Developing the methods of factor analysis of the socio-economic diagnosis of the status of transboundary regions

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Abstract. The article discusses the socio-economic diagnosis of the status of transboundary regions on the basis of the factor analysis method. Also, it provides our analysis of the mathematical properties of an index method, considering the proposed index model of the socio-economic diagnosis of the status of transboundary regions.

Keywords: factor analysis, region, cross-border regions, socio-economic processes

1. Introduction

In the modern world, in the context of globalization and the growing global financial crisis, the state has to not only improve its structure, but also use the entire domestic resource, geopolitical, and any other potential to optimize the country's economy and realize its national interests in the international arena. In such conditions, it is the cross-border region that acquires special importance in the priorities of the state, since it becomes the field of political and economic interaction where states are able to effectively fulfill their needs for various international investments.

In the field of management of socio-economic processes of cross-border regions, an important element determining management efficiency is the equipment of managers and analysts with appropriate methods for describing and mathematical modeling of processes and phenomena for which factor analysis methods play an important role.

The purpose of the study is to improve the methods of factor analysis based on multiplicative index models in relation to the diagnosis of the socio-economic status of cross-border regions.

In accordance with this goal, the work solves the problem of developing a mathematical description of factor models. The work also solves the problem of obtaining (using graph theory) algorithms for deriving formulas for calculating the values and increments of the effective indicator as a function of the increment of factors.

2. Materials and Methods

Factor analysis assumes further deepening of knowledge about a region, obtained as a result of applying the method of systemic and correlation analysis [1]. For a variety of indicators, a small number of cores, as a rule, directly unobservable, but defining the characteristic properties and characteristics of the object indicators is often difficult to determine. In this case, the need arises, on

the one hand, to select the smallest possible number of hidden common factors. On the other hand, the desire arises so that the selected factors approximate the observed variables as precisely as possible, or more precisely describe the relationships between them.

Information is a key element of the management process. In this regard, the urgent task is the improvement of information processing methods. In the field of management of socio-economic processes, the equipment of managers and analysts with appropriate methods for describing and mathematical modeling of processes and phenomena (for which the important role played by the methods of factor analysis) is an important element determining the effectiveness of management [2].

3. Results

First, a number of particular problems exist, their solution is absent in well-known works. For example, these are the tasks related to obtaining requirements for the accuracy of the presentation of initial data and estimating the influence of random factors. In this regard, the problem of the reliability of the results arises.

Second, at the present time, the need to improve the mathematical software arises. This is due to the improving characteristics of computer equipment. Computing technology is able to quickly process and store large amounts of information. The use of personal computers allows one to analyze a wider front by solving problems, using various models and methods, and to work with the "beam" of models.

Currently, a variety of factor analysis methods are widely accepted. At the same time, it must be admitted that the known methods have their own specific areas of application and do not always allow to effectively solve special tasks due to the presence of limitations of the most diverse nature, incompleteness and errors in the collection of source data.

A number of specialists addressed the problem of increasing the efficiency of factor analysis. The following Russian scientists can be distinguished among them: V. A. Borisov, V. V. Nalimov, S. V. Tkachev, G. F. Filaretov, Yu. P. Yurachkovsky, and others. Among foreign scholars we should mention such authors as D. Jones, I. Fisher, J. Fisher, G. Whipple, R. Miers, A. Cole, etc.

4. Discussion

A qualitative and quantitative study of the influence of factors on generalizing economic indicators is one of the main tasks of this area of analysis. This problem is solved in several stages [3]:

1. The formation of the factor system, i.e. the set of indicators that have the most significant impact on the generalizing indicator in the analyzed period;
2. Construction of a mathematical model of dependence of the level of a generalizing indicator on the levels of indicators-factors;
3. Quantitative assessment of the influence of each factor or their group on the change in the synthesis indicator.

This paper discusses the issues of solving problems of factor analysis using the concepts of graph theory.

Let's consider the option of factor analysis based on multiplicative index models. A characteristic feature of this analysis method is that it does not provide for the usual commutativity, i.e. the results of estimating the increments by factors depend on the order of the factors of the factors.

The noted property represents a certain difficulty in selecting the order of the factors. To overcome this drawback and to obtain a formally rigorous result, in this work, a special study is conducted of the properties of the method of factor analysis when presenting a generalizing resultant in a multiplicative form. In this case, the task of obtaining a correct result that would satisfy the requirement of commutativity is posed.

The algorithm for solving using multifactor index models is known from the literature [4]. We will consider it in general form to calculate the absolute increase and growth rate of the effective index due to individual factors.

The economic-mathematical model of dependence in general form in a multiplicative form is represented as follows:

$$Y = a \cdot b \cdot c \cdot, \ldots, \cdot k,$$

where Y is the effective sign; $a, b, c, \ldots, k$ are the factors.

The calculation of the growth rate of the performance indicator due to individual factors is made by subtracting the corresponding conditional growth rates of the performance indicator. The conditional growth rate implies the growth rate of the effective indicator taking into account only the change of the factor "a", the factors "a" и "b", and so on. The conditional growth rates of the effective indicator will be presented respectively by indices. $I_a, I_a I_b, \ldots, I_k$.

The overall growth rate of the effective indicator due to the factor "a" is:

$$\Delta I_{ya} = I_a - 1, \quad (1)$$

due to the factor "b"

$$\Delta I_{yb} = I_a I_b - I_a, \quad (2)$$

due to the factor "k"

$$\Delta I_{yk} = I_a I_b, \ldots, I_k - I_a I_b I_{k-1}, \quad (3)$$

where individual indices take into account the influence of factors. The above solution algorithm is applicable for models that have a quantitative indicator in the first place. In the case when the quality indicator is located in the first place of the index model, when solving them, the weighting of the factor values to the left of the indicator being studied is based on baseline data, and those to the right are based on reported data.

In this case, the graph-analytical method can be used for calculations. For example, to calculate growth rates by factors, the following directed graph (which is shown in Figure 1)is to be built.

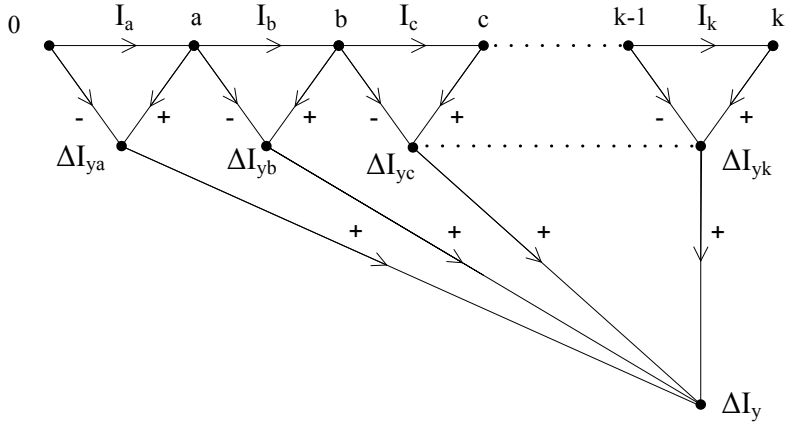


Figure 1. Graph for calculating growth rates.

We would like to explain the principle of graph construction.

The first principle: The signs "+" and "-" mean the weights of arcs +1 and -1.

The second principle: The presented graph displays the system of equations. The first equation (1) can be represented by the graph shown in Figure 2 (a). The second equation (2) is shown in the form of a graph shown in Figure 2 (b). If (1) and (2) are combined into a system, then we get the graph, which is shown in Figure 2 (c). This becomes possible due to the fact that the graphs in Figures 3, a and 3, b are weakly connected graphs, they have only one common arc - I_a. In addition, the calculations are recursive. To get the increment calculation expressions, we need to calculate the

weights of the paths from the vertex (the source) to the corresponding vertices of interest. For example, for the vertex ΔI_{yc}, we get the following expression:

$$\Delta I_{yc} = P\{I_a, I_b, I_c, 1\} + P\{I_a, I_b, -1\} = I_a I_b I_c - I_a I_b, \quad (4)$$

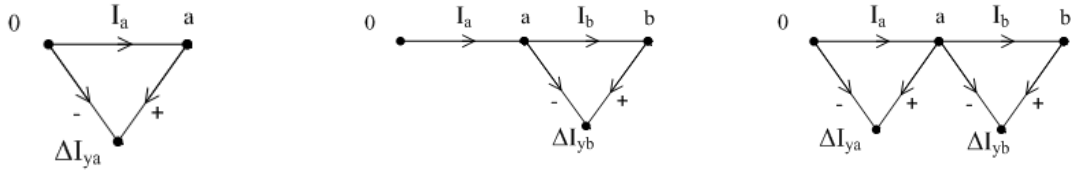
where $P\{\ \}$ are the weights indicated by the arcs of the ways.

Using the procedure described, we can construct a graph for any real number of factors.

The topological formula for calculating the cumulative growth rate is a generalization of the expression (4):

$$\Delta I_y = \sum_i P_i,$$

where P_i are the weights of all possible paths from the vertex of the source "0" to the vertex ΔI_y.



)a) (b) (c)

Figure 2. The order of constructing the graph of the system of equations.

It should be emphasized that the graph in Fig. 1 allows to consider the consequences of the order of the individual indices in the works (1) ... (3). This is achieved by appropriate permutations of arcs $I_a, I_b, \ldots, I_k$ in the graph under consideration. Graph theory allows in a compact form to display the area of change of the analyzed functional dependence.

When choosing the type of graphs, the following question is important: What means to display factors (indices)? Vertices or arcs? In the study of this, we found that preference should be given to the image of the indices in the form of arcs, the weights of which are equal to the values of the corresponding displayed indices. Consider the problem in a number of examples presented in Figures 3, 4(a), and 4(b).

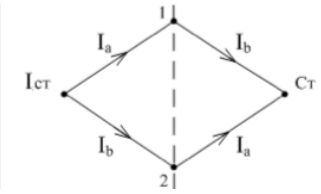


Figure 3. Graph of permutation variants for a two-factor model.

For two-factor models, the graph of solutions is presented in Fig. 3. The cross section of the graph, highlighted by a dot-dash line, shows that two solutions are possible. Each solution is determined by the path from the source (S) to the drain (D), and the order of the indices in the product is described by the order of the arcs in the paths. There are two pieces $I_a I_b$ и $I_b I_a$ here.

To clarify this, we consider the image of the graph in Fig. 3 using the notion of a subgraph (see Figure 4(a) and 4(b), where the subgraphs $G(I_a)$ and $G(I_b)$ – and this is an extreme case – in effect represent the arcs I_a и I_b. This confirms the generality of the decision:

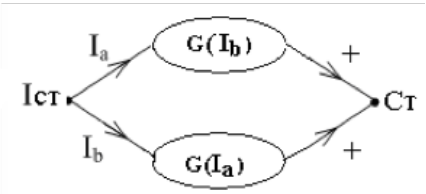


Figure 4a. Graph of permutation variants for the two-factor model.

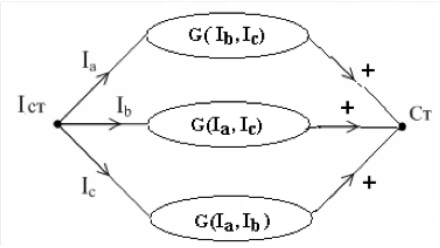


Figure 4b. Graph of permutation variants for the three-factor model.

Thus, by analogy, the properties of the procedure for calculating the influence of factors for any number of influencing quantities can be displayed as graphs.

5. Conclusion

Summarizing the above, we can say that when solving problems of factor analysis based on multiplicative index models, in order to take into account the non-transitivity of the applied calculation algorithms, we proposed to present the permutation variants in the form of directed graphs. This is an effective tool for programming and verifying analysis programs.

The use of the author's method of factor analysis will enable the most accessible and less laborious to draw a conclusion on the change in the socio-economic status of cross-border regions. Also, the use of the author's method of factor analysis will most accurately assess the influence of factors on the change in the studied indicator in the economic system and its trends to change.

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References

[1] Ivakhnenko A G, and Yurachkovsky Yu P 1987 *Modeling of complex systems based on experimental data* (Radio and communication)
[2] Iberla K 1980 *Factor analysis* (Statistics)
[3] Kizub N 1972 Questions of index methodology of factor analysis *Bulletin of Statistics* **11** pp 16-18
[4] Caves P 1990 *Index theory and practice of economic analysis* (Moscow: Finance and statistics)
[5] Ctoan A M 2016 Analysis of the constitutionality of economy on the basis of applying the methods of cluster and factor analysis *Journal of Statistical Issues* **4** pp 35-40
[6] Suvorov N V 2017 Balance and factor models as a tool for analysis and forecasting of the structure of the economy *Journal of Scientific Works: Institute of National Economic Forecasting RAS* 15 pp 50-75
[7] Suvorov N V, and Balashova E E 2009 Model tools for predictive and analytical studies of the dynamics of intersectoral relations of the domestic economy *Journal of Problems of Forecasting* **6** pp 16-33
[8] Suvorov N V 2015 Actual directions and problems of improving the model tools of macroeconomic analysis *Journal of Problems of Forecasting* **5** pp 25-39