

# Method for a Multi-Objective Selection of a Fluid Power Drive Based on Analysis of Its Design, Operational and Dynamic Characteristics

P. Kazakov

Information Technologies Department  
Bryansk State Technical University  
Bryansk, Russia  
pvk\_mail@list.ru

V. Kazakov

Power engineering and Electronics Department  
Bryansk State Technical University  
Bryansk, Russia

**Abstract**— The approach for a multi-objective selection of fluid power drive based on information of its design, operational and dynamic characteristics is proposed. It uses the optimal Pareto variants of a fluid power drive found at the stage of multi-objective optimization. The operational and dynamic characteristics are evaluated for the optimal fluid power drive selection related of the current problem of design.

**Keywords**— fluid power drive, multi-objective optimization, mathematical modeling, dynamic characteristics

## I. INTRODUCTION

The operational efficiency of a fluid power drive (FPD) depends much upon results of its design combining the selection, calculation, optimization of design and operational parameters of a unit. The existing conventionally used methods of FPD design [1, 2] are mainly focused on the calculation of the geometric parameters values of FPD elements for the follow-up selection of equipment from the range of production-run available standard sizes [1]. The advantage of such approach is its commonality, the opportunity to implement in special libraries of structural elements form in different CAD systems. However, there is a number of problems where the formal FPD selection from the available model bank may be insufficiently because of the demand to design systems with high requirements for the technical capabilities of FPD in the direct operation (speed, power density, transient process time and attainment the values of the working parameters) [3; 4]. In this case, the design process of FPD should be considered as a research problem has been formalized in the form of an optimization procedure. It aims at solving the FPD construction with the values of the parameters coming up to the performance specification and having optimal operational, dynamic behaviors. In the article, the process of fluid power drive design is considered as a multistage method of its multi-objective selection. There are three stages in the method: the optimal design of FPD according to several quality criteria, evaluating dynamics of the found FPD variants taking into account their possible operation in changing environment, determining the FPD variant with the best compliance of

characteristics to a performance specification and assigned by the expert selection criteria.

## II. MATHEMATICAL MODEL OF THE MULTI-OBJECTIVE OPTIMIZATION PROBLEM OF A FLUID POWER DRIVE

The process of FPD optimal design is considered as a multi-objective optimization problem (MOP) [5], its solving let to obtain the set of Pareto-optimal variants of FPD with a various combination of values of parameters. These are a diameter and stroke of the piston of hydraulic actuator, the sizes of a effective forcing on the rod and normalized to it weight load, the traverse speed of the output segment in a working stroke mode, the sizes of plunger and hydrodistributor windows, etc.

To formalize the criteria of the FPD MOP and the mathematical formulation of dynamic properties of FPD elements, the shown in Fig. 1 FPD design model is used. Also the differential equation of forces balance having an influence upon a hydraulic cylinder, the consumptions equation, the motion equation of the hydraulic cylinder model with linearized characteristics were used [2].

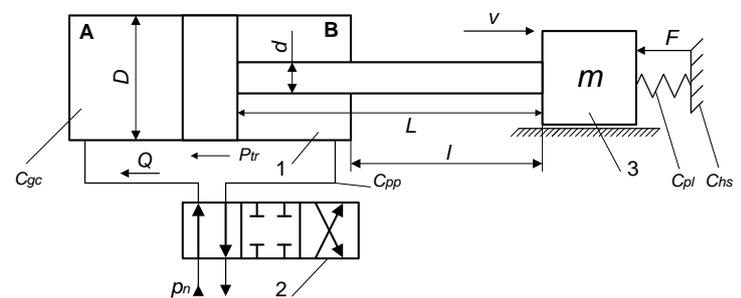


Fig. 1. The design scheme for the fluid power drive MOP: 1 - cylinder, 2 - valve, 3 - added mass

As per chosen scheme the FPD has the hydraulic cylinder with a one-way rod which is used in the most power plants of various machines (asphalt pavers, graders, forklift trucks, excavators, etc.). The selected optimality criteria of FPD reflect upon the most important operating characteristics of

such machines, namely a dynamic stability and power. Quantitatively these characteristics can be expressed through amounts of the damping value. That is inversely to the transient process time (the damping coefficient  $f_{\xi}$ ) [2], and the power density coefficient  $f_{\bar{N}}$  correspondingly. Optimization of these criteria lets to determine the best values of FPD design parameters.

$$f_{\bar{N}} = F \frac{(Q + K_{Qp} \frac{2}{3} p_n)}{m \sqrt{\frac{p_n}{\rho}} S_A} \sqrt{\frac{2 \left( p_n - \frac{F}{S_B} \right)}{\rho}} \rightarrow \max$$

$$f_{\xi} = \frac{K_{Qp}}{2S_A^2} \sqrt{\frac{m}{\frac{1}{C_{gc}} + \frac{1}{C_{hs}} + \frac{1}{C_{pp}}}} \rightarrow \max$$

where  $K_{Qp} > 0$  - the slope coefficient of the throttling characteristic of a hydraulic valve, determines its size and area of the working window ( $w$ ),  $m^4s/kg$ ;

$$\omega = \frac{Q + K_{Qp} \frac{2}{3} p_n}{\mu \sqrt{\frac{p_n}{\rho}}} - \text{the working window area of a hydraulic valve, } m^2;$$

$\mu = 0.7$  - the discharge coefficient of the working window hydraulic valve [2];

$$s_A = \frac{\pi D^2}{4} - \text{the effective area of piston, } m^2;$$

$$D = \sqrt{\frac{4F}{\pi p_n \eta_{mc} \eta_{gc}}} - \text{the piston diameter, } m;$$

$$s_B = \frac{\pi}{4} (D^2 - d^2) - \text{the effective area of piston rod side, } m^2;$$

$$Q = \frac{\pi D^2}{4} \frac{v}{\eta_{vc}} - \text{a working fluid flow in the discharge line, } m^3/s;$$

$v$  - the working stroke speed of piston rod,  $m/s$ ;

$F$  - the piston rod stress of hydraulic cylinder,  $N$ ;

$p_n$  - the nominal pressure in a discharge line,  $Pa$ ;

$\eta_{mc} = 0.93...0.97$  - the mechanical efficiency coefficient of hydraulic cylinder, taking into account a mechanical friction loss;

$\eta_{gc} = 0.85...0.90$  - the hydraulic efficiency coefficient, taking into account the energy loss in the hydraulic system of hydraulic actuator;

$\eta_{vc} = 0.96...0.98$  - the cylinder volumetric efficiency coefficient of hydraulic cylinder, taking into account the loss of energy from the working fluid leaks;

$\rho$  - the density of working fluid,  $kg/m^3$ ;

$m$  - inertial mass, including added elements of the construction and the volume of working fluid,  $kg$ ;

$L$  - the maximum piston rod stroke,  $m$ ;

$l$  - the length of piston rod working stroke,  $m$ ;

$C_{gc}, C_{hs}, C_{pp}$  - the coefficients of stiffness of the hydraulic cylinder, its holding support and associated piping correspondingly,  $N/m$ .

In addition to the listed expressions, the mathematical model of FPD MOP contains the set of constraints, taking into account the regulation of the values of a number of parameters according to GOST and fulfillment of strength condition in compression and the stability evaluating of the rod of hydraulic cylinder [1] and the result of verification of the real values of realizing efforts on rod in fact ( $F_r$ ), the piston rod movement speed ( $v_r$ ). All variables and constants of the mathematical model can be divided into input  $\{F, v, l, m, \eta_{mc}, \eta_{gc}, \eta_{vc}\}$ , output  $\{D, d, p_n, L, w, F_r, v_r\}$  and varying at the stage of optimization of FPD  $\{d, p_n, K_{Qp}, L, m\}$ .

The selected optimality criteria of FPD are conflicting, that is improvement of indicators of FPD on one criterion results in their decrease on another. Therefore, the expert is given for the analysis the set of Pareto-optimal hydraulic drives constructions with a differing combination of the values of the specific power and damping criteria. This lets explore all the possible variants of the FPD system, most accurately to determine the limits of their capabilities, to obtain data for a decision making on the appropriateness of further efforts to improve some FPD quality indices. Later on the expert, focusing on the specific operating conditions of FPD, its design and dynamic characteristics, should choose one - the best variant FPD.

### III. MATHEMATICAL MODELING OF SOME DYNAMIC CHARACTERISTICS OF A FLUID POWER DRIVE

To reduce the number of Pareto-optimal variants of FPD, it is important to take into account the possible dynamic characteristics of FPD in addition to information about their already known design and weight-size parameters. As ones can be used indicators of the transient process dynamics of the executive organ of FPD - the piston of a power cylinder.

In a general form, the transient process effect in the dynamics of oscillatory and aperiodic second-order links and more complex schemes of the fluid power drive with throttle control are studied, described and presented as a system of differential equations in monograph [2]. In this work added to the form of the generalized differential equation [2] the differential equations of hydraulic cylinder piston motion, balance of fluid consumptions and operating forces, working

characteristics of throttle hydrodistributor were taken as a basis for the mathematical modeling of dynamics of investigated processes:

$$\frac{mV_0}{E_{gc}S_A K_{Qx}} \frac{d^2v}{dt^2} + \frac{S_A}{K_{Qx}} \left( \frac{mK_{Qp}}{S_A^2} + \frac{k_{tr}V_0}{E_{gc}S_A^2} \right) \frac{dv}{dt} + \frac{S_A}{K_{Qx}} \left( 1 + \frac{V_0 C_{pl}}{E_{gc}S_A^2} + \frac{k_{tr}K_{Qp}}{S_A^2} \right) v(t) + \frac{K_{Qp}C_{pl}}{K_{Qx}S_A} v(t) t = X_{vs},$$

where  $V_0$  – the volume of effective cavity of a hydraulic cylinder, m<sup>3</sup>;  $E_{gc}$  – the reduced coefficient of elasticity of a hydraulic cylinder, Pa;  $k_{tr}$  – a coefficient of viscosity friction, m<sup>2</sup>/c;  $C_{pl}$  – the hardness of a hydraulic cylinder piston load, H/m;  $K_{Qx}$  – the consumption-falling characteristic parameter of throttle hydrodistributor, m<sup>2</sup>/c;  $X_{vs}$  – the valve spindle motion of a throttle hydrodistributor, m.

According to the practice of studying non-stationary, transient and unsteady processes in plants, machines and their aggregates, the main characteristics of the dynamic behaviors of the equipment units are associated with the laws of speed change (acceleration) of a final control element movement [2]. Therefore, the goal function of the exploration of the FPD dynamic characteristics in the transition period of its operation the function  $v(t)$  of changing the speed of piston working stroke have been adopted.

To be able to research and analyze the behavior of the goal function of the velocity in the transition period, the original differential equation (1) was converted to a general form. It is convenient for integration with respect to function  $v(t)$  and taking into account the specific conditions and operation parameters of the power-output segment of the hydraulic drive of the further considered types of machines. In addition, the following assumptions and boundary conditions were accepted: the leakage and flow of the working fluid are not taken into account; the compressibility of the fluid and dissolved gases in it, the deformation of the pipe walls and hydraulic cylinder are taken into account by the  $E_{gc}$  modulus of elasticity; the calculation of viscous friction forces is made according to the theory of hydrodynamic resistance in the boundary layer of the working fluid; load resistance is simulated by the corresponding value of the stiffness coefficient  $C_{pl}$ ; the valve spindle motion  $X_{vs}$  corresponds to the found optimal values of the opening area of the hydrodistributor window  $\omega$  and the parameter  $K_{Qp}$ ; at the initial time (at  $t = 0$ ), the piston speed is assumed to be zero.

Thus, the dimensionless form of the differential equation of the FPD dynamics was obtained; solving of it allows obtaining additional information about the transient process time of the explored FPD variant, the power reserve and the time to achieve the required speed of a working stroke:

$$K_1 \frac{d^2v}{dt^2} + K_2 \frac{dv}{dt} + K_3 v(t) + K_4 v(t) \cdot t = 1, \quad (2)$$

where 
$$K_1 = \frac{ml(D^2 - d^2)}{2.88 \cdot 10^9 p_n D^2 K_{Qp}};$$

$$K_2 = \frac{0.7m}{p_n D^2} + \frac{vl^3(D^2 - d^2)}{0.068 \cdot 10^9 p_n D^2 K_{Qp}};$$

$$K_3 = \frac{0.44D^2}{p_n K_{Qp}} + \frac{0.44(D^2 - d^2)^2 \psi}{p_n D^2 K_{Qp}} + \frac{30vl^2}{p_n D^2};$$

$$K_4 = \frac{0.89 \cdot 10^9 (D^2 - d^2) \psi}{p_n D^2 l};$$

$$\psi = C_{pl}/C_{gc} - \text{FPD load stiffness coefficient}; \quad C_{gc} = \frac{E_{gc} S_B^2}{V_0} -$$

the reduced stiffness of a hydraulic cylinder, N/m.

The got differential equation of the 2nd order can be solved by the Runge-Kutta numerical method using Maple software. The analysis of the results is performed by a graphic-analytical manner in the form of a study of the obtained dependences of the speed change  $v(t)$  of the output segment for the found FPD variants.

#### IV. RESULTS OF THE MULTI-OBJECTIVE SELECTION OF A HYDRAULIC DRIVE FOR DIFFERING DESIGN DATA

As examples of the use of the developed by the authors' method of a multi-criteria FPD selection the problems of hydraulic drive design for the technical systems of asphalt paver, forklift truck and excavator bucket were solved. To obtain the set of Pareto-optimal variants of a hydraulic drive, the methods of evolutionary modeling were used, in particular, the developed genetic algorithm with a cluster modification [6]. Here, the relations of the values of the genetic algorithm control parameters were determined; the better Pareto front approximation for various initial data of the FPD MOP is achieved with them. It was enabled to analyze various combinations of the values of the hydraulic cylinder and hydrodistributor geometric parameters effect on the selected quality criteria of FPD. The results of the multi-objective optimization of a hydraulic drive for the initial data corresponding to differing machines types and their operating conditions are presented in Tables I-III.

To select the best FPD variant, additional information should be got from the expert. It contains the operating conditions of the system with a hydraulic drive and expert's preferences on the importance of keeping the FPD indicators of power density, response speed and stability when changing the external conditions of operation, the designed-out impacts beginning, etc. To evaluate these features in the FPD study, the dynamics, the transient process time and the time of achieve the desired operating conditions, in particular the working stroke speed should be analyzed. The differential equations of the form (2) have been solved numerically for the selected FPD design problems using the obtained results of FPD multi-objective optimization and the families of the corresponding curves of the function  $v(t)$  were obtained taking into account the working operational parameters, including the load stiffness  $\psi$  (Fig. 2).

**TABLE I. PARETO-OPTIMAL VARIANTS OF THE HYDRAULIC DRIVE OF ASPHALT PAVER**

$$\{F = 52000; v = 0.012; l = 0.41; m \in [450, 550]; \eta_{mc} = 0.95; \eta_{gc} = 0.85; \eta_{vc} = 0.96\}$$

Variant number	$D$ , mm	$d$ , mm	$p_n$ , MPa	$L$ , mm	$F_r$ , N	$v_r$ , m/s	$\omega$ , mm <sup>2</sup>	$f_{\bar{N}}$	$f_{\xi}$
1	180	125	2.5	630	53530	0.012	17.33	1.251	0.0097
2	63	36	20	500	51768	0.012	0.94	1.300	0.0057
3	90	63	10	500	53565	0.012	2.15	1.327	0.0052
4	63	40	20	630	52108	0.012	0.88	1.398	0.0044
5	75	30	16	500	57855	0.012	1.16	1.436	0.0038
6	75	35	16	630	58146	0.012	1.1	1.459	0.0030
7	50	40	32	630	53575	0.012	0.38	1.474	0.0026

**TABLE II. PARETO-OPTIMAL VARIANTS OF THE HYDRAULIC DRIVE OF FORKLIFT TRUCK**

$$\{F = 10000; v = 0.1; l = 0.4; m \in [2400, 2600]; \eta_{mc} = 0.95; \eta_{gc} = 0.85; \eta_{vc} = 0.96\}$$

Variant number	$D$ , mm	$d$ , mm	$p_n$ , MPa	$L$ , mm	$F_r$ , N	$v_r$ , m/s	$\omega$ , mm <sup>2</sup>	$f_{\bar{N}}$	$f_{\xi}$
1	80	22	2.5	500	10210	0.1	35.53	0.416	0.676
2	80	28	2.5	500	10252	0.1	34.1	0.419	0.630
3	80	40	2.5	500	10366	0.1	30.95	0.423	0.531
4	80	45	2.5	500	10425	0.1	29.6	0.426	0.488
5	80	50	2.5	630	10492	0.1	28.62	0.429	0.408
6	80	56	2.5	630	10581	0.1	27.36	0.433	0.372
7	50	40	6.3	500	10548	0.1	6.54	0.44	0.247

**TABLE III. PARETO-OPTIMAL VARIANTS OF THE HYDRAULIC DRIVE OF EXCAVATOR BUCKET**

$$\{F = 52000; v = 0.4; l = 0.9; m \in [1000, 1300]; \eta_{mc} = 0.95; \eta_{gc} = 0.85; \eta_{vc} = 0.96\}$$

Variant number	$D$ , mm	$d$ , mm	$p_n$ , MPa	$L$ , mm	$F_r$ , N	$v_r$ , m/s	$\omega$ , mm <sup>2</sup>	$f_{\bar{N}}$	$f_{\xi}$
1	180	125	2.5	900	53530	0.4	593.37	17.86	0.44
2	90	63	10	900	53565	0.4	73.89	18.65	0.21
3	63	40	20	1000	52108	0.4	29.13	18.77	0.18
4	50	30	32	900	52322	0.4	14.50	19.01	0.15
5	75	35	16	1000	58146	0.4	36.54	19.39	0.12
6	50	40	32	1000	53575	0.4	12.19	19.55	0.10

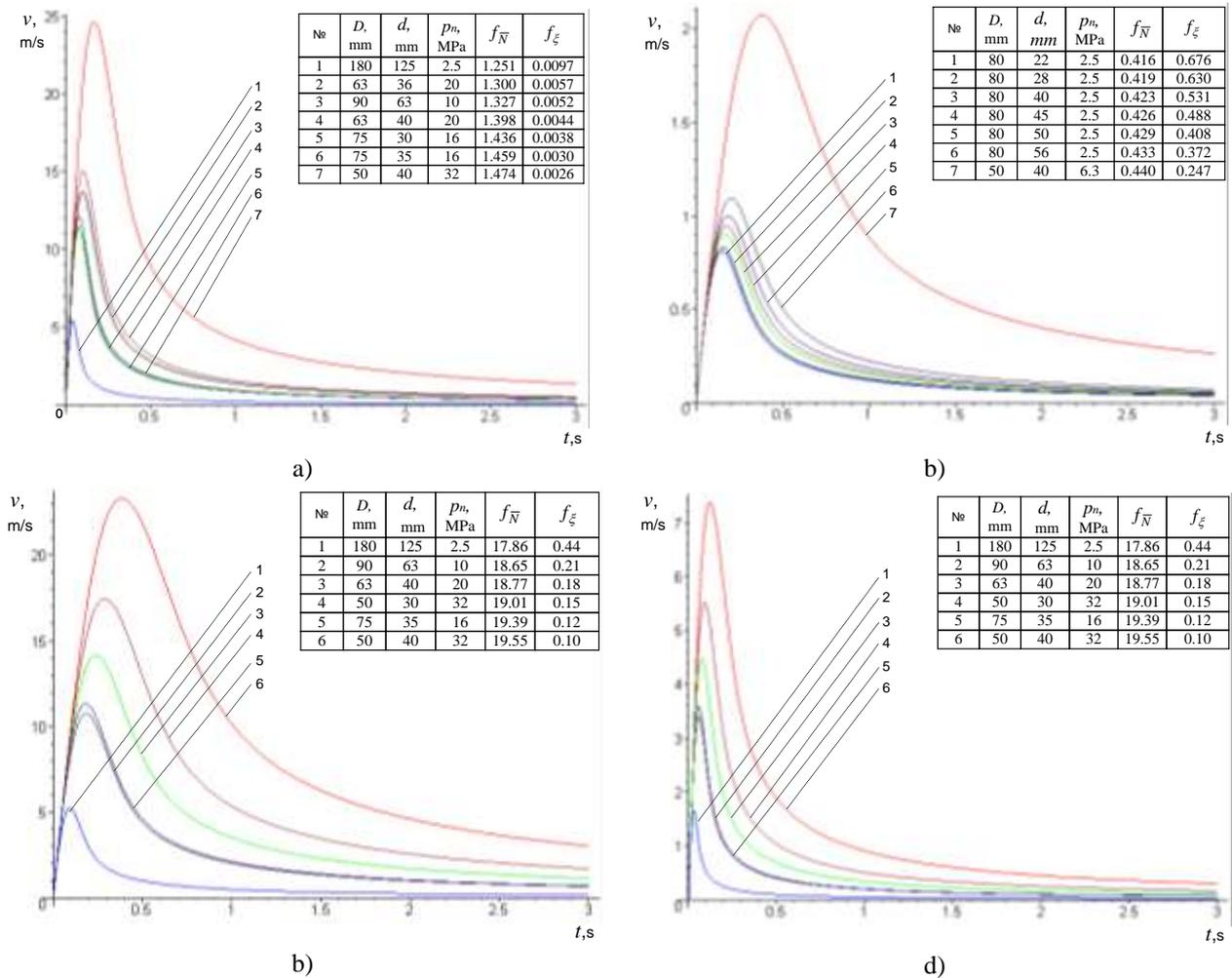


Fig. 2. The dynamics of FPD transient process depending on the design characteristics and load stiffness: a - asphalt paver ( $\psi=0.01$ ); b - forklift truck ( $\psi=0.01$ ); c - excavator bucket ( $\psi=0.01$ ); d - excavator bucket ( $\psi=0.1$ )

Analysis of the curves of the piston speed change from the time factor lets to evaluate the duration  $T$  of the FPD transition process at the initial stage of its working, depending on the design and operating characteristics. Also the dependence of the time moment of a steady movement beginning of the piston on the specific values of power density and damping capacity of its cylinder is revealed. Thus, the following conclusions can be drawn from the comparison of the plotted curves. The FPD power density increase goes with a decrease in the cylinder piston geometry, entails an increase in its reduced stiffness and a decrease in the damping features, together with an increase in the load stiffness results in a significant reduction in the transient process time. This, in turn, facilitates to higher quality (in terms of stability) and accuracy compliance to the values of the hydraulic drive output parameters. On the other hand, an increase in the damping coefficient reduces the stiffness of the system and causes the operation of the hydraulic drive with a longer duration of the transition process, when the mode of beyond-design technical conditions of operation is possible, in which the value  $T$  becomes longer than the working stroke time  $t_{ws}$ .

To prevent this situation, each FPD variant can be evaluated for the effect of load stiffness, which is expressed by the coefficient  $\psi$ . Its different values may reflect the changing operating conditions of the FPD, which in combination with the modelling of the dynamics of FPD makes it possible to predict the preservation of the normal operating mode, namely the time of cessation of self-oscillations, the period of reaching the speed of operation and the power reserve to maintain the required speed. Indeed, if we assume that the FPD with the maximum specific power at a given stiffness ( $\psi_1$ ) allows for a minimum time to transfer the kinetic energy to the attached load, then the FPD power with an increase in the load stiffness ( $\psi_2$ ) may not be enough to complete the working stroke at the same speed (Fig. 2 b, d)

The availability of information on the design and operational FPD parameters in combination with data on its dynamic behavior allows the expert to formulate selection criteria and to evaluate their importance for making the optimal decision. Concerning to the FPD design problems under consideration, this stage of FPD selection can be represented in the test table form (Table IV).

TABLE IV. TEST TABLE FOR THE DECISION MAKING FOR BETTER FPD VARIANT SELECTION

№	Type of machine with a fluid power drive	Selection criteria, their importance for an expert						Number of FPD variant
		I	II	III	IV	V	VI	
1	Asphalt paver	A	H	H	A	L	A	4
2	Forklift truck	L	A	H	H	L	H	2
3	Bucket excavator	H	A	L	N	H	L	2

**Notes:** 1. Selection criteria: I – power density, II – damping, III – responsivity, IV – transient process dynamics ( $T/t_{ws}$ ), V – load stiffness, VI – weight-size indicator. 2. The importance of selection criteria: H – high, A – average, L – low, N – indifferently.

By the example of FPD selection for an excavator bucket the conclusion can be draw that for its running the indicators of power density, ability to overcome the load stiffness on FPD in combination with an enough damping ability has the highest priority values. To a greater extent, the FPD variant number two fits the same requirements, since in this case the operating characteristics with increase load stiffness are kept, FPD has a high power density and enough damping capacity. For the asphalt paver FPD the most important in operation is to ensure stroke smoothness and responsivity. This can be achieved by selecting the FPD variant number four, differing by a maximum damping value and a minimum transient process time. For the forklift truck manipulator the responsivity, the dynamics of a transition process and weight-size indicators should take into account first on the FPD variant selection. To implement these requirements, the FPD variant number two may be recommended. Further, after specifying the values of the corresponding parameters (the pressure in a hydraulic cylinder, the volume flow of working fluid, the hydraulic characteristics of hydrodistributor and pipeline system) the power, type of pump and its drive can be assigned using existing methods [1].

The considered FPD design problems have the preliminary specifications data sufficiently limited reflecting the required characteristics of the system, where a fluid power drive will be used. Therefore, on the better FPD variant selection, the accordance to the most significant criteria was taken into account in the first instance. A comprehensive analysis of the

system with FPD may require the use of a different set of criteria and a more complex expert's system preferences. It can be used models of decision-making, using specialized methods of alternatives ranking according to the importance value and their selection in condition of indeterminacy. In combination with the use of automatization decision-making support software, this all will improve the quality of design system with a fluid power drive, to determine the better control and operation modes for them.

## V. CONCLUSION

The proposed method of FPD multi-criteria selection expands the potential of common used methods for calculation and selection of a FPD construction, for instance from the catalogues of mass-produced products. It is achieved by obtaining the Pareto-optimal FPD variants and selecting the alternative of them, taking into account the found design values corresponding to a technical specification for design and optimal operational characteristics. In addition the selection criteria for the expert are supplemented with information about the FPD dynamic properties that ensure the ability to keep the required modes of operation under changing the working conditions of the system with FPD. The developed method lets to increase the accuracy of decisions in the design and operation of machines with a hydraulic drive. The study of the FPD construction with a one-way rod does not limit the applicability of the considered technique in engineering practice; also it can be adapted to other schemes of a fluid power drive.

## References

- [1] V. Sveshnikov, Machine hydraulic drives: Handbook. Moscow, RU: Machine Building, 2004.
- [2] D. Popov, Mechanics of hydraulic and pneumatic drives. Moscow, RU: N. Eh. Bauman MGTU, 2001.
- [3] S. Habibia, G. Singha, "Derivation of Design Requirements for Optimization of a High Performance Hydrostatic Actuation System," in International Journal of Fluid Power, vol. 1, issue 2, 2000, pp. 11-27.
- [4] O. Gomis-Bellmunta, F. Campanileb, S. Galceran-Arellanoa, D. Montesinos-Miraclea, J. Rull-Duran, "Hydraulic actuator modeling for optimization of mechatronic and adaptronic systems," in Mechatronics, vol. 18, issue 10, 2008, pp. 634-640.
- [5] R. Marler, S. Arora "Survey of multi-objective optimization methods for engineering," in Struct Multidisc Optim, №26, 2004, pp. 369-395.
- [6] V. Averchenkov, P. Kazakov, "Application of evolutionary algorithm to multiobjective optimization of hydraulic actuation system," in proceedings of International Conference on Mechanical Engineering, Automation and Control systems (MEACS), IEEE, 2015, pp. 1 - 4.