, and carried out the finite element analysis of flexible spring under the condition of dynamic load, and proposed the dimensionless design curve of scroll profile. Chen Nan and others[5-7] of Shanghai Jiaotong University put forward the design method of circular involute vortex plate spring based on CAD analysis of overseas plate spring profile, and further studied the performance of vortex plate spring by numerical and theoretical methods. Gan Zhihua et al[8]., Zhejiang University combined with finite element analysis and experimental results, the circular involute is more suitable for the construction of the vortex line, and the eccentric plate spring has the advantages of larger axial stiffness ratio and better stress distribution.

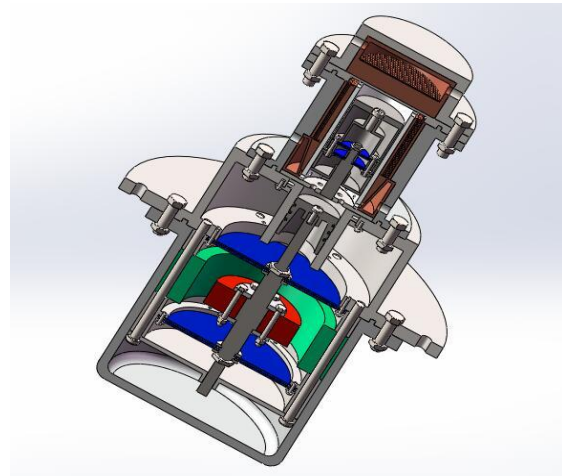


Figure 1. General structure of Stirling heat pump

In this paper, CAD, SolidWorks and other tools are used to build a three-dimensional model of three groove eccentric involute plate spring with different eccentricities. The range of eccentricity in the design of eccentric plate spring is

analyzed by derivation and calculation. Based on the finite element analysis model, the axial and radial stiffness of plate spring with different eccentricity are analyzed by ANSYS software.

## II. DETERMINATION OF THE VALUE RANGE OF ECCENTRICITY

When designing the three groove eccentric plate spring, it is necessary to construct the eccentric involute. The eccentric involute occurs on three different base circles with the same radius. The included angle between the center of two adjacent base circles and the line connecting the plate spring center is  $120^\circ$ ; and the length of the line section is the eccentricity. The eccentricity has a certain value range. When the eccentricity is too large, the two vortex grooves will intersect and the leaf spring will be damaged. In the design process of eccentric plate spring, it is necessary to determine the maximum value of eccentricity, so as to get the value range of eccentricity. The value range formula of eccentricity can be determined according to the minimum distance between adjacent vortex tanks.

When the normal lines of two adjacent eccentric involutes are on the same line, the length of the line connecting the intersection point of the line and the two involutes is the longest or shortest distance between the two involutes, as shown in fig. 2. In this paper, we only need to find the shortest distance formula, that is, the length of line segment EF. According to the length of the line segment EF, the range of values of the eccentric amount to be finally obtained can be obtained. Suppose the length of line segment EF is  $l$ , the radius of the base circle is  $r$ , and the eccentricity is  $x$ .

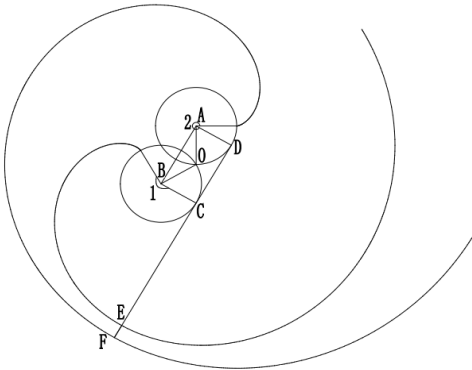


Figure 2. Schematic diagram of two adjacent eccentric involutes

It is easy to prove that the quadrilateral ABCD is a rectangle, so we can get:

$$l_{CD} = l_{AB} = \sqrt{3}x \quad (1)$$

Observe the rotation angles of two involutes, and through calculation and derivation, we can get:

$$\angle 1 = 7\pi/6, \quad \angle 2 = 11\pi/6 \quad (2)$$

According to the property of involute, the length of  $l_{CE}$  and  $l_{DF}$  can be obtained:

$$l_{CE} = \frac{7\pi/6}{2\pi} \cdot 2\pi r = 7\pi r/6 \quad (3)$$

$$l_{DF} = \frac{11\pi/6}{2\pi} \cdot 2\pi r = 11\pi r/6 \quad (4)$$

Therefore, the shortest distance formula between two adjacent involutes:

$$x = l_{EF} = l_{DF} - l_{CD} - l_{CE} = 2\pi r/3 - \sqrt{3}l \quad (5)$$

In order to ensure that the plate spring can be used, the minimum distance between two eccentric involutes should be greater than 0, which can be obtained:  $l < 2\sqrt{3}\pi r/9$ . According to this formula, the range of the eccentricity can be accurately obtained. According to the range of the eccentricity, a lot of time can be saved when designing the eccentric plate spring.

## III. DESIGN PARAMETERS AND 3D MODEL OF PLATE SPRING

In the design process of eccentric plate spring, the eccentricity has an obvious influence on the spring stiffness. In this study, the finite element analysis is carried out for the plate springs with different eccentricities and all other parameters are the same, and the change rule of radial and axial stiffness with eccentricity is summarized, which provides reference for the design of eccentric involute flexible spring. In addition to eccentricity, design parameters of plate spring are shown in Table 1. Considering the complicated working condition of the Stirling heat pump, the plate spring is required to have good elasticity, anti-magnetic property, corrosion resistance and the like. Taking into account the superior elastic properties of beryllium bronze, it is the best high-grade elastic material in copper alloy, and has the reputation of the king of non-ferrous metal elasticity. Finally decided to choose the material of the bronze material Qbe2 as a plate spring.

TABLE I. DESIGN PARAMETERS OF PLATE SPRING

diameter of bore (mm)	16
Plate spring diameter (mm)	166
Plate spring thickness (mm)	1
Radius of involute base circle (mm)	6
Involute phase difference ( $^\circ$ )	30
Vortex slot width (mm)	$\pi$
Initial angle of involute ( $^\circ$ )	144
Involute end angle ( $^\circ$ )	594
Plate spring material	Beryllium bronze in tempered state

After the rest of the design parameters of the plate spring are determined, according to the design parameters of the leaf spring, three different models of the plate spring under different eccentricities are established by using SOLIDWORKS by selecting different eccentricities, as shown in Fig. 3.

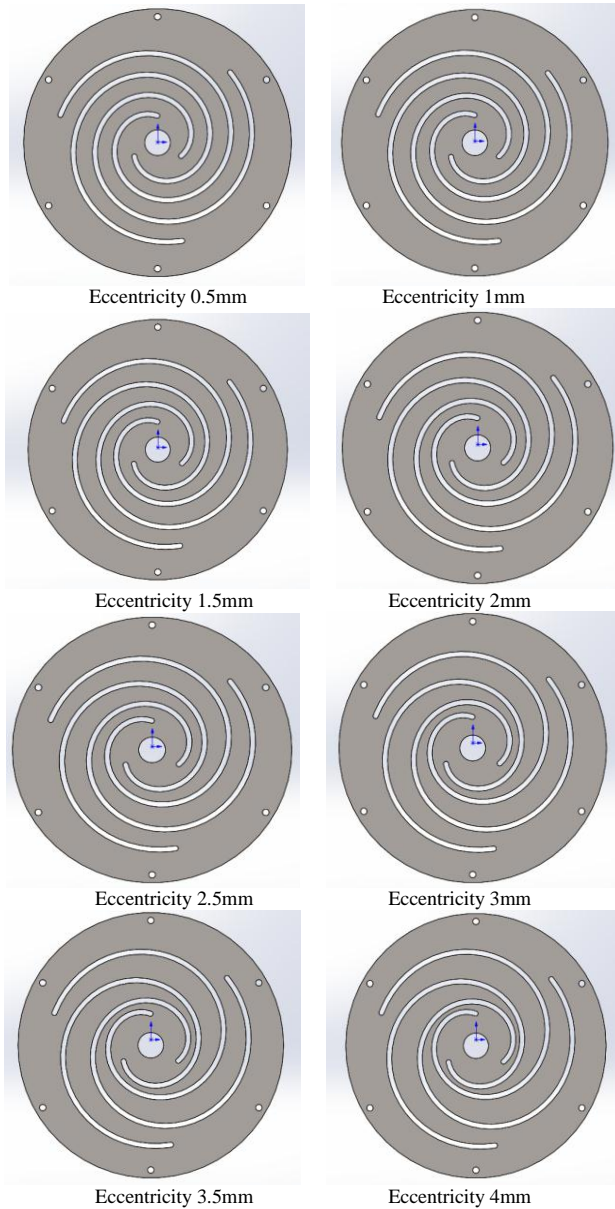


Figure 3. Three dimensional model of plate spring with different eccentricity

#### IV. INFLUENCE OF ECCENTRICITY ON RADIAL AND AXIAL STIFFNESS OF PLATE SPRING

The finite element method is used to analyze the radial and axial stiffness of plate spring[9-11]. The finite element method has been quoted in many literatures about the performance of plate spring, and it has been proved to be an

effective analysis method. In this paper, ANSYS software is used for analysis[12-14].

In the process of analyzing the axial radial stiffness of the plate spring, the plate spring material should be defined first, and beryllium bronze in tempered state should be selected. Then divide the Mesh and click "Mesh". This paper adopts global grid control. Select "Mechanical" structural field in "physics Preference". Grid accuracy is "relevant" input 100, and the range of grid accuracy is -100 ~ 100. The more negative the grid is, the coarser the grid will be, and the worse the grid quality will be. The further you go, the finer the mesh, the higher the quality. The meshing result of the plate spring is shown in Fig. 4.

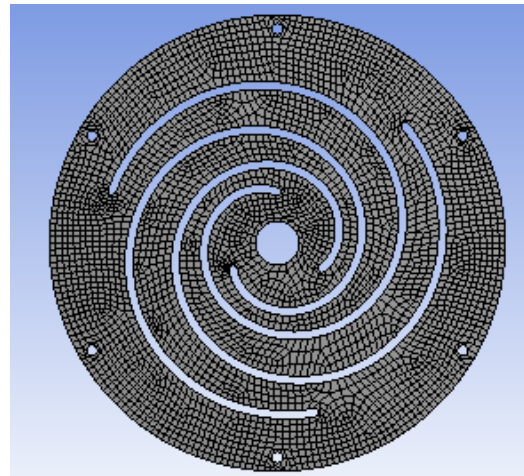


Figure 4. Meshing of plate springs

After mesh generation, constraints and load forces need to be added according to the service conditions of plate springs. In the free piston Stirling heat pump system, it can be considered that the outer edge of the leaf spring is fixed to the shell of the Stirling heat pump part, and the screw hole is fixed to the screw nail and the leaf spring spacer ring. Therefore, the constraint condition is to add reinforcement constraint to the cylindrical surface where the outer edge of the model is located and six screw holes, as shown in Fig.5.

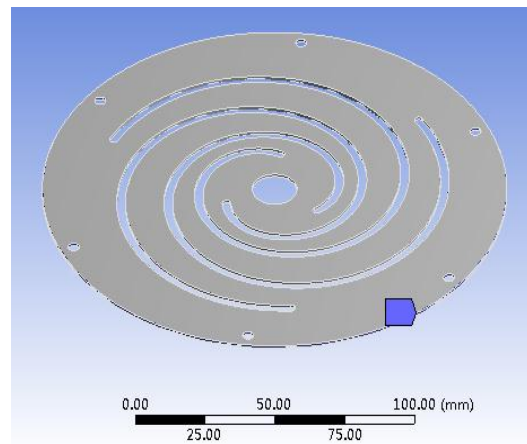


Figure 5. Schematic diagram of leaf spring constraints

When analyzing the axial stiffness of each plate spring under different eccentricity, apply 5N axial force on the inner surface of the central hole of the plate spring to solve the problem, and observe the cloud chart of the axial displacement of the plate spring under different eccentricity, as shown in Fig. 6.

It can be seen from the results of finite element analysis that with the increase of eccentricity, the axial displacement of plate spring decreases first and then increases. Under this design parameter, when the eccentricity is 2.5mm, the axial displacement is the smallest. Therefore, the axial stiffness of plate spring increases first and then decreases with the increase of eccentricity. The whole process changes smoothly. The change rule of axial stiffness with eccentricity is shown in Fig.7.

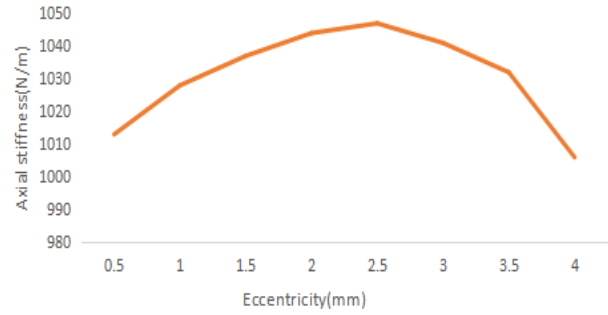


Figure 7. Variation of axial stiffness with eccentricity

When analyzing the radial stiffness of each plate spring under different eccentricity, apply 5N radial force on the inner surface of the central hole of the plate spring to solve the problem, and observe the radial displacement nephogram of the plate spring under different eccentricity, as shown in Fig. 8.

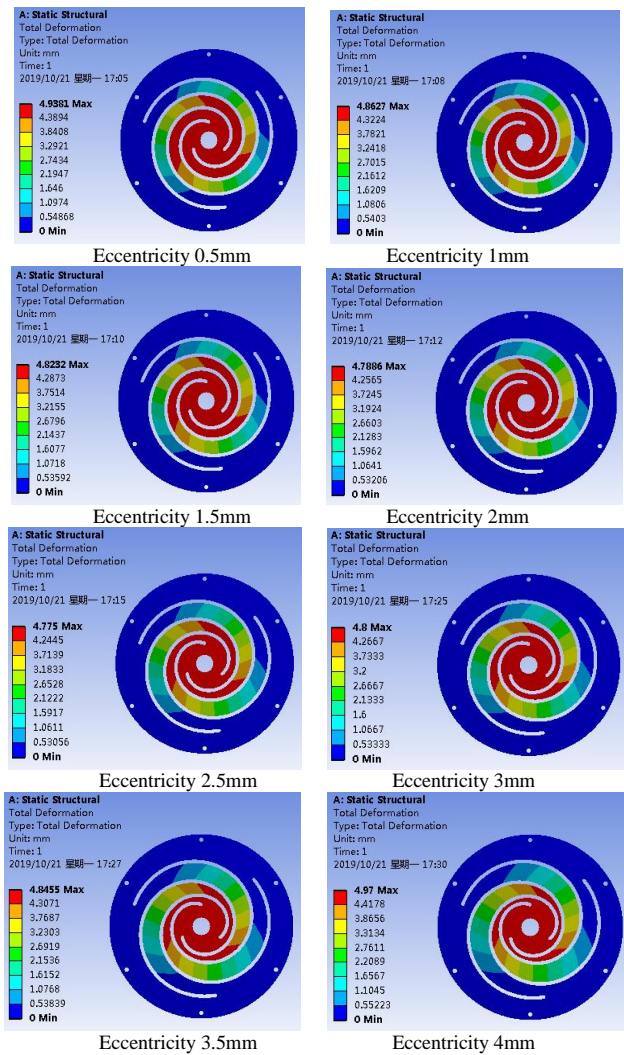


Figure 6. Axial displacement nephogram of plate spring with different eccentricity

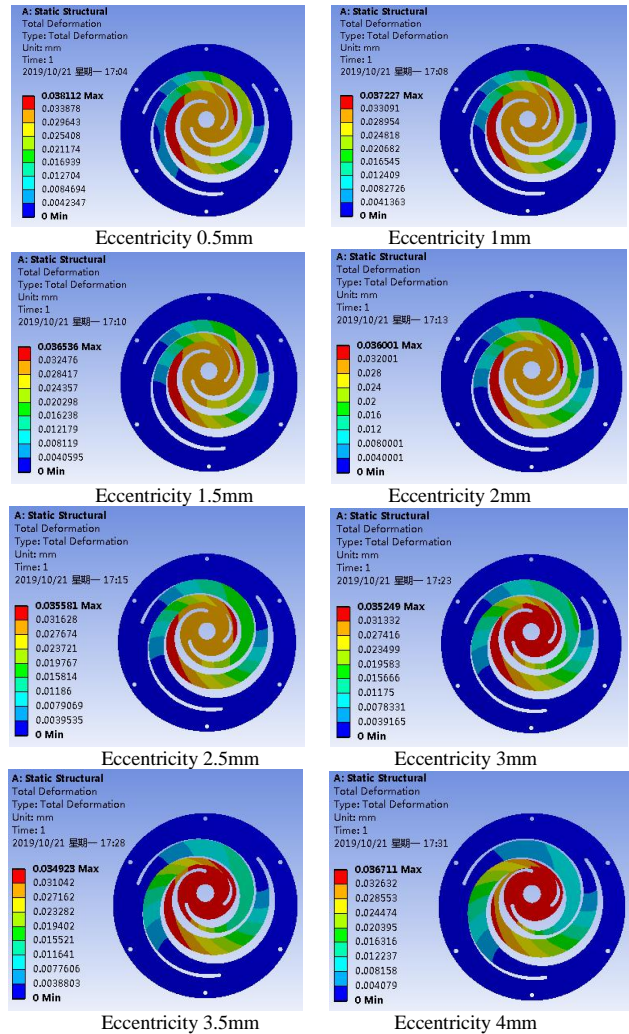


Figure 8. Radial displacement nephogram of plate spring with different eccentricity

It can be seen from the results of finite element analysis that at the beginning, the radial displacement of the plate spring decreases with the increase of eccentricity, and the radial stiffness increases; when the eccentricity reaches about 3.5mm, the radial displacement starts to increase and the speed of increase is obvious, and the radial stiffness rapidly decreases, and the change rule of radial stiffness with eccentricity is shown in Fig. 9.

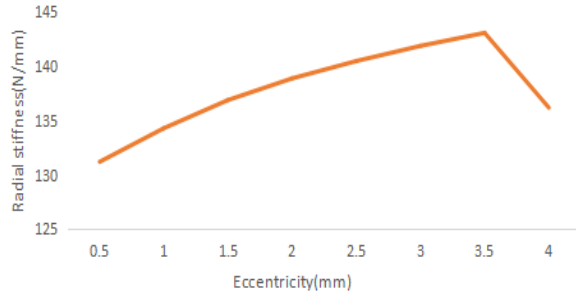


Figure 9. Variation of radial stiffness with eccentricity

## V. CONCLUSION

Based on the analysis of the eccentric involute and the axial stiffness of the spring diameter of the eccentric plate under different eccentricity, combined with the finite element results, the following conclusions are concluded: Through formula, the maximum value of eccentricity can be obtained under different radius of base circle, thus the range of eccentricity can be obtained, which is used to guide the design of eccentric involute plate spring. When the other design parameters of the plate spring are the same, the axial stiffness of the plate spring increases first and then decreases with the increase of eccentricity, the whole process changes smoothly, and the turning point is in the middle of the value range of eccentricity. The radial stiffness of the leaf spring increases first and then decreases with the increase of the eccentric amount. The turning point is at the larger value of the value range of eccentricity, and the radial stiffness decreases at an obvious rate. Therefore, in the design process of eccentric plate spring, after using the radius of the base circle to determine the value range of eccentricity, according to the change rule of radial and axial stiffness with eccentricity and according to the required stiffness under the use condition of plate spring, the eccentricity can be easily determined.

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