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# Simplified Viscoelastic Plastic Model of Ore Particle Movement Along the Vibration Organ

V.L. Lapshin
Irkutsk National Research Technical University
Irkutsk, Russia
lapshin@istu.edu

S.A. Bogidaev
Irkutsk Research Institute of Precious and Rare Metals and Diamonds
Irkutsk, Russia
fluorit2001@mail.ru

V.P. Myazin
Transbaikal State University
Chita, Russia
myazinvpchita@mfil.ru

***Abstract*—The viscoelastic plastic model of vibration movement of ore particles along the vibration organ was described. The model includes viscoelastic and elastic plastic blocks. The first one describes elastic deformations of the system and measures energy losses using a damper. The second block describes plastic (residual) deformations. The differential equations of model motion at different stages of ore particle movement were analyzed. Two mathematical models were presented using second and third order differential equations. Recommendations to simplify the model at various stages of movement were suggested. Research models of various levels of complexity were developed. At the stage of particle movement along the vibration organ, it is not advisable to simplify the viscoelastic plastic model in the direction normal to the organ. It is advisable to simplify the model in the direction of the working surface of the vibration organ at the stage of non-slip interaction by eliminating the elastic-plastic block. When the particle is sliding along the vibration organ and flying, the model should be simplified to a rigid body. These simplifications facilitate the task of developing algorithms and software for the vibration process under study.**

***Keywords*— vibration separation of materials, modeling of vibration processes.**

## I. INTRODUCTION

In the ore processing industry, vibration processes are crucial. Dry vibration separation methods have been developed, and new vibration effects have been studied and described [1–8]. New vibratory machines for size distribution have been designed [9].

The raw material separation method using a vibrating surface is one of the vibration methods (Fig. 1). It is based on the effect of separation of ore particles by their properties. The particles are distributed by their size, shape, friction coefficient, elasticity and other physical and mechanical characteristics [8,10,11].

To ensure high technological equipment performance, it is necessary to assess the efficiency of mineral raw materials processing and identify rational operating modes and vibration equipment parameters based on physical and mechanical properties of materials.

Theoretical and applied studies [5, 6, 7, 12-17] are devoted to the size distribution of bulk materials. The most common approach in bulk media mechanics is mathematical modeling and computer research which allows for equipment optimization by numerical methods [1,3,11,15-21]. For each component of mineral raw materials, an individual research model has been developed. When studying the dynamics of interaction of a particle and a vibration organ, the efficiency of separation is evaluated, and the equipment is optimized by its parameters and operation modes. The most perfect models are mechanorheological ones consisting of bodies of vibrational rheology – elastic-inertial, viscous-inertial, and plastic-inertial ones [17,18,21]. The numerical characteristics of model parameters are related to the ones of the initial mineral raw materials and the vibration organ surface.

A system of viscoelastic mechanorheological models of vibrational movement of particles was developed [17]. The models can be used to study the dynamics of interaction of particles and a vibration organ at all stages of movement and calculate vibration parameters: a dynamic load on the vibration organ, movement trajectories and speed, average transport speed.

A more general viscoelastic plastic model has been developed. It includes an additional shear element which measures energy losses caused by shock interaction [22]. The model is based on the viscoelastic plastic model developed for theoretical studies of shock processes. A method for determining the elastic modulus has been developed. However, the model is rather complicated which makes it difficult to use it for computer research on vibration processes. Therefore, it is necessary to develop research models on its basis.

Fig.1. The working body of a vibration separator

## II. Problem Statement

The vibration movement includes several stages. When interacting with the vibration organ, the particle has a certain movement speed. The particle can slide along the vibration organ. When the force of normal reaction of the particle becomes equal to zero, the particle is separated from the vibration organ. At the stage of flight, the particle moves over the vibration organ. Let us consider the shock interaction of a particle with a working surface of the vibration organ in the direction normal to the organ.

This process can be presented as follows. At the initial moment of contact, the particle-working surface system experiences elastic deformations. Irreversible processes (plastic deformations, crushing and destruction of asperities) can develop. These processes can have a significant impact on the dynamics of particle movement and vibration separation in general. When unloading the system, the contact force decreases to zero and only elastic deformations disappear. The potential energy of elastic deformations transforms into the kinetic energy of the particle.

Let us describe the mechanorheological model of deformation of the particle-vibration organ system during shock interaction. When studying deformations, the most important properties are elasticity, plasticity, and viscosity which characterize the dissipation of impact energy during elastic deformations. In general, the body may experience elastic and plastic deformations.

## III. Theory

When interacting with the vibration organ, the mechanorheological model can be presented as a viscoelastic plastic system (Fig. 2). Let us consider a horizontally located vibration organ. The *X'Y'* coordinate system is a fixed system, the *XY* coordinate system is associated with the vibration organ, and the $X_1Y_1$ coordinate system is associated with the particle. The coordinate system $X_2Y_2$ allows us to separate elastic and plastic deformations. If the particle does not move along the vibration organ (there is no sliding motion), the *XY* and $X_1Y_1$ coordinate systems coincide.

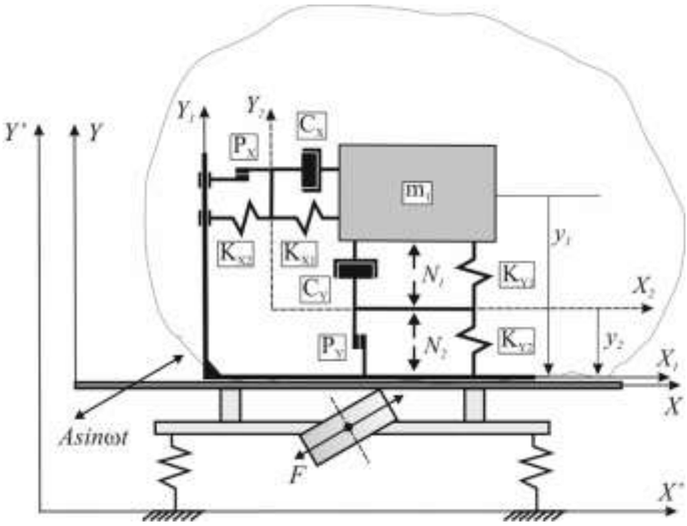


Fig.2. The scheme of viscoelastic plastic model

Let us consider the operation of the model along the Y axis. The model describes the movement of the particle's center of gravity ($m_1$) and includes two blocks: the viscoelastic block $K_{Y1} - C_Y$ and the elastic plastic block $K_{Y2} - P_Y$. The block $K_{Y1} - C_Y$ describes elastic deformations and measures energy losses using a damper, where $K_{Y1}$ is the stiffness coefficient of the elastic element of the viscoelastic block; $C_Y$ is the viscosity coefficient of a viscous element of the viscoelastic block. The block $K_{Y2} - P_Y$ describes plastic (residual) deformations and measures energy losses, where $K_{Y2}$ is the stiffness coefficient of the elastic element of the elastic plastic block; $K_{Y2}$ is the shear coefficient of the elastic plastic block.

The operation of the viscoelastic plastic model is as follows. At the initial stage of shock interaction, elastic and plastic deformations occur. It is assumed that plastic deformations occur simultaneously with elastic ones. This is permissible for shock processes, since the zone of elastic deformations is usually insignificant. Therefore, the viscoelastic $K_{Y1} - C_Y$ and elastic-plastic $K_{Y2} - P_Y$ blocks are deformed. One can use a more complex version of the model which allows it to set the force corresponding to the beginning of plastic deformations.

At the unloading stage, the force of shock interaction reaches its maximum $N_{MAX}$. Only the viscoelastic block operates. It describes the disappearance of elastic deformations. The elastic plastic block remains deformed, since it describes plastic (residual) deformations.

Let us consider the simplest option when all model elements have linear characteristics. This approach is widely used in viscoelastic mechanorheological models [17,18,21,23,24,25] for ore and other stone materials. For example, experimental studies of granite under uniaxial compression identified a linear dependence of elastic deformations on the load [26]. The plastic component can be proportional to the acting force [23,24]. The unloading process can be described by the law of linear elasticity [23].

Based on the Lagrange equations, differential equations describing the non-slip contact of a particle and a vibration organ have been built:

$$m_1 \ddot{y}_1 + C_Y(\dot{y}_1 - \dot{y}_2) + K_{Y1}(y_1 - y_2) = -m_1 g - m_1 \ddot{y}' \quad (1)$$

$$m_1 \ddot{x}_1 + C_X(\dot{x}_1 - \dot{x}_2) + K_{X1}(x_1 - x_2) = -m_1 \ddot{x}', \quad (2)$$

where $y_1, \dot{y}_1, \ddot{y}_1, x_1, \dot{x}_1, \ddot{x}_1$ - displacement, speed and acceleration $m_1$ relative to the vibration organ; $\ddot{y}'$, $\ddot{x}'$ - vibration organ acceleration.

## IV. Discussion

The additional coordinate system $X_2Y_2$ makes it difficult to solve equations (1), (2), since there are additional variables $x_2$, $y_2$ determined by the elastic-plastic block. Let us make transformations and solve equation (1).

The resistance force of viscoelastic deformations (along the Y axis) is determined by formula

$$N_1 = F_{VIS} + F_{EL1};\ F_{VIS} = C_Y(\dot{y}_1 - \dot{y}_2);\ F_{EL1} = K_{Y1}(y_1 - y_2).$$

The resistance force of elastic plastic deformations is determined by formula

$$N_2 = F_{PL} + F_{EL2};\ F_{EL2} = K_{Y2}\, y_2;\ F_{PL} = P_Y\, y_2;\ N_1 = N_2.$$

Thus, we can write

$$C_Y(\dot{y}_1 - \dot{y}_2) + K_{Y1}(y_1 - y_2) = K_{Y2} y_2 + P_Y y_2;$$

$$m_1(\ddot{y}_1 + \ddot{y}' + g) = -(K_{Y2} + P_Y) y_2;$$

$$y_2 = -\frac{m_1}{K_{Y2} + P_Y}(\ddot{y}_1 + \ddot{y}' + g);$$

$$\dot{y}_2 = -\frac{m_1}{K_{Y2} + P_Y}\frac{d(\ddot{y}_1 + \ddot{y}' + g)}{dt} = -\frac{m_1}{K_{Y2} + P_Y}(\dddot{y}_1 + \dddot{y}').$$

By substituting the expressions for $y_2, \dot{y}_2$ in equation (1), we have

$$\dddot{y}_1 D_1 + \ddot{y}_1(m_1 + D_2) + \dot{y}_1 C_Y + y_1 K_{Y1} =$$

$$= -\dddot{y}' D_1 - \ddot{y}'(m_1 + D_2) - g(m_1 + D_2),$$

where $D_1 = \dfrac{m_1 C_Y}{K_{Y2} + P_Y}$; $D_2 = \dfrac{m_1 K_{Y1}}{K_{Y2} + P_Y}$.

We obtain a third-order differential equation. Equation (2) can be transformed in a similar way.

To solve the resulting equation, one can use the fourth order numerical Runge-Kutta method which is accurate, simple and convenient. It allows for changes related to the vibration law, the type of a mechanorheological model and acting forces.

The equation can be solved using the Runge-Kutta method:

$$\dddot{y}_1 = B_1 \ddot{y}_1 + B_2 \dot{y}_1 + B_3 y_1 - \dddot{y}'(t) + B_4 \ddot{y}'(t) + B_5;$$

$$B_1 = -\frac{K_{Y1} + K_{Y2} + P_Y}{C_Y};\ B_2 = -\frac{K_{Y2} + P_Y}{m_1};$$

$$B_3 = -\frac{K_{Y1}(K_{Y2} + P_Y)}{C_Y m_1};$$

$$B_4 = B_1;\ B_5 = gB_1;$$

$$b_1 = dt^3(B_1 \ddot{y}_1 + B_2 \dot{y}_1 + B_3 y_1 - \dddot{y}'(t) + B_4 \ddot{y}'(t) + B_5)/6;$$

$$b_2 = dt^3(B_1(\ddot{y}_1 + 3b_1/dt^2) + B_2(\dot{y}_1 + \ddot{y}_1 dt/2 + 3b_1/4dt) +$$
$$+ B_3(y_1 + \dot{y}_1 dt/2 + \ddot{y}_1 dt^2/8 + b_1/8) - \dddot{y}'(t + dt/2) +$$
$$+ B_4 \ddot{y}'(t + dt/2) + B_5)/6;$$

$$b_3 = dt^3(B_1(\ddot{y}_1 + 3b_2/dt^2) + B_2(\dot{y}_1 + \ddot{y}_1 dt/2 + 3b_1/4dt) +$$
$$+ B_3(y_1 + \dot{y}_1 dt/2 + \ddot{y}_1 dt^2/8 + b_1/8) - \dddot{y}'(t + dt/2) +$$
$$+ B_4 \ddot{y}'(t + dt/2) + B_5)/6;$$

$$b_4 = dt^3(B_1(\ddot{y}_1 + 6b_3/dt^2) + B_2(\dot{y}_1 + \ddot{y}_1 dt + 3b_3/dt) +$$
$$+ B_3(y_1 + \dot{y}_1 dt + \ddot{y}_1 dt^2/2 + b_3) - \dddot{y}'(t + dt) +$$
$$+ B_4 \ddot{y}'(t + dt) + B_5)/6;$$

$$b_5 = (9b_1 + 6b_2 + 6b_3 - b_4)/20;\ b_6 = b_1 + b_2 + b_3;$$

$$b_7 = (b_1 + 2b_2 + 2b_3 + b_4)/2;$$

$$y_1 = y_1 + \dot{y}_1 dt + \ddot{y}_1 dt^2/2 + b_5;$$

$$\dot{y}_1 = \dot{y}_1 + \ddot{y}_1 dt + b_6/dt;\ \ddot{y}_1 = \ddot{y}_1 + 2b_7/dt^2.$$

Setting the initial values of parameters $y_1 = y_H$, $\dot{y}_1 = \dot{y}_H$, $y_1, \dot{y}_1, \ddot{y}_1$ at the moment $t=t+dt$ are calculated. At the next stage, the calculations are repeated.

An additional mass $m_2 << m_1$ between the viscoelastic and elastic-plastic blocks can be introduced (Fig. 3). At $m_2 \to 0$, this mass will not have a significant effect on the dynamics of movement; however, missing differential equations of movement for mass $m_2$ can be built:

$$m_2 \ddot{y}_2 + K_{Y2} y_2 + P_Y y_2 + C_Y(\dot{y}_2 - \dot{y}_1) + K_{Y1}(y_2 - y_1) = -m_2 g - m_2 \ddot{y}'; \qquad (3)$$

$$m_2 \ddot{x}_2 + K_{X2} x_2 + P_X x_2 + C_X(\dot{x}_2 - \dot{x}_1) + K_{X1}(x_2 - x_1) = -m_2 \ddot{x}'. \qquad (4)$$

where $y_2, \dot{y}_2, x_2, \dot{x}_2$ are movement and speed of mass $m_2$.

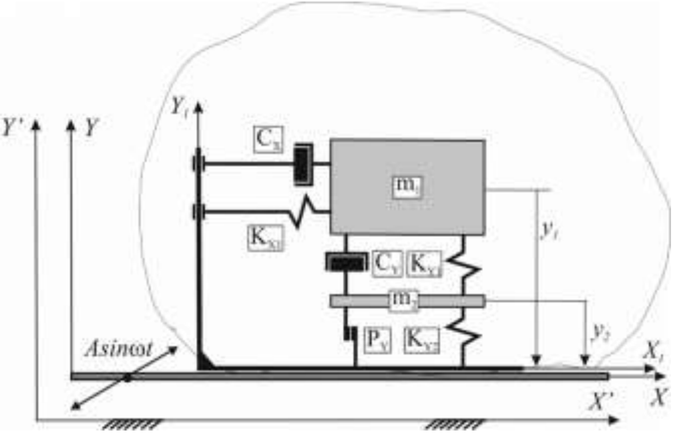


Fig.3. Scheme of the simplified viscoelastic plastic model at the stage of non-slip contact interaction

As a result, we have systems of two equations (1), (3) and (2), (4) describing the dynamics of movement of the two-mass model along the $Y$ and $X$ axes which can be solved by numerical methods.

## V. Recommendations

Let us simplify the model at various stages of particle movement. At the stage of particle contact with the vibration organ, the value and regularity of the change in the force of normal reaction $N = f(t)$ are important. They determine the force of friction, the beginning of sliding, and the moment of separation of the particle during the transition to the flight. As a result, the trajectory and velocity of the particle are determined; therefore, parameter $N = f(t)$ is accurately calculated, and model simplification along the Y axis is not advisable.

Along the X-axis, the beginning of sliding a particle along the vibration organ is determined. It corresponds to the condition $F_{CD} > F_{FR} = f_{FR} N$, where $f_{FR}$ is the friction coefficient. Force $F_{CD}$ can be calculated by the viscoelastic block:

$$F_{CD} \approx N_{X1} \approx N_{X2}; \; N_{X1} = C_X(\dot{x}_1 - \dot{x}_2) + K_{X1}(x_1 - x_2);$$
$$N_{X2} = P_X x_2 + K_{X2} x_2.$$

Therefore, it is advisable to simplify the model along the X axis at the non-slip contact stage by eliminating the elastic-plastic block (Fig. 3).

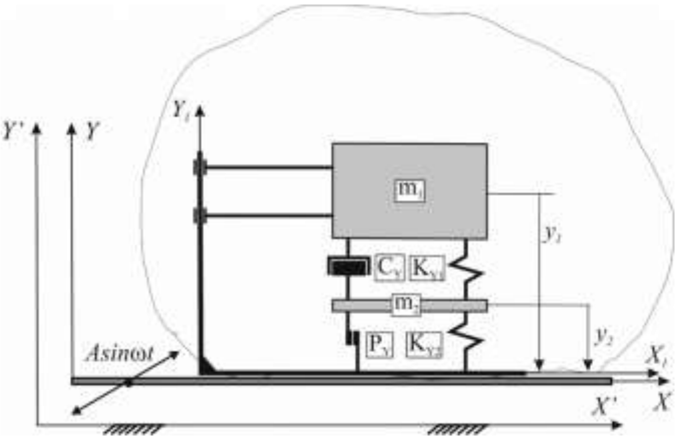


Fig. 4. Scheme of a simplified viscoelastic plastic model at the sliding stage

At the stage of particle sliding along the X axis, it is advisable to simplify the model to a rigid body (Fig. 4). When small particles slide along the vibration organ, deformations will be insignificant, and the dynamics of movement of an viscoelastic body will differ insignificantly from the dynamics of movement of an absolutely rigid body. The movement equation for a particle (an absolutely rigid body) at the stage of sliding along the X axis is as follows:

$$m_1 \ddot{x}_1 = -m_1 \ddot{x}' - F_{FR}, \qquad (5)$$

where $F_{FR} = f_{CK} \cdot N \cdot Sign(\dot{x})$;

$$Sign(\dot{x}_1) = \begin{cases} 1, & npu \;\; \dot{x}_1 > 0 \\ -1, & npu \;\; \dot{x}_1 < 0 \end{cases},$$

where $f_{CK}$ are the friction coefficients.

The end of the sliding stage meets the condition $\dot{x}_1 = 0$.

At the flight stage, the particle can be described as a solid body. Transition from the joint movement stage to the flight stage is carried out under the condition $N = 0$.

Movement equations at the flight stage are as follows:

$$m_1 \ddot{y}_1 = -m_1 \ddot{y}' - m_1 g - a_y(\dot{y}' + \dot{y}_1)^2; \qquad (6)$$

$$m_1 \ddot{x}_1 = -m_1 \ddot{x}' - a_x(\dot{x}' + \dot{x}_1)^2, \qquad (7)$$

where $a_y, a_x$ describe aerodynamic resistance to the movement and depend on the shape and size of particles, air density and movement speed.

## VI. Conclusion

The mathematical model (equations (1) - (7)) can be used to study the influence of elastic, viscous (dissipative) and plastic properties of the material on the dynamics of shock interaction of a particle and a vibration organ. Research models of different complexity describe the movement of ore particles. The simplifications facilitate the task of developing algorithms and software for studying the vibration process and improve the efficiency of practical application of the viscoelastic plastic model for research purposes (identification of rational operating modes and equipment parameters and evaluation of vibration separation efficiency).

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