

# Research on Complex Tooth Surface Forming Technology of Spiral Face Gear

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**Abstract**—In order to meet the needs of special structural gears in the development of aviation, aerospace and automobile industries, the existing gear processing methods can not meet the processing precision. If it is necessary to pay a large cost to meet the processing precision, a new Processing method - hobbing processing method of spiral face gear. On the basis of a large number of previous studies, based on the differential geometry and spatial meshing principle, the tooth surface equation of the helical face gear is derived and the three-dimensional model is simulated by coordinate transformation and meshing relationship; considering the assembly mode and motion of the hob and the helical face gear Relationship, the machining coordinate system is established, and the helical face gear tooth surface and meshing equation are obtained from the hob-based worm tooth surface equation and the Archimedes worm hob is designed. Finally, the conclusion is given and the research work that needs further development is proposed.

**Keywords**-Spiral Face Gear; Hobbing Machining

## I. INTRODUCTION

Spiral face gear transmission is a new type of spatially staggered non-parallel shaft gear transmission consisting of an involute helical pinion and a helical face gear. Spiral face gear transmission has a large degree of coincidence, compact structure, high transmission ratio, low noise, small wheel floating installation, strong carrying capacity, etc., in the transmission system of helicopters, automobiles, machine tools, robots and other devices widely used.

At present, most of the domestic small-modulus spiral face gears are manufactured by die-casting using a mold developed abroad. Through the modification of the cavity profile, the cast spiral face gear and the cylindrical gear are in point contact. Due to the processing precision of the mold and the influence of thermal deformation, the manufactured spiral face gear has a large tooth surface deviation and a rough tooth surface appearance, resulting in a decrease in

meshing performance, which is difficult to meet the needs of a precision transmission system. Considering the existing processing methods of face gears at home and abroad, this paper proposes a hobbing processing method using a hob to machine spiral face gears to improve the topography quality of die-cast spiral face gears and improve spiral face gears and involute spiral pinions. The contact performance and lubrication characteristics of the aviation, aerospace, automotive and other industries are of great significance.

## II. PRINCIPLE OF TOOTH SURFACE GENERATION OF SPIRAL EVALUATION GEAR

### A. Tooth surface equation of pinion

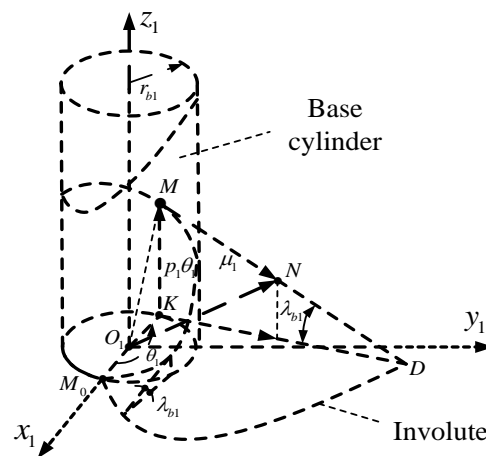


Figure 1. Formation of the side r of the right-handed involute helical tooth

The position vector  $O_1N$  of the flow point on the side r of the right-handed involute helical tooth can be expressed as

$$O_1N = O_1K + KM + MN \quad (1)$$

Tooth surface equation and flank unit normal vector of the right-handed involute helicoid:

$$\begin{cases} x_1 = r_b \cos \theta + \mu \cos \lambda_b \sin \theta \\ y_1 = r_b \sin \theta - \mu \cos \lambda_b \cos \theta \\ z_1 = -\mu \sin \lambda_b + p\theta \end{cases} \quad (2)$$

$$\mathbf{n}_1 = \frac{\frac{\partial \mathbf{r}_1}{\partial \theta} \times \frac{\partial \mathbf{r}_1}{\partial \mu}}{\left| \frac{\partial \mathbf{r}_1}{\partial \theta} \times \frac{\partial \mathbf{r}_1}{\partial \mu} \right|} = \begin{bmatrix} -\sin \lambda_b \sin \theta \\ \sin \lambda_b \cos \theta \\ -\cos \lambda_b \end{bmatrix} \quad (3)$$

The involute profile of the pinion end face is shown in Figure 2, and the plane  $x_s = 0$  is the symmetry plane of the pinion cogging. The tooth surface equation and the tooth surface unit vector of the pinion gear corresponding to the tooth profile I and the tooth profile II are respectively given below:

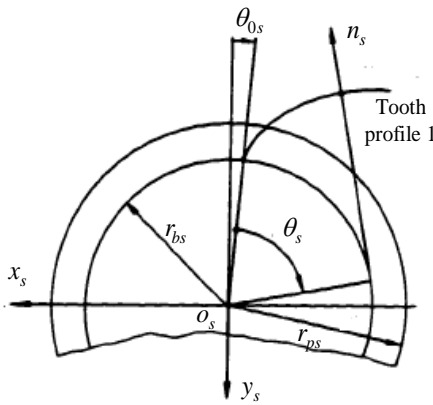


Figure 2. Involute profile of the pinion

$$\mathbf{r}_{s1} = \begin{bmatrix} -r_{bs} \sin(\theta_{0s} + \theta_s) + u_s \cos \lambda_{bs} \cos(\theta_{0s} + \theta_s) \\ -r_{bs} \cos(\theta_{0s} + \theta_s) - u_s \cos \lambda_{bs} \sin(\theta_{0s} + \theta_s) \\ -u_s \sin \lambda_{bs} + p_s \theta_s \\ 1 \end{bmatrix} \quad (4)$$

Where  $\theta_s$  and  $\mu_s$  are the tooth surface parameters in the Gaussian coordinate system;  $r_{bs}$  is the base circle radius;  $\lambda_{bs}$  is the base circle lead angle;  $p_s$  is the spiral parameter;  $\theta_{0s}$  is the difference between the half-angle of the upper end face of the pinion index circle and the involute angle. The relationship between the lead angle on the index circle and the lead angle on the base circle is  $\tan \lambda_{bs} = \tan \lambda_{ps} / \tan \alpha_{ts}$ , where  $\alpha_{ts}$  is the end pressure

angle. The relationship between the spiral parameter  $p_s$  and the index circle radius and the helix angle is  $p_s = r_{ps} \cot \beta$  in the formula,  $r_{ps}$  is the index circle radius of the small cylindrical gear, and  $\beta$  is the helix angle at the index circle.  $\theta_{0s}$  is determined by the following formula  $\theta_{0s} = \omega_1 / 2 \times r_{ps} - \text{inv} \alpha_{ts}$ .

### B. Spiral face gear development principle

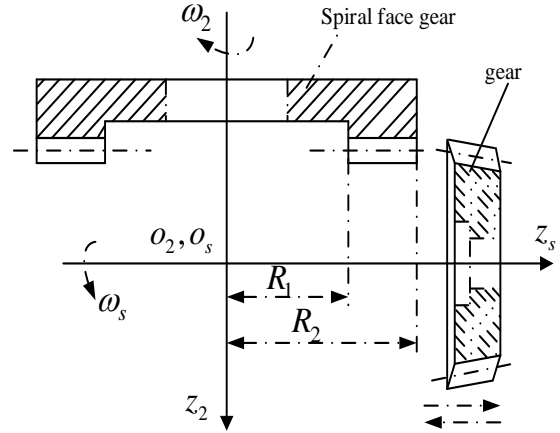


Figure 3. Schematic principle of spiral plane gear

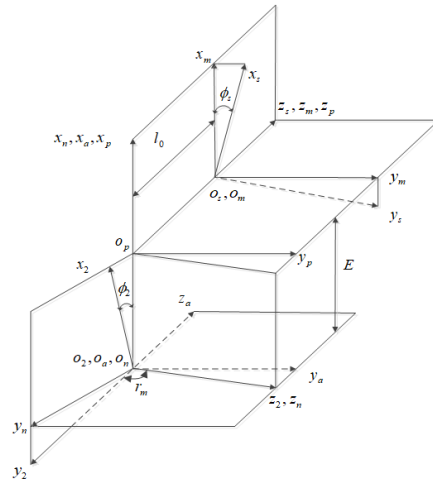


Figure 4. Established machining coordinate system

To derive the tooth surface equation  $\Sigma_{2s}$  of the helical face gear, we use the coordinate system shown in Figure 4:  $S_s(O_s, x_s, y_s, z_s)$ ,  $S_2(O_2, x_2, y_2, z_2)$  are moving coordinate systems respectively fixed to the pinion and the helical face gear;  $S_n(O_n, x_n, y_n, z_n)$ ,  $S_a(O_a, x_a, y_a, z_a)$ ,  $S_p(O_p, x_p, y_p, z_p)$  and  $S_m(O_m, x_m, y_m, z_m)$  are fixed coordinate systems; E is the distance between the axis of rotation of the spiral

plane gear and the axis of rotation of the pinion,;  $\gamma_m$  is the angle between the helical plane gear axis  $z_2$  and the pinion axis  $z_s$ ;  $\phi_s$ ,  $\phi_2$  is the corner of the pinion and the helical face gear respectively;  $l_0$  determines the distance of the coordinate origin  $S_s$  of the pinion coordinate system  $O_s$  with respect to the coordinate origin  $S_2$  of the helical plane gear coordinate system  $O_2$ ,  $R_1$ ,  $R_2$  is the minimum inner radius and the largest outer radius of the helical face gear, respectively.

According to the meshing theory, the helical face gear tooth surface can be expressed by the following equation:

$$\begin{cases} \vec{r}_2(\mu_c, \theta_c, \varphi_c) = M_{2c}(\varphi_c) \vec{r}_c(\mu_c, \theta_c) \\ f_{2c}(\mu_c, \theta_c, \varphi_c) = \vec{n}_c \cdot \vec{v}_c^{(c2)} = 0 \end{cases} \quad (5)$$

Where  $M_{2c}$  is the homogeneous coordinate transformation matrix of the hypothetical pinion tool envelope spiral plane gear

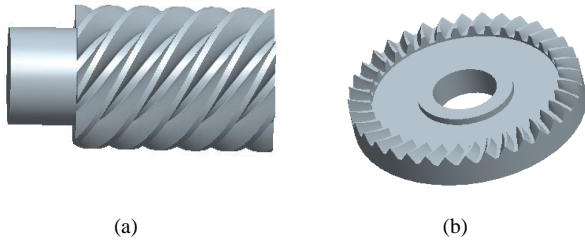


Figure 5. Involute spiral cylindrical gear and Small modulus spiral flat gear

### III. SPIRAL PLANE GEAR HOBBIING PROCESSING PRINCIPLE

#### A. Establishment of the base worm equation

Consider two production lines I and II, and the two production lines are shown in the moving coordinate system  $\Sigma_u$  as shown in Fig. 6. The moving coordinate system  $\Sigma_u$  makes a spiral motion around the worm axis, and the two forming lines respectively form the tooth faces on both sides of the worm tooth groove. The coordinate system  $\Sigma_u$  is fixed to the worm.

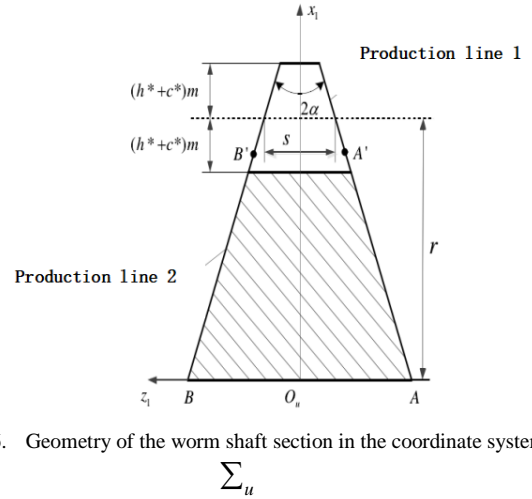


Figure 6. Geometry of the worm shaft section in the coordinate system

The machined tooth surface is represented by the following matrix equation in the coordinate system  $\Sigma_1$ .

$$\vec{r}_1(u, \theta) = M_{1u}(\theta) \vec{r}_u(u) \quad (6)$$

Where  $M_{1u}$  -- is the transformation matrix of the hob-based worm

The vector equation of any point A on the production line I in the coordinate system  $\Sigma_u$  is

$$\vec{r}_u = x_u \vec{i} + y_u \vec{j} + z_u \vec{k}$$

$$x_u = u \cos \alpha$$

$$y_u = 0 \quad (7)$$

$$z_u = u \sin \alpha - (r \tan \alpha + s/2)$$

Where  $\alpha$  --tooth angle, often take  $\alpha = 20^\circ$

#### B. Spiral face gear hobbing principle

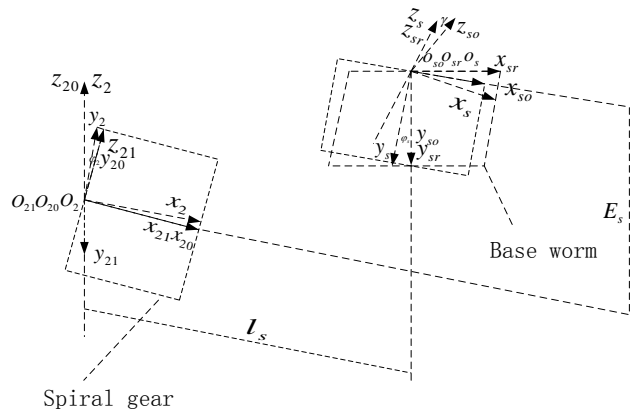


Figure 7. Hobbing machining spiral plane gear coordinate system

Figure 7 is the motion coordinate system of the helical plane gear and the base worm. In the figure, the coordinate system  $S_s(O_s, x_s, y_s, z_s)$  is the coordinate system fixed to the base worm; the coordinate system  $S_{sr}(O_{sr}, x_{sr}, y_{sr}, z_{sr})$  is the coordinate system after the actual installation time base worm is tilted.  $z_{sr}$  coincides with the axis of the base worm;  $S_{s0}(O_{s0}, x_{s0}, y_{s0}, z_{s0})$  is the auxiliary coordinate system, the  $Z_{s0}$  axis is on the normal surface of the base worm; and the coordinate system  $S_{21}(O_{21}, x_{21}, y_{21}, z_{21})$  is also the auxiliary coordinate system. It is used to represent the coordinate system of the movement of the base worm; the coordinate system  $S_{20}(O_{20}, x_{20}, y_{20}, z_{20})$  is a fixed coordinate system; the coordinate system  $S_2(O_2, x_2, y_2, z_2)$  is a follow-up coordinate system fixed to the spiral plane gear, The helical face gear rotates about the axis  $O_2z_2$  the angular velocity is  $\omega_2$ , and the corner is  $\varphi_2$ . The two independent parameters representing the movement of the hob-based worm are the parameter  $O_s z_s$  that rotates about the axis  $\varphi_s$  and the parameter  $l_s$  that moves along the axis. According to the meshing theory, the tooth surface equation of the helical face gear can be expressed by the following equation:

$$\begin{aligned} \vec{r}_2 &= \vec{r}_2(\theta_s, u_s, \varphi_2, l_s, \varphi_s) \\ f(\theta_s, u_s, \varphi_2, l_s, \varphi_s) &= 0 \\ g(\theta_s, u_s, \varphi_2, l_s, \varphi_s) &= 0 \end{aligned} \quad (8)$$

### C. Spiral worm gear hob basic worm design

The modulus is 0.65, the number of worm heads is 1, the tooth angle is 20, the height of the tooth tip is 1, and the head clearance coefficient is 0.25. According to the hob-based worm coordinate equation, the coordinates of the point on the hob can be calculated. Through these points, the contour of the hob-based worm can be obtained, and the calculated vector coordinate output can be obtained as a data point file, and the hob can be established by Pro/E function. The three-dimensional model of the base worm is fitted into the line by the point cloud data, the shape is generated, and then the hob-based worm is obtained. The final effect diagram is shown in figure 8.

TABLE I. CALCULATION PROJECT AND FORMULA

Calculation project	Calculation formula and selection project	Calculation example
Imitation	Known	$m = 0.65$
Outer diameter	Reference tool design manual	$d_{a0} = 25mm$
Aperture length	Reference tool design manual	$D = 4mm$
Axle diameter	Reference tool design manual	$D_1 = 15mm$
Axle length	Reference tool design manual	$a = 3mm$
Number of chip pockets	According to GB/T6083-2001	$z_k = 12$
High tip	$h_{a0} = (h_a^* + c^*)m$	$h_{a0} = 1.35mm$
Root height	$h_{f0} = (h_a^* + c^*)m$	$h_{f0} = 0.8775mm$
Full height of teeth	$h_0 = h_{a0} + h_{f0}$	$h_0 = 1.755mm$
Normal pitch	$p_{n0} = \pi m$	$p_{n0} = 2.0420mm$
Normal tooth thickness	$s_{n0} = \pi m / 2$	$s_{n0} = 1.0210mm$
Shovel back	$K = \frac{\pi d_{a0}}{z_k} \tan \alpha_0$	$K = 1mm$
Chip groove depth	$H_K = h_0 + K + 0.5$	$H_K = 2.775mm$
Groove bottom radius	$r = \frac{\pi (d_{a0} - 2H_K)}{10z_k}$	$r = 0.5092mm$

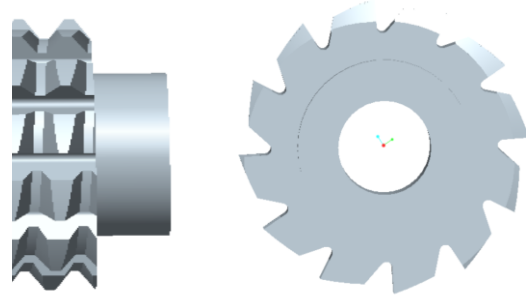


Figure 8. Spiral plane gear hob

### IV. CONCLUSION

- According to the principle of spiral plane gear machining, the machining coordinate system of spiral face gear is established, and the meshing equation and tooth surface equation of helical plane gear machining are derived. The tooth width of the helical face gear is determined by the root undercut and the tip of the spiral face gear. The conditional equation of the root of the helical face gear is not cut and the tip of the tooth is not pointed. The minimum inner radius and the largest outer radius of the helical face gear. The computer program for solving

the discrete points of the tooth surface was programmed by MATLAB software, and the three-dimensional geometric model of the helical face gear and the involute spiral pinion was established.

- According to the principle of spatially interlaced shaft meshing, a hobbing method for small-modulus helical face gears is proposed. The hobbing coordinate system of the helical face gear is established, and the expression, meshing equation and tooth surface equation of the relative motion speed of the hobbing helical face gear are derived. And a simple tool design is given, which provides a way of thinking and method for the future research of hobbing and processing spiral gears.

#### ACKNOWLEDGMENT

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