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Analysis of the Load Sharing of the Flexible Ring Gear Planetary Gear Train

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***Abstract*—The lumped mass model-finite element hybrid model with flexible ring gear is proposed. The dynamic model of planetary transmission system with flexible ring gear structure is established. The calculation of internal meshing time variation between flexible ring gear and planetary gear is proposed. The method of meshing stiffness. According to Newton's second law, the planetary gear train dynamics load model considering time-varying meshing stiffness, damping, comprehensive meshing error and thin-walled sleeve inner ring gear is derived. The planetary gear train with flexible ring gear structure is established. The dynamic equations are solved by numerical methods. The dynamic load-carrying characteristics of the planetary gear trains with different inner sleeves and inner ring gears with different sleeve thicknesses are analyzed. The results show that the inner ring gear sleeve wall The thinner the thickness, the greater the flexibility of the inner ring gear, the smaller the meshing stiffness during internal meshing, the better the load sharing performance of the system, and the internal meshing load factor is more affected.**

Keywords-Planetary Train; Flexible Ring Gear; Loaded

I. INTRODUCTION

The planetary gear train has a series of advantages such as large transmission ratio, high power density and strong carrying capacity, and has become an important transmission form in helicopter power transmission. However, the installation and manufacturing errors of the structural components may cause uneven power transmission of the planetary gear trains, which will further reduce the bearing capacity, reduce the life of the planetary gears and the dynamics of the impact vibration, thus seriously affecting the reliability of the helicopter transmission system. Sex and smooth running. Therefore, it is necessary to adopt a uniform configuration that can compensate for the comprehensive error of the planetary gear train. The flexible ring gear planetary gear train compensates the planetary gear train installation and manufacturing error through the flexible deformation of the ring gear. The configuration, the flexible ring gear structure is light and thin, which is beneficial to achieve weight reduction and load sharing. Based on this, domestic and foreign scholars have conducted extensive and in-depth research on the planetary gear train system considering the flexibility of the inner ring gear. In 2001, Kahraman[1] used a combination of finite element and semi-analytical nonlinear contact mechanics to simulate the various components of a planetary gear train in a typical automotive automatic transmission, and analyzed the flexibility of the ring gear to the planetary gear set. The influence of static characteristics lays the foundation for the analysis of the influence of ring gear flexibility on the load sharing performance. Tanna et al.[2] established a finite element model for predicting the free vibration characteristics of the member from the vibration characteristics of the flexible ring gear, and obtained four typical vibration modes under the free vibration of the ring gear. The effects of different restraint modes and structural simplification on the inherent characteristics of the ring gear are discussed. After that, they further analyzed the influence of design parameters on the free vibration characteristics of the ring gear. Abousleiman. V[3] established a more accurate model than the lumped mass model—the rigid-flexible coupling model. The model is mainly to flex the inner ring gear through the finite element software. The other components are regarded as rigid bodies. The concentrated mass method is used to couple the two models together through corresponding transformations, which provides the influence of the component flexibility on the dynamic characteristics of the system. Theoretical support. According to the rigid body discrete analysis method, the domestic scholar Zhang Jun[4] divides the inner ring gear into several rigid bodies, and the segments are connected by springs, which similarly express the flexibility of the inner ring gear, and then analyze the force of the inner ring gear. Then, the dynamic model is established, and the change of the spring stiffness between the ring gear and the change of the inherent characteristics of the system are explored. In summary, domestic and foreign scholars have carried out a lot of research on the influence of the inner ring gear flexibility on the dynamic characteristics of the planetary gear transmission system, and obtained some mature conclusions.

Li Lei[5] Analyze and study the design technology of the planetary gear train's flexible internal ring gear

Therefore, this paper takes the planetary gear train with thin-walled sleeve inner ring gear as the research object. Considering the flexibility of the inner ring gear structure, a concentrated mass model-finite element hybrid model with flexible ring gear is proposed, based on the energy method and Timshenko beam. The method of combining the theory of Timshenko beam is to calculate the time-varying meshing stiffness of the gear teeth, and to explore the influence of the flexibility of the ring gear on the dynamic load-carrying performance of the planetary gear train, and provide theoretical support for the structural design of the ring gear.

II. ESTABLISHMENT OF DYNAMIC PHYSICAL MODEL OF PLANETARY GEAR TRAIN WITH THIN INNER SLEEVE FLEXIBLE RING GEAR

The structure of the planetary gear transmission system with the thin ring sleeve inner ring gear is shown in Figure 1. The whole system is mainly composed of an input shaft D connected to the motor, a sun gear s, N planetary gears p, a carrier c, and a thin-walled sleeve type inner ring gear r. In addition, when the ring gear is coupled with the body, the torsion stiffness of the ring gear is zero, and the ring gear is of a thin-walled sleeve type, which has a certain flexibility, so the ring gear has a certain radial support stiffness.

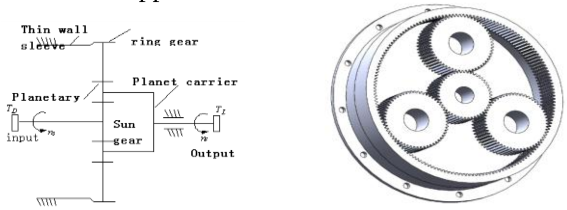


Figure 1. (a)System structure motion diagram and(b) 3D solid model diagram

The physical model of the planetary gear train with the thin-walled sleeve-type flexible internal ring gear structure is shown in Fig. 2, with the origin O as the fixed installation center and the coordinate system xoy with the rotation of the planet carrier. Where k_{pn} is the planetary gear support stiffness (n=1, 2, 3, ..., N); ψ_n is the angle between the x-axis forward and the n th planetary gear center and the coordinate origin ($\psi_n = 2\pi(n-1)/N$); k_{ij} is the support Stiffness($i = s,c; j = x, y$); k_{iu} is the torsional stiffness ($i = s, c$); x_i , y_i , u_i are the transverse, radial and torsional vibration line displacements of the members except the ring gear, and ($u_i = r_i\theta_i; i = c, s, 1, 2, \ldots, N$) where r_i is the radius of gyration of the member; k_{spi} For the external meshing time-varying stiffness between the sun gear and the planet gear, k_{rpi} is the time-varying meshing stiffness between the ring gear and the planet gear.

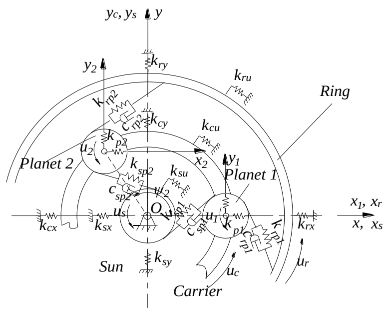


Figure 2. Physical model of planetary gear train with thin-walled sleeve type flexible ring gear structure

III. TIME-VARYING MESH STIFFNESS ANALYSIS

A. *Calculation of gear insertion stiffness based on energy method*

For a pair of gears with a certain degree of coincidence, as time changes, it is possible to set the meshing stiffness of the driving wheel to represent k_1 , and the meshing stiffness of the driven wheel to represent k_2 , then the magnitude of the integrated meshing stiffness k of the pair of gears is

Expressed as: $$k = \frac{k_1 k_2}{k_1 + k_2}$$

In this paper, the influence of the flexible ring gear on the dynamic characteristics of the planetary gear train is studied. When calculating the meshing stiffness of the sun gear and the planetary gear, the base part of the gear is treated as a rigid body, and only the deformation of the gear teeth is considered. Therefore, the single tooth meshing rigidity of the external gear is mainly calculated by the deformation of the gear teeth and the contact deformation of the gear teeth. According to the Hertz contact deformation theory, the contact stiffness of the two teeth is calculated, and the magnitude of the contact stiffness does not change with the change of the meshing position due to the assumption. That is to say, the magnitude of the contact stiffness(k_h) is a fixed value, which is only related to the parameters of the gear itself. Its calculation formula is:

$$K_h \frac{\pi EB}{4(1-\nu^2)}$$

Where E is the modulus of elasticity; B is the tooth width: v is the Poisson's ratio.

In the planetary gear transmission process, the general coincidence degree is greater than 1. Therefore, the formula for calculating the external stiffness of the outer gear between the sun gear and the planetary gear is as follows:

$$K = \sum_{i=1}^{2} \frac{1}{1/(\frac{1}{K_{bs,i}} + \frac{1}{K_{ss,i}} + \frac{1}{K_{as,i}} + \frac{1}{K_{bp,i}} + \frac{1}{K_{sp,i}} + \frac{1}{K_{ap,i}} + \frac{1}{K_h})}$$

Where, k_{si}、k_{bi}、$k_{ai}(i = s、p)$ respectively represent the bending stiffness, shear stiffness and axial compression stiffness of the gear.The subscript s represents the sun gear and p represents the planet gear. Indicates the number of pairs of teeth that are engaged. i indicates the number of teeth that are engaged.

The internal meshing stiffness and the external meshing stiffness are calculated consistently, when the rim of the ring gear is particularly thick and not the base body portion of the ring gear can also be processed into a rigid body by a load-carrying structure such as a thin-walled sleeve. At this time, the total meshing rigidity of the pair of gear teeth of the ring gear-planetary gear is

$$K = 1/(\frac{1}{K_{br}} + \frac{1}{K_{sr}} + \frac{1}{K_{ar}} + \frac{1}{K_{bp}} + \frac{1}{K_{sp}} + \frac{1}{K_{ap}} + \frac{1}{K_h})$$

B. Calculation of deformation of flexible ring gear base based on Timshenko beam theory

For the inner ring gear structure of the thin-walled sleeve, this paper draws on the calculation method of the base deformation of the literature[6], and uses the theory of iron-wood-simple beam, and then according to the superposition principle, the inner ring gear with thin-walled sleeve is simplified to the same outer diameter. Two thin-walled rings with different inner diameters are used to study the inner ring gear with thin-walled sleeve structure, revealing the influence of the deformation of the inner ring gear base on the internal meshing stiffness, and analyzing the stability of the inner ring gear to the uniform load characteristics of the system. influences.

Since the inner ring gear structure studied in this paper is fixed on the body by uniform bolts, it can be assumed that the inner ring gear has a total of n bolt supports. In the process of meshing the ring gear with the planetary gears, it is assumed that there are m concentrated external forces at the same time. Or torque. At this time, the inner ring gear containing the thin-walled sleeve can be divided into the $m+n$ section uniformly curved Timoshenko beam, as shown in Fig.3.In the figure, the numbers 1, 2, 3, ..., m+n represent the position of each section of the Timoshenko beam, the connection position of the adjacent two-section beam, that is, the position of the bolt and the force constraint is represented by the symbols a,b,c, ..., x; The angular position of the calculated cross section on the k segment beam is represented by θ_k , and the $k+1$th segment is represented by θ_{k+1}; α_k、 α_{k+1} represents its corresponding central angle.

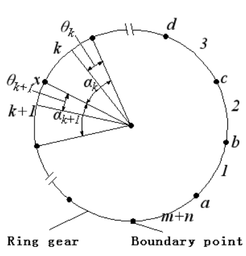
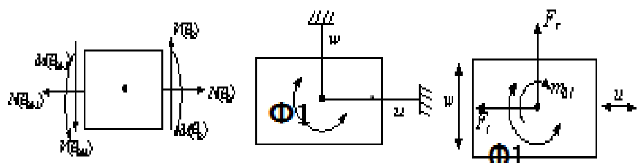


Figure 3. A ring of teeth that is evenly bent into a number of sections

For the research object of this paper, it can be determined that the boundary condition is fixed support. At this time, the segments of the ring gear are fixed by bolts, and the degrees of freedom in the radial direction and the circumferential direction are zero, and only the degree of freedom in the direction of rotation. In the actual transmission process of the planetary gear, the inner ring gear and the planetary gears mesh with each other, thereby generating a certain amount of meshing force. At this time, the generated meshing force is a binding force condition for the fixed ring gear. In addition, the meshing force acts as a concentrated force on the teeth and is always perpendicular to the tooth profile of the teeth. For ease of analysis, the analysis can be performed first on a ring gear base with a toothed structure. Therefore, for a ring gear with a gear tooth structure, the magnitude of the concentrated force can be equivalent to the radial force F and the tangential force F and the concentrated moment M of the point of action of any section of the beam, combined with the Timoshenko beam. The boundary condition of the beam and the actual boundary conditions of the ring gear structure can be obtained as shown in Fig. 4.



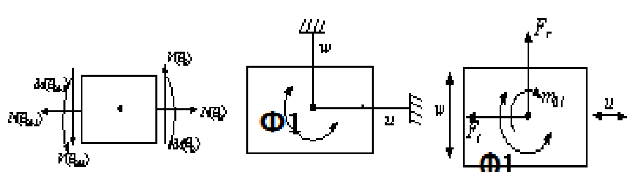
(a) Internal force and moment (b)Fixed support (c)Equivalent analysis of meshing force

Figure 4. Schematic diagram of the ring gear with gear teeth

Therefore, considering the internal force and moment action of the boundary point, as shown in Fig. 4(a), and the support form of the inner ring gear in this paper, as shown in Fig. 4(b) and the external force, that is, the meshing force, as shown in Fig. 4(c) , the boundary condition can be expressed as

$$u(\theta_k) = u(\theta_{k+1});\ w(\theta_k) = w(\theta_{k+1});\ \phi(\theta_k) = \phi(\theta_{k+1});\ M(\theta_{k+1}) - M(\theta_k) = m_0;$$
$$V(\theta_{k+1}) - V(\theta_k) = F_r;\ N(\theta_{k+1}) - N(\theta_k) = F_t.$$

The above is the analysis of the deformation of the ring base with the tooth portion of the ring gear. It can be seen from the related literature [7] that the partial deformation of the thin-walled sleeve is mainly due to the torsional deformation caused by the concentrated moment m_{02} , assuming that the radial direction is ignored. The force F_r and the tangential force F_t bring the radial displacement and the circumferential displacement of the thin-walled sleeve, and the boundary diagram of the thin-walled sleeve structure is as shown in Fig. 5.



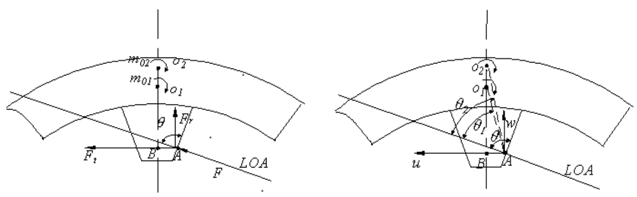
(a)Internal force and moment (b)Fixed support (c)Equivalent analysis of meshing force

Figure 5. Boundary diagram of a thin-walled sleeve

At this time, the boundary condition of the thin-walled sleeve structure can be expressed as

$u(\theta_k)=0;\quad u(\theta_{k+1})=0;\quad w(\theta_k)=0;\quad w(\theta_{k+1})=0;$
$\phi(\theta_k)=\phi(\theta_{k+1});\ M(\theta_{k+1})-M(\theta_k)=m_{02};$

In order to solve the deformation size of the ring gear structure, it is necessary to calculate the radial force F_r and the tangential force F_t and the magnitude of the concentrated moment m_{01}, m_{02}. When calculating the influence of the ring gear deformation on the meshing stiffness, the internal tooth is regarded as a rigid body, and the meshing force F acts on the ring gear in the form of the circumferential force F_t, the radial force F_r and the moment m_{01}、m_{02}, as shown in Fig. 6(a). The corresponding displacement is shown in Figure 6(b)



(a)Decomposition of external force F, (b)Displacement of the contact point on the tooth profile on the meshing line

Figure 6. Gear teeth caused by deformation of the ring gear are displaced along the mesh line

As can be seen from Fig.6

$F_r=F\cos\theta, F_t=F\sin\theta, m_{01}=F_t|O_1B|-F_r|AB|,\ \ m_{02}=F_t|O_2B|-F_r|AB|$

Based on the calculation of the ring gear flexible deformation, the circumferential displacement u, the radial displacement w and the rotation angle ϕ_1、 can be obtained, as shown in Fig. 6(b), the displacement of the contact point on the tooth profile caused by the ring gear deformation on the meshing line for

$$\delta_r = w\cos\theta + u\sin\theta + \phi_1|O_1A|\sin\theta_1$$

The displacement of the contact point on the tooth profile caused by the deformation of the thin-walled sleeve on the meshing line is

$$\delta_t = \phi_2|O_2A|\sin\theta_2$$

Therefore, the stiffness of the meshing line caused by the flexible deformation of the ring gear is

$$K_{fr}=\frac{F}{\delta_r+\delta_t}$$

So far, according to the planetary gear system parameters shown in Table Ⅰ, based on the energy method to obtain the external meshing time-varying stiffness between the sun gear and the planet gear, as shown in Fig. 7. Then based on the energy method and the Timshenko beam theory, the internal meshing time-varying mesh stiffness between the six inner ring gears of the planetary wheel and the sleeve wall thickness t, respectively $t=3mm$, $t=6mm$, $t=9mm$, $t=12mm$, $t=15mm$, $t=18mm$, is shown in Fig. 8.

TABLE I. THIS PAPER STUDIES THE PLANETARY GEAR SYSTEM PARAMETERS

Basic parameters	Planet carrier	Planetar y wheel	Ring gear	Sun gear
Number of teeth	/	57	162	48
Tooth width (mm)	/	88	88	88
Sleeve height (mm)	/	/	66	/
Modulus (mm)	/	3.5	3.5	3.5
pressure angle(°)	/	20	20	20
quality(kg)	10.749	8.835	30.618	9.503
Moment of inertia(kg m^2)	0.392	0.063	/	0.056
Support stiffness(N/m)	1e8	4.5e7	5e9	1e8
Torsional stiffness(N/m)	0	/	0	1e8

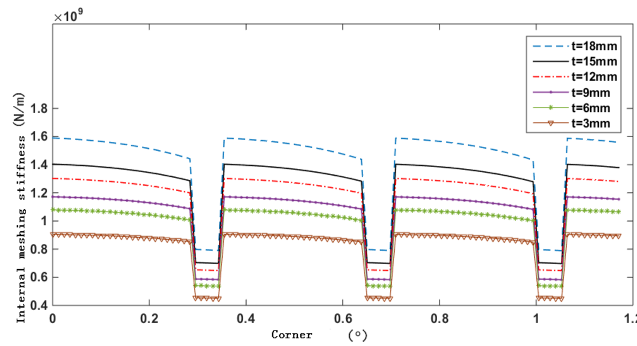


Figure 7. Internal meshing time-varying mesh stiffness with different sleeve thickness

C. Establishment of Dynamic Load Equation of Planetary Gear Train with Flexible Inner Ring Gear

The system dynamics equation is

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{F\}$$

$[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the transmission system stiffness matrix, $\{X\}$ is the transmission system degree of freedom column vector, and F is the excitation column vector.

By reading a large number of literatures on the planetary gear trains, it is known that the description of the load-carrying characteristics of the planetary gear train system is described by one of the internal and external meshing pairs [8], and some scholars also use it. These two average loading coefficients are described[9]. In order to further explore the influence of the flexibility of the ring gear inner ring structure on the whole system, this paper decided to adopt the latter, and the planetary gear system parameters are shown in Table 1, with the sleeve thickness t being 6mm and 9mm respectively. For example, according to the established dynamics load equation, then the equation is dimensionless, eliminate the rigid displacement, and finally through the MATLAB programming, the Runge Kutta method is used to solve the equation, the system external meshing load factor and internal The results of the meshing load factor are shown in TableⅡ.

TABLE II. INTERNAL AND EXTERNAL MESHING LOAD FACTOR OF PLANETARY GEAR TRAIN SYSTEM

Sleeve thickness t(mm)	External meshing load factor	Internal meshing load factor
6	1.058	1.073
9	1.092	1.123

When the inner ring gear sleeve thickness t=6mm, the dynamic load sharing coefficient of the system is shown in Fig. 8. When the thickness of the inner ring gear sleeve is t=9mm, the dynamic load sharing coefficient of the system is shown in Fig. 8.

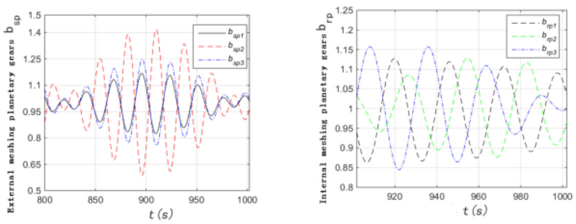


Figure 8. External and Internal meshing planetary gears

IV. INFLUENCE OF THE STABILITY OF THE RING GEAR ON THE DYNAMIC LOAD OF THE PLANETARY GEAR TRAIN

The average load factor of the outer meshing pair is larger than the load sharing coefficient of the inner meshing pair, which explains to some extent that the flexibility of the thin-walled sleeve contributes to the improvement of the load-carrying performance of the inner meshing. In addition, as the internal meshing stiffness increases, the overall load-carrying coefficient of the external mesh does not change as a whole. This is because only the internal meshing stiffness is changing, and the external meshing stiffness is not changed, so the internal meshing stiffness is changed to the external meshing. The average load factor has little effect. However, as the internal meshing stiffness increases, the load sharing coefficient of the internal meshing is obviously increasing. Therefore, the greater the flexibility of the ring gear, the better the load sharing performance.

V. CONCLUSION

In order to explore the influence of the flexibility of the thin-walled sleeve inner ring gear on the load sharing performance of the system, this paper proposes a method based on the energy method combined with Timshenko beam theory to calculate the flexible ring gear. Time-varying stiffness with internal engagement between the planet wheels. Under the excitation of time-varying mesh stiffness, a general dynamic load-carrying model of planetary gear train with thin-walled sleeve type inner ring gear is established. Considering time-varying meshing stiffness, damping, and comprehensive meshing error, The numerical method is used to solve the above-mentioned dynamic equation of the planetary mass system considering the flexible ring gear, and the average load performance and dynamic response of the planetary gear train are obtained. The specific conclusions are as follows:

- An external meshing time-varying meshing rigidity in consideration of factors such as mounting manufacturing error of the gear pair, gear modification, distribution along the contact line load, and the like are obtained.
- The relationship between the inner ring gear sleeve thickness and the internal meshing time-varying meshing stiffness is analyzed. The larger the inner ring gear sleeve thickness is, the smaller the inner ring gear is, the smaller the internal ring gear is. The greater the variable meshing stiffness.
- The kinetic load sharing coefficient is solved for the specific example. The influence of the inner ring gear flexibility on the load sharing performance of the system is analyzed. The results show that the greater the inner ring gear is, the better the system load performance is, and the internal meshing is The load factor is more affected.

VI. ACKNOWLEDGMENT

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REFERENCES

[1] A. Kahraman, S. Vijayakar, Effect of Internal Gear Flexxibility on the Quasi-Static Behavior of a Planetary Gear Set[J]. Journal of Mechanical Design, 2001, 123(8): 409-415.

[2] TANNA R P, LIM T C. Effects of boundary conditions on the natural modes of transmission ring gear structure[C]/ /SAE Noise& Vibration Conference &Exposition . Michigan: SAE Technical Paper, 2001: 2001 — 01 — 1 416.

[3] Abousleiman .V, P. Velex. A Hybrid 3D Finite Element/Lumped Parameter Model for Quasi-static and Dynamic Analyses of Planetary Gear Sets[J]. Mechanism and Machine Theory, 2006, 41(6): 725-748.

[4] ZHANG Jun, SONG Yimin, WANG Jianjun. Dynamic Modeling of Straight Tooth Planetary Transmission Considering Ring Gear Flexibility[J]. Journal of Mechanical Engineering, 2009, 45(12): 29-36.

[5] Li Lei, Shen Yunbo, Research on the design technology of the flexible internal ring gear of the planetary gear train [D]. Xi'an: Xi'an University of Technology, 2019

[6] KAHRAMAN, LIGATA, SINGH. Influence of Ring Gear Rim Thickness on Planetary Gear Set Behavior[J]. Journal of Mechanical Design, 2010, 132(2):021002.

[7] ZHANG Jun, SONG Yimin, WANG Jianjun. Dynamic Modeling of Straight Tooth Planetary Transmission Considering Ring Gear Flexibility[J]. Journal of Mechanical Engineering, 2009, 45(12): 29-36.

[8] Zaigang Chen, Yimin Shao. Mesh stiffness of an internal spur gear pair with ring gear rim deformation[J]. Mechanism and Machine Theory, 2013, 69(1): 1-12.

[9] Li Siqian, Wu Shijing, Wang Xiaosun. Dynamic Load Sharing Characteristics Analysis of Two-stage Planetary Gear Transmission System[J]. Mechanical Transmission, 2016(10): 11-16.