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The Modeling of Mechanical Engineering Facilities to Reduce the Impact of External Vibration Effects

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***Abstract*—The article considers free and forced oscillations of a system with six degrees of freedom with symmetrical and asymmetrical mass-inertial characteristics. The conditions of single-frequency oscillations are obtained. Initial deviations and initial velocities must be interconnected through one of the amplitude distribution coefficients. The differential equations of free vibrations of the system in principal coordinates are six independent linear differential equations of second order. Their common solution is obtained. We examine the possibility of reduction in the quantity of degrees of freedom of the considered system from six to two ones. Passing from six degrees of freedom to two was substantiated with the aid of the numerical experiments. According to calculations, linear and angular deflections of the transportation vehicle, which has six and two degrees of freedom, differ not more than by 10%. This fact indicates that it is possible to replace a study of a system with six degrees of freedom with a system with two degrees of freedom. An example of the transport system, which has three degrees of freedom, is considered. The obtained differential equation is solved in the final form with the specific relationship of the parameters. Analogously, there are equations derived and solutions obtained for the systems, which have one, one-and-a-half and two degrees of freedom. The equations obtained allow you to define transformations of right parts depending on the considered case of external influences applied to the system and on the number of degrees of freedom.**

Keywords—mechanical engineering; vibration effects; modeling; forced oscillations; mass-inertial characteristics

I. INTRODUCTION

Modeling machine engineering objects usually begins with the solution of the simplest tasks: modeling systems with one degree of freedom [1-3]; systems with unilateral constraints; controlled mechanical systems; systems with additional constraints. A further development of research is a system with many degrees of freedom. The systems under consideration have a complex structure; therefore, for an analytical solution, it is necessary to reduce systems to the simplest form and to the normal coordinates.

In practice [1, 4-10], the determination of the normal coordinates is a task of the same order of difficulty as a complete study of the free oscillations of this system in generalized coordinates. However, the normal coordinates are convenient for studying forced oscillations of the system [11-14], since the motion of the system in this case is differential equations that do not depend on each other.

It is advisable when considering a system with many degrees of freedom to consider a single-frequency mode, i.e. oscillations of the system, in which all points of the system under consideration oscillate with the same frequency.

The possibility of transition to a simplified system with two degrees of freedom makes it possible to obtain analytical calculations to estimate forced vibrations of a transportation vehicle. And reducing the system of differential equations, which describe the movement of engineering objects, to a

single higher order equation sometimes makes it easier to obtain analytical solutions [13-16].

II. SYSTEMS WITH SIX DEGREES OF FREEDOM

We consider free and forced oscillations of a system with six degrees of freedom with symmetrical and asymmetrical mass-inertial characteristics. When setting up a mathematical model of a mechanical system, standard assumptions for rolling stock dynamics were adopted. The model includes a transportation vehicle body having elastic and dissipative elements with intermediate load (Fig. 1).

With the help of the Lagrange equation of the second kind, a system of differential equations for the transportation vehicle model was set up:

$$\begin{cases} m_K\ddot{z}_K+(\beta_{11}+\beta_{12})\dot{z}_K-\beta_{11}\dot{z}_{T1}-\beta_{12}\dot{z}_{T2}+ \\ +(\beta_{11}L_1-\beta_{12}L_2)\dot{\varphi}_K+(c_{11}+c_{12})z_K- \\ -c_{11}z_{T1}-c_{12}z_{T2}+(c_{11}L_1-c_{12}L_2)\varphi_K=0; \\ m_{T1}\ddot{z}_{T1}-\beta_{11}\dot{z}_K+(\beta_{11}+\beta_{21}+\beta_{22})\dot{z}_{T1}-\beta_{11}L_1\dot{\varphi}_K+ \\ +(\beta_{21}L_{21}-\beta_{22}L_{22})\dot{\varphi}_{T1}-c_{11}z_K+(c_{11}+c_{21}+c_{22})z_{T1}- \\ c_{11}L_1\varphi_K+(c_{21}L_{21}-c_{22}L_{22})\varphi_{T1}=0; \\ m_{T2}\ddot{z}_{T2}-\beta_{12}\dot{z}_K+(\beta_{12}+\beta_{31}+\beta_{32})\dot{z}_{T2}+\beta_{12}L_2\dot{\varphi}_K+ \\ +(\beta_{31}L_{31}-\beta_{32}L_{32})\dot{\varphi}_{T2}-c_{12}z_K+(c_{12}+c_{31}+c_{32})z_{T2}+ \\ c_{12}L_2\varphi_K+(c_{31}L_{31}-c_{32}L_{32})\varphi_{T2}=0; \\ I_K\ddot{\varphi}_K+(\beta_{11}L_1-\beta_{12}L_2)\dot{z}_K-\beta_{11}L_1\dot{z}_{T1}+\beta_{12}L_2\dot{z}_{T2}+ \\ +(\beta_{11}L_1^2+\beta_{12}L_2^2)\dot{\varphi}_K+(c_{11}L_1-c_{12}L_2)z_K- \\ -c_{11}L_1z_{T1}+c_{12}L_2z_{T2}+(c_{11}L_1^2+c_{12}L_2^2)\varphi_K=0; \\ I_{T1}\ddot{\varphi}_{T1}+(\beta_{21}L_{21}-\beta_{22}L_{22})\dot{z}_{T1}+(\beta_{21}L_{21}^2+\beta_{22}L_{22}^2)\dot{\varphi}_{T1}+ \\ +(c_{21}L_{21}-c_{22}L_{22})z_{T1}+(c_{21}L_{21}^2+c_{22}L_{22}^2)\varphi_{T1}=0; \\ I_{T2}\ddot{\varphi}_{T2}+(\beta_{31}L_{31}-\beta_{32}L_{32})\dot{z}_{T2}+(\beta_{31}L_{31}^2+\beta_{32}L_{32}^2)\dot{\varphi}_{T2}+ \\ +(c_{31}L_{31}-c_{32}L_{32})z_{T2}+(c_{31}L_{31}^2+c_{32}L_{32}^2)\varphi_{T2}=0, \end{cases} \quad (1)$$

where the following generalized coordinates are introduced: Z_K, Z_{T1}, Z_{T2} are current vertical displacements of the transportation vehicle's center of gravity, first and second intermediate loads, φ_K, φ_{T1}, φ_{T2} are current angular displacements of the transportation vehicle's rotation and intermediate masses relative to their main inertia axes perpendicular to the longitudinal plane in which movement is examined.

It is quite difficult to obtain an analytical solution of equations (1). Therefore, we use the mathematical software package MathCAD to study them. On the resulting model in the MATHCAD package, the effect of the suspension parameters on the natural oscillations was studied. As a result of studying oscillations of a system with symmetrical and asymmetrical mass-inertial characteristics [1], it was concluded that the mass-inertial parameters of the system significantly infuence the dynamic behavior of the transportation vehicle. The obtained vibration frequencies of the conservative system can be taken as the natural vibration frequencies of real transportation vehicles, including dampers. At the same time, even though resistances to oscillations are not very high, they strongly affect the amplitudes.

The differential equations of free vibrations of the system in normal coordinates are six independent linear differential equations of second order. Their common solution is obtained.

The conditions of single-frequency oscillations are obtained. Initial deviations and initial velocities must be interconnected through one of the amplitude distribution coefficients.

The method of obtaining the main coordinates is considered, which allows considering each of the obtained differential equations separately.

The general solution of the system of differential equations (2) is obtained.

$$\begin{cases} z_K(t)=q_1+q_2+q_3+q_4+q_5+q_6; \\ z_{T1}(t)=\mu_{11}q_1+\mu_{12}q_2+\mu_{13}q_3+\mu_{14}q_4+\mu_{15}q_5+\mu_{16}q_6; \\ z_{T2}(t)=\mu_{21}q_1+\mu_{22}q_2+\mu_{23}q_3+\mu_{24}q_4+\mu_{25}q_5+\mu_{26}q_6; \\ \varphi_K(t)=\mu_{31}q_1+\mu_{32}q_2+\mu_{33}q_3+\mu_{34}q_4+\mu_{35}q_5+\mu_{36}q_6; \\ \varphi_{T1}(t)=\mu_{41}q_1+\mu_{42}q_2+\mu_{43}q_3+\mu_{44}q_4+\mu_{45}q_5+\mu_{46}q_6; \\ \varphi_{T2}(t)=\mu_{51}q_1+\mu_{52}q_2+\mu_{53}q_3+\mu_{54}q_4+\mu_{55}q_5+\mu_{56}q_6, \end{cases} \quad (2)$$

containing arbitrary constants $q_1=q_{10}$; $\dot{q}_1=\dot{q}_{10}$; $q_2=q_{20}$; $\dot{q}_2=\dot{q}_{20}$; $q_3=q_{30}$; $\dot{q}_3=\dot{q}_{30}$, $q_4=q_{40}$; $\dot{q}_4=\dot{q}_{40}$; $q_5=q_{50}$; $\dot{q}_5=\dot{q}_{50}$; $q_6=q_{60}$; $\dot{q}_6=\dot{q}_{60}$, which must be determined by the initial values of the generalized coordinates $z_K=z_{K0}$; $z_{T1}=z_{T10}$; $z_{T2}=z_{T20}$, $\varphi_K=\varphi_{K0}$, $\varphi_{T1}=\varphi_{T10}$, $\varphi_{T2}=\varphi_{T20}$ and generalized speeds $\dot{z}_K=\dot{z}_{K0}$; $\dot{z}_{T1}=\dot{z}_{T10}$; $\dot{z}_{T2}=\dot{z}_{T20}$, $\dot{\varphi}_K=\dot{\varphi}_{K0}$, $\dot{\varphi}_{T1}=\dot{\varphi}_{T10}$, $\dot{\varphi}_{T2}=\dot{\varphi}_{T20}$.

III. TRANSITION TO A SYSTEM WITH TWO DEGREES OF FREEDOM

The above is a system with six degrees of freedom. Let us examine the possibility of reduction in the quantity of degrees of freedom of the considered system from six to two ones [2].

Passing from six degrees of freedom to two was substantiated with the aid of the numerical experiments. Suppose that the transport system under investigation is asymmetric with respect to the main central axes of inertia. As a result of the calculations, we obtained that the period, amplitude and frequency of vertical oscillations of the object are $T_1\approx 0,458$ (sec), $\lambda_1\approx 1,47\cdot 10^{-2}$ (m) and $n_1\approx 2,183$ (Hz), respectively. The characteristics of natural oscillations on the basis of the solution obtained are the following: $T_2\approx 0,4585$ (sec), $\lambda_2\approx 0,25^0$, $n_2\approx 2,181$ (Hz).

Since the intermediate masses m_{T1} and m_{T2} are small

compared to the mass of the transportation vehicle m_K, we study a simplified model of the system.

We consider a transportation vehicle as a solid supported at two points on successively set springs with stiffnesses c_{11}, c_{12}, c_{21}, c_{22}, c_{31}, c_{32} (fig. 2).

The reduced stiffness of these springs is (3)

$$\tilde{c}_{11} = \frac{c_{11}(c_{21}+c_{22})}{c_{11}+c_{21}+c_{22}}, \quad \tilde{c}_{12} = \frac{c_{12}(c_{31}+c_{32})}{c_{12}+c_{31}+c_{32}} \tag{3}$$

Under the accepted assumptions, the system configuration can be determined using two independent variables: z_K is the vertical movement of the center of gravity and φ_K is the angular movement of the transportation vehicle's rotation.

$$\begin{cases} m_K\ddot{z}_K + (\tilde{c}_{11}+\tilde{c}_{12})z_K + (\tilde{c}_{11}L_1 - \tilde{c}_{12}L_2)\varphi_K = 0; \\ I_K\ddot{\varphi}_K + (\tilde{c}_{11}L_1^2 + \tilde{c}_{12}L_2^2)\varphi_K + (\tilde{c}_{11}L_1 - \tilde{c}_{12}L_2)z_K = 0. \end{cases} \tag{4}$$

As a result of the calculations, the following characteristics of the natural oscillations of the intermediate load were obtained: the period, amplitude and frequency of oscillations are $T_1 \approx 0,459$ (sec), $\lambda_1 \approx 1,47 \cdot 10^{-2}$ (m) and $n_1 \approx 2,179$ (Hz), respectively; the period, amplitude and frequency of the vehicle are as follows: $T_2 \approx 0,459$ (sec), $\lambda_2 \approx 0,25^0$, $n_2 \approx 2,179$ (Hz) (fig. 3-4).

In fig. 3 and 4 the solid line indicates the movement of the transport system with six degrees of freedom, and the dotted line - with two ones.

The deviation of the linear and angular coordinates of a system with six degrees of freedom from a system with two degrees of freedom was estimated according to three norms [2]:

$\|x\|_1 = \max_i |x_i|$ is the first norm, $\|x\|_2 = \sum_{i=1}^{n} |x_i|$ is the second norm, $\|x\|_3 = \sqrt{\sum_{i=1}^{n} x_i^2}$ is the third norm.

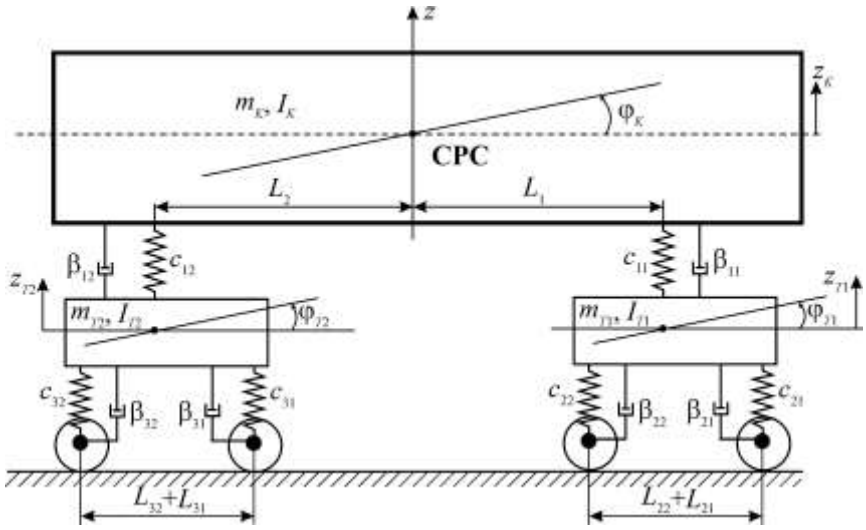

Fig. 1. The computational scheme of the transportation vehicle oscillations

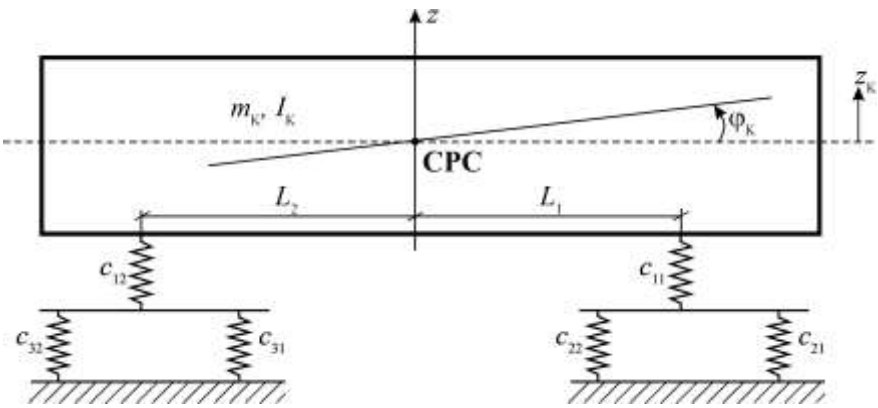

Fig. 2. The computational scheme of oscillations of a transportation vehicle with two degrees of freedom

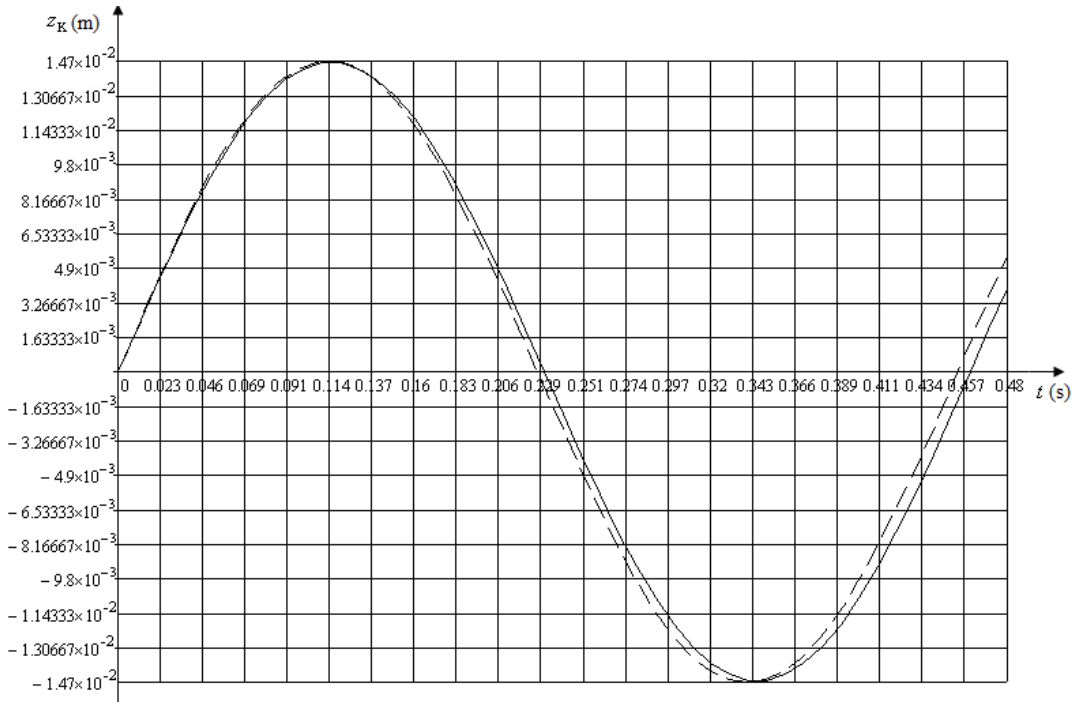

Fig. 3. Vertical movements of the transport system with six and two degrees of freedom

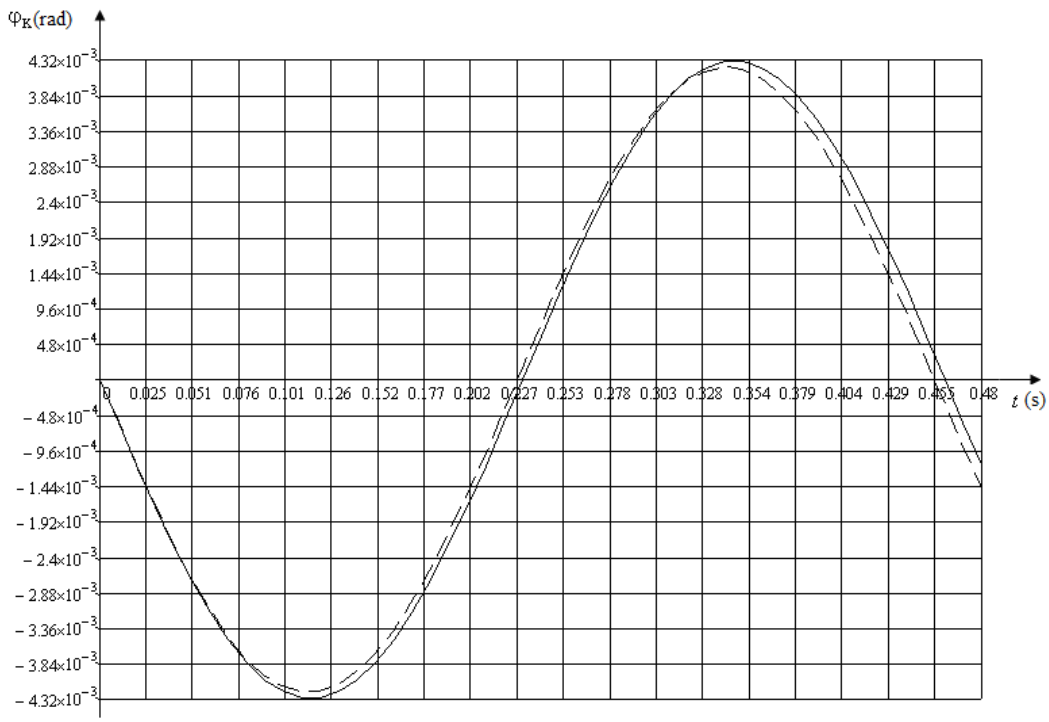

Fig. 4. Angular movements of the transport system with six and two degrees of freedom

In the second and third norms *n* are equal to the number of points calculated on the period of the function.

Comparison of the results of calculations for a system with two and six degrees of freedom showed that the linear deviation calculated on the first norm is 4.6 %, and the angular one is 0.6 %. On the second norm, linear and angular deviations are, respectively, 1.4 % and 4.3 %. On the third norm they are 1.3 % and 3.8 %.

For a simplified system with two degrees of freedom, the differential equations describing the forced oscillations of the system (Fig. 5) take the form (5):

$$\begin{cases} m_K \ddot{z}_K + \left(\tilde{c}_{11} + \tilde{c}_{12}\right) z_K + \left(\tilde{c}_{11} L_1 - \tilde{c}_{12} L_2\right) \varphi_K = F_1; \\ I_K \ddot{\varphi}_K + \left(\tilde{c}_{11} L_1^2 + \tilde{c}_{12} L_2^2\right) \varphi_K + \left(\tilde{c}_{11} L_1 - \tilde{c}_{12} L_2\right) z_K = F_2, \end{cases} \quad (5)$$

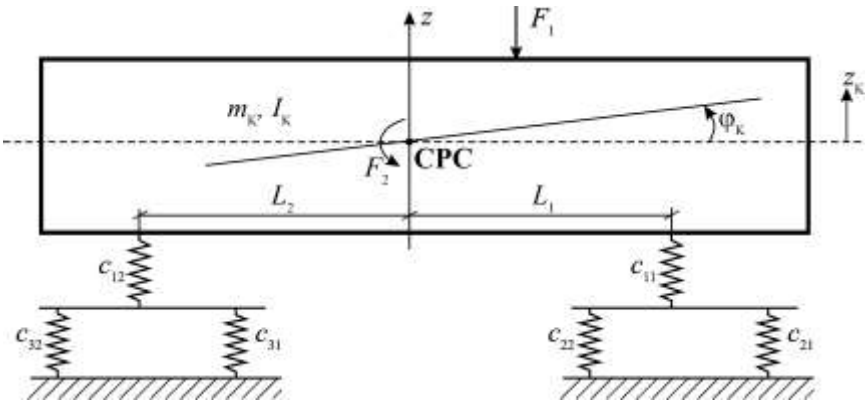

Fig. 5. The simplified dynamic system with two degrees of freedom

Here $F_1 = H_1 \sin pt$, $F_2 = H_2 \sin pt$, and, accordingly, the amplitude frequency characteristics response for the linear amplitude of the transportation vehicle oscillations A_1 will be determined by the expression.

Solving this equation, we get:

$$A_1(p) = \left| \frac{H_1\left(C_{22} - A_{22}p^2\right) - H_2 C_{12}}{\left(C_{11} - A_{11}p^2\right)\left(C_{22} - A_{22}p^2\right) - C_{12}} \right| \tag{6}$$

for the angular amplitude of transportation vehicle vibrations A_2 is determined by the expression:

$$A_2(p) = \left| \frac{H_2\left(C_{11} - A_{11}p^2\right) - H_1 C_{21}}{\left(C_{11} - A_{11}p^2\right)\left(C_{22} - A_{22}p^2\right) - C_{12}} \right| \tag{7}$$

where

$$\begin{gathered} A_{11} = m_K,\ C_{11} = \tilde{c}_{11} + \tilde{c}_{12}\ , \\ C_{12} = C_{21} = \tilde{c}_{11}L_1 - \tilde{c}_{12}L_2,\ A_{22} = I_K\ ,\ C_{22} = \tilde{c}_{11}L_1^2 + \tilde{c}_{12}L_2^2 \end{gathered} \tag{8}$$

According to calculations linear and angular deflections of the transportation vehicle, which has six and two degrees of freedom, they differ not more than by 10%. This fact indicates that it is possible to replace a study of a system with six degrees of freedom with a system with two degrees of freedom.

In addition, a compelling force that influences the system and has a periodic (and not harmonic) nature is considered [1, 3].

The problem of finding the steady-state forced oscillations caused by a periodic piecewise constant driving force is considered.

$$Q(t+T) = Q(t) \tag{9}$$

where T is the period of change of force (Fig. 6).

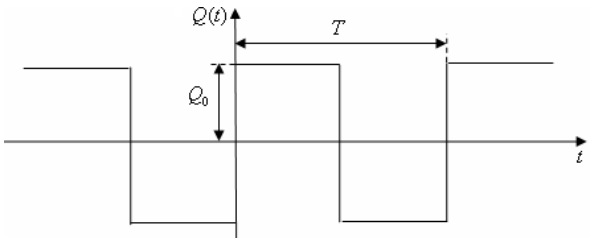

Fig. 6. The form of applied driving force

$$Q(t) = Q_0 \text{ with } 0 < t < \frac{T}{2},\ Q(t) = -Q_0 \text{ with } \frac{T}{2} < t < T\ .$$

We obtain the system of differential equations:

$$\begin{cases} A_{11}\ddot{z}_K + C_{11}z_K + C_{12}\varphi_K = Q(t); \\ A_{22}\ddot{\varphi}_K + C_{21}z_K + C_{22}\varphi_K = 0, \end{cases} \tag{10}$$

where the coefficients are determined by the formula (8).

The normal coordinates of the system are obtained.

As a result of the research, differential equations of motion of the system have been set up and an analytical solution has been obtained for the case of application of an external piecewise constant driving force.

IV. Reduction of the System of Differential Equations of a Mechanical Engineering Facility to the Simplest Form

The above mechanical engineering facilities are described by a system of differential equations. We consider an example of a transport system that has three degrees of freedom (Fig. 7).

For the generalized coordinates we take vertical displacement z and angular displacements φ and ψ. We assume that the stiffnesses $\tilde{c}_{11}$, $\tilde{c}_{12}$, $\tilde{c}_{21}$ and $\tilde{c}_{22}$ are reduced and are calculated using c_{11}, c_{12}, c_{21}, c_{22}, c_{31}, c_{32}, c_{33} and c_{34}, respectively.

The system of equations (11) describes the motion of the considered transport system.

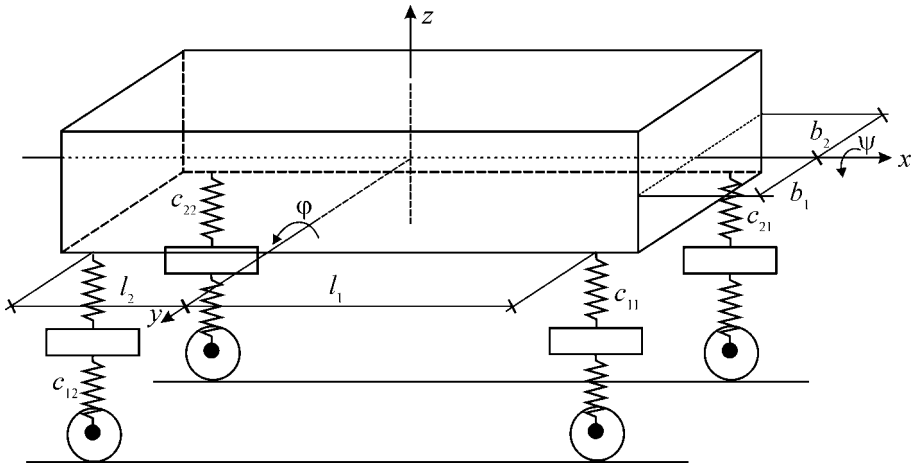

Fig. 7. The spatial scheme of the transportation vehicle

For the generalized coordinates we take vertical displacement z and angular displacements φ and ψ. We assume that the stiffnesses $\tilde{c}_{11}$, $\tilde{c}_{12}$, $\tilde{c}_{21}$ and $\tilde{c}_{22}$ are reduced and are calculated using c_{11}, c_{12}, c_{21}, c_{22}, c_{31}, c_{32}, c_{33} and c_{34}, respectively.

The system of equations (11) describes the motion of the considered transport system.

$$\begin{cases} m\ddot{z} + \left(\tilde{c}_{11} + \tilde{c}_{21} + \tilde{c}_{22} + \tilde{c}_{12}\right) z + \\ \quad + \left(\tilde{c}_{11}L_1 + \tilde{c}_{21}L_1 - \tilde{c}_{22}L_2 - \tilde{c}_{12}L_2\right)\phi + \\ \quad + \left(\tilde{c}_{21}b_2 + \tilde{c}_{22}b_2 - \tilde{c}_{11}b_1 - \tilde{c}_{12}b_1\right)\psi = f_1; \\ I_y\ddot{\phi} + \left(\tilde{c}_{11}L_1 + \tilde{c}_{21}L_1 - \tilde{c}_{22}L_2 - \tilde{c}_{12}L_2\right) z + \\ \quad + \left(\tilde{c}_{11}L_1^2 + \tilde{c}_{21}L_1^2 + \tilde{c}_{22}L_2^2 + \tilde{c}_{12}L_2^2\right)\phi + \\ \quad + \left(\tilde{c}_{21}b_2L_1 - \tilde{c}_{11}b_1L_1 - \tilde{c}_{22}b_2L_2 + \tilde{c}_{12}b_1L_2\right)\psi = f_2; \\ I_x\ddot{\psi} + \left(\tilde{c}_{21}b_2 + \tilde{c}_{22}b_2 - \tilde{c}_{11}b_1 - \tilde{c}_{12}b_1\right) z + \\ \quad + \left(\tilde{c}_{21}b_2L_1 - \tilde{c}_{11}b_1L_1 - \tilde{c}_{22}b_2L_2 + \tilde{c}_{12}b_1L_2\right)\phi + \\ \quad + \left(\tilde{c}_{11}b_1^2 + \tilde{c}_{21}b_2^2 + \tilde{c}_{22}b_2^2 + \tilde{c}_{12}b_1^2\right)\psi = f_3. \end{cases} \tag{11}$$

After simplification, the system (11) takes the form:

$$\begin{cases} \ddot{z} + a_1 z + a_2\phi + a_3\psi = f_1; \\ \ddot{\phi} + s_1 z + s_2\phi + s_3\psi = f_2; \\ \ddot{\psi} + d_1 z + d_2\phi + d_3\psi = f_3. \end{cases} \tag{12}$$

Having done the obvious operations on the equations of system (12), we obtain:

$$E_1\psi^{\mathrm{VI}} + E_2\psi^{\mathrm{IV}} + E_3\ddot{\psi} + E_4\psi = F_5 \tag{13}$$

where the coefficients E_1, E_2, E_3, E_4 and F_5 are expressed in terms of the parameters of the original system of differential equations.

The obtained differential equation (13) is solved in the final form with a certain ratio of parameters.

Analogously, there are equations derived and solutions obtained for the systems, which have one, one-and-a-half and two degrees of freedom. The equations obtained allow you to define transformations of right parts depending on the considered case of external influences applied to the system and on the number of degrees of freedom.

V. EXPERIMENTAL RESEARCH

Conducting experiments is an important part of scientific research. The main purpose of the experimental study is to experimentally confirm the dynamic properties obtained in the modeling of dynamic properties.

Processing and analysis of the obtained experimental results when compared with analytical calculations and modeling performed in MathCad confirmed the hypotheses that the parameters of oscillatory processes depend strongly on external perturbing influences and symmetry breaking.

The technique of conducting the experiment was developed. It allowed drawing conclusions about the validity of the proposed mathematical models of mechanical engineering facilities. Antiresonance modes recommended by analytical studies were confirmed in experimental studies.

Experimental studies were carried out on a construction vibration table with an adjustable rotor speed. Various models of mechanical engineering facilities were installed on the stand. To obtain data on the oscillatory processes of these objects, the vibration measuring apparatus TV2 Series Vibration Pen, the vibration inverter AP2019 and the spectrum analyzer A17-U2 were used.

The discrepancy between analytical calculations, mathematical modeling and experiments does not exceed 12%.

VI. CONCLUSION

Applying the concept of mathematical modeling to the transport system, we note that the model reflects the basic properties of the object under study to the extent necessary to assess its dynamic qualities. Free and forced oscillations of the system are considered for a given initial deviation from the equilibrium position. In practice, the problems like that are solved in the design of transportation vehicles. By changing the speed of the transport system, it is possible to determine the critical speeds and the corresponding maximum displacements in the suspension system. By changing the parameters of suspension, one can outline ways to reduce negative phenomena.

The obtained results of the dependence of the lateral displacement and the angle of the transportation vehicle, and of the first and second masses on the time indicate that the natural oscillations of the masses of transportation vehicle and the masses, which are the sum of two harmonic oscillations, are non-damping [15] since while solving the problem we disregarded the action of the inelastic resisting forces of the dampers. Since the periods of oscillations of the transport vary little in $T_1 \approx 0.7$ (sec), $T_2 \approx 0.66$ (sec), the oscillations z_{T1} and z_{T2} have the form of beatings. In case of symmetry breaking, the center of masses of the transportation vehicle has shifted by 7%, the movement has become a beating. As a result of studying oscillations of a system with symmetrical and asymmetrical mass-inertial characteristics [2], it was found that the amplitudes of oscillations with asymmetrical characteristics decreased. Thus, the mass-inertial parameters of the system significantly influence the dynamic behavior of the transport system.

Studies of systems with six degrees of freedom and systems with two degrees of freedom showed that they do not differ by more than 10%, which means that a system with two degrees of freedom describes the movement of a mechanical system with six degrees of freedom. A method is presented that allows one to obtain a closed solution for steady-state forced oscillations. The simplified model is further developed and supplemented, and the simulation results serve as the initial estimate for more complex models.

Processing and analysis of the obtained experimental results when compared with analytical calculations and modeling performed in MathCad confirmed the hypotheses that the parameters of oscillatory processes depend strongly on external perturbing influences and symmetry breaking. Antiresonance modes recommended by analytical studies were confirmed in experimental studies.

References

[1] A.I. Karlina, V.E. Gozbenko, “Modeling of objects of mechanical engineering to reduce the influence of external vibration influence”, Bulletin Irkutsk State Technical University vol. 20, N 10 (117), pp. 35-47, 2016.

[2] V.E. Gozbenko, S.K. Kargapoltsev, Yu.I. Karlina, A.I. Karlina, A.I. Artyunin, “Automation of the motion process of the materials which are subject to the incoming quality control”, Advances and Applications in Discrete Mathematics, vol. 19, N 3, pp. 289-297, 2018.

[3] T. Tsisovski Improving the control systems of oscillations of the rolling stock of railways: a dissertation for the degree of Doctor of Technical Sciences: 05.22.07.

[4] A.E. Balanovskiy, “Visualization of the process of metal heating and melting in the anode zone in an arc discharge with a tungsten electrode”, High Temperature, vol. 54, N 5, pp. 627-631, 2017.

[5] A.E. Balanovskiy, “Digital visualisation of the process of heating and melting of metal in arc discharge with a non-consumable electrode”, Welding International, vol. 31, N **6,** pp. 467-476, 2017.

[6] A.E. Balanovskii, V.V. Huy, “Plasma surface cementation using graphite coating”, Letters on Materials, vol. 7(2), pp. 175-179, 2017.

[7] A.E. Balanovskii, V.V. Huy, “Estimation of wear resistance of plasma-carburized steel surface in conditions of abrasive wear”, Journal of Friction and Wear, vol. 39(4), pp. 311-318,2018.

[8] S.I. Medvedev, A.E. Nezhivlyak, M.V. Grechneva, A.E. Balanovsky, V.L. Ivakin, “Optimization of plasma hardening conditions of the side surface of rails in pur-1 experimental equipment”, Welding International, vol. 29(8), pp. 643-649, 2015.

[9] A.E. Balanovskii, “Study of the attachment stage of a welding arc discharge of direct-current straight polarity on aluminum surface”, High Temperature, vol. 56(4), pp. 486-495, 2018.

[10] V.A. Nikolaev, 2003 Development of methods for analytical design of quasi-invariant systems of spring suspension of railway carriages, 2003, p. 371.

[11] M.F. Verigo, Dynamics of cars. Lecure notes, Moscow: Transport Publ., 1988, p. 174.

[12] S.V. Vershinsky, V.N. Danilov, V.D. Khusidov, Dynamics of the rail car, Moscow: Transport Publ., 1991, p. 360.

[13] V.K. Garg, R.V. Dukkipati, Dynamics of the rolling stock. Transl. from English, Moscow: Transport Publ., 1988, p. 391.

[14] Ya.G. Panovko, Introduction to the theory of mechanical oscillations, Moscow: Nauka, 1989, p. 252.

[15] K.S. Yolkin, D.K. Yolkin, M.G. Shtayger, A.D. Kolosov, N.A. Ivanov, “Technologies, which allow to reduce an impact of metal silicon production on the environment”, IOP Conference Series: Materials Science and Engineering, vol. 411, pp. 012028, 2018.

[16] A.G. Savitsky, Integrated system for automated control of the sorting process: a Ph.D, 2005.