

Refined Method of Analyzing Potential Defects in Engineering Products

V.V. Miroshnikov

Department of Quality Management, Standardization and Metrology
Bryansk State Technical University
Bryansk, Russia
e-mail: v.v.miroshnikov@mail.ru

N.M. Borbač

Department of Quality Management, Standardization and Metrology
Bryansk State Technical University
Bryansk, Russia
e-mail: borbact@mail.ru

A.V. Morozova

Department of Computer Technologies and Systems
Bryansk State Technical University
Bryansk, Russia
e-mail: niotiostu@gmail.com

Abstract— The analysis of the types and consequences of potential defects in engineering products based on the application of the theory of fuzzy sets is considered. The formulas of the corresponding linguistic variables are proposed. The calculation of the risk priority number based on mathematical operations with fuzzy numbers is given. The experimental verification of the proposed refined method of FMEA-analysis was carried out by the authors in order to improve the design of the car steering hydraulic booster. The use of elements of fuzzy sets theory in the FMEA procedure made it possible to move to such a powerful decision-making tool as a fuzzy logical conclusion, which is an approximation of the “inputs-outputs” relationship based on linguistic statements “if-then” as well as logical operations on fuzzy sets.

Keywords— FMEA, defects, analysis, products, engineering, fuzzy sets, linguistic variables, fuzzy inference.

I. INTRODUCTION

One of the common methods of ensuring and improving the quality of technical systems being developed, aimed at preventing the occurrence of defects (inconsistencies) and reducing the negative effects of them, is the analysis of types and effects of potential defects (FMEA - Potential Failure Mode and Effects Analysis) [5, 6, 8]. FMEA has been used in the manufacture of complex technical objects for more than half a century, from space, aviation and military equipment to household appliances. The method can be used both in the development of new products and their production technology, as well as in the improvement of products already launched into production [2, 3, 4, 7, 17, 18, 21]. In addition, when conducting research, combinations of FMEA with other modern methods are often used [10, 12, 19], including the usage of linguistic terms [11, 20]. This makes it possible to

apply this type of analysis, for example, in conditions of uncertainty [16], when developing maintenance programs for capital goods [15], when assessing the sustainability of production systems [1, 14], to determine the effect of several failure modes [13].

II. METHODS AND MATERIALS

In accordance with the FMEA methodology, the analysis is carried out by a special expert group (FMEA – team). In the course of analysis possible types of defects for a specific technical object or production process, are determined. For all identified types of potential defects, their consequences are determined and evaluated by a score of seriousness S , which varies from 1 for the least significant damage defects to 10 for the most significant defects. Also, for each defect, potential causes are determined and, for each cause, the probability score of occurrence O is set, which varies from 1 for the most rarely occurring defects to 10 - for defects that occur almost always. Thereafter, a detection score D is determined for each defect during the intended manufacture. The detection score varies from 10 for practically undetectable defects (causes) to 1 for almost reliably detectable.

After receiving expert assessments S , O and D , the complex defect risk is calculated as the risk priority number (RPN):

$$RPN = S \cdot O \cdot D \quad (1)$$

Then a list of defects / reasons for which the calculated RPN value exceeds a predetermined RPN_{bound} limit value is made. It is for them that the design and / or the production process will be finalized.

At the same time, despite the fact that the probability of a defect occurrence can be determined on the basis of available statistical data on similar objects (processes) or using methods

accepted in calculations of reliability, in the end it is still expressed in the form of rank (score). However, as it is known from metrology, the scores are only designations of reference points on the order scale, and therefore no mathematical operations should be carried out with them, as a result of which formula (1) is incorrect.

However, the need to use formula (1) is associated with the features of the method. First, as it has been noted already, the analysis is carried out by a group of experts using their knowledge and experience, as well as information, which in many cases cannot be expressed in strict quantitative form. Secondly, the analysis result should be presented in the form of some complex index characterizing the criticality of the potential defect as well as indicating the need to improve the object of analysis. The function of such an indicator can be performed by *RPN*. To resolve this problem situation, the authors propose to apply the theory of fuzzy sets when conducting FMEA with subsequent defuzzification of the analysis result.

III. RESULTS

Let *A* be a random event consisting in the appearance of a defect of the object under study. However, unlike the traditional probabilistic approach, the $P[A]$ probability of an *A* event will be considered as a linguistic variable, i.e. a variable whose values are words or sentences of a natural or artificial language [22, 23, 24]. Thus, there is a linguistic probability of an *A* event. The base variable for linguistic probability *P* is a *p* variable that takes values from the universal set *U*, which is a unit interval [0, 1].

Formally, any linguistic variable is described by the five parameters: $\langle X, T, U, G, M \rangle$, where *X* is the variable name; *T* is a term-set, each element of which is given by a fuzzy set on a universal set; *U* and *G* is a syntactic rule that generates the names of terms; *M* - semantic rules that define the membership functions of fuzzy terms generated by a syntax rule *G*.

Thus, for the linguistic variable “probability of occurrence of a defect” (*P*) it is necessary to specify a term-set, as well as to formulate syntactic and semantic rules.

In accordance with the FMEA methodology, it is advisable to set the following term set to describe the defect probability: $T(P) = \text{probably} -$

- *probably enough* - *quite probably* -
- *more or less probably* - *not very probably* -
- *improbably* - *extremely improbably* -
- *absolutely improbably* ,

where the term “probably” is the primary term, and the syntactic rule *G* is the simplest - new terms are obtained using the appropriate quantifiers.

Since each term is the name of the corresponding fuzzy subset of the universal set *U*, the membership function should be specified for the primary term [25]. The problem of

constructing membership functions that adequately fix the values of linguistic terms is not the task of the theory of fuzzy sets as such, but belong to the general problem area of knowledge acquisition. Currently, various methods have been developed for constructing membership functions of fuzzy sets [9].

Since the analysis by the FMEA method is carried out by a specially selected group of experts, it is advisable to use the method when FMEA team members are asked to set the membership functions of the primary terms before the analysis begins. In this case, each of the experts fills out a special questionnaire using binary estimates $b_{j,i} \in \{0; 1\}$, where “1” indicates that the *i*-th element has a universal set of *U* properties of the *j*-th fuzzy set, and “0” indicates their absence.

According to the survey results of the survey, the degree of belonging to a fuzzy set \tilde{A}_j is calculated by the formula:

$$\mu_{\tilde{A}_j}(u_i) = \frac{1}{m} \sum_{k=1}^m b_{j,i}^k, \quad (2)$$

where *m* – total number of experts; $b_{j,i}^k$ – a mark given by the *k*-th expert about the presence of the of the *j*-th fuzzy set properties of the *i*-th element.

For practical purposes, it is convenient to set the membership function of the primary term in a parametric form using a Gaussian function of the form:

$$\mu(u) = \exp\left(-\frac{(u-b)^2}{2c^2}\right), \text{ where } b \text{ is the maximum coordinate; } c - \text{ concentration ratio.}$$

Then, as an approximation of the membership function of the term “probably,” obtained by an expert method, the function defined by the expression

$$\mu_{\text{probably}}(p) = \begin{cases} \exp\left(-\frac{(p-0.5)^2}{0.0134}\right), & \text{with } p < 0.5, \\ 1, & \text{with } p \geq 0.5, \end{cases}$$

with $b=0.5$ and $c=0.082$ parameters can be used.

In the first approximation, semantic rules *M* can be specified in the form given in Table 1.

The graphs of the membership functions of the terms “probably”, “probably enough” and “quite probably” are shown in Fig. 1. The relationships between the defect probabilities according to the FMEA methodology and the proposed linguistic probabilities are given in Table. 2

TABLE I. SEMANTIC RULES FOR THE “PROBABILITY OF DEFECT APPEARANCE” LINGUISTIC VARIABLE

Quantifier	b_j and c_j parameters with respect to the b and c parameters of the primary term membership function	Base variable range
Enough A	$b_j = b/2$; $c_j = c - 0.01$	$[0, 1]$
Quite A	$b_j = b/4$; $c_j = c - 0.03$	$[0, 1]$
More or less A	$b_j = b/40$; $c_j = c/20$	$[0, 1]$
Not very A	$b_j = b/1000$; $c_j = c/500$	$[0, 1]$
Little A	$b_j = b/7500$; $c_j = c/2000$	$[0, 1]$
Extremely little A	$b_j = b/75000$; $c_j = c/15000$	$[0, 1]$
Not at all A	$b_j = 750000$; $c_j = c/150000$ $\mu(p) = 1$	$[b_j, 1]$ $[0, b_j]$

TABLE II. RELATIONSHIPS BETWEEN THE DEFECT PROBABILITIES BY THE FMEA METHODOLOGY AND THE VALUES OF THE LINGUISTIC VARIABLE “PROBABILITY OF DEFECT APPEARANCE”

The probability of a defect according to the FMEA methodology	Probable defect frequency	Score O	Linguistic variable value	Fuzzy carrier
Very high: a defect is almost inevitable	More than 1 of 2	10	Probably	$[0.5, 1]$
	$\gg 1$ of 3	9	Probably enough	0.25
High: recurring defects	More than 1 of 8	8	Quite probably	0.125
	$\gg 1$ of 20	7		
Moderate: occasional defects	More than 1 of 80	6	More or less probably	0.0125
	$\gg 1$ of 400	5	Not very probably	0.0005
	$\gg 1$ of 2000	4		
Low: relatively few defects	More than 1 of 15000	3	Improbably	$6.67 \cdot 10^{-5}$
	$\gg 1$ of 150000	2	Extremely improbably	$6.67 \cdot 10^{-6}$
Minor: defect is improbable	Less than 1 of 1500000	1	Absolutely improbably	$[0, 6.67 \cdot 10^{-7}]$

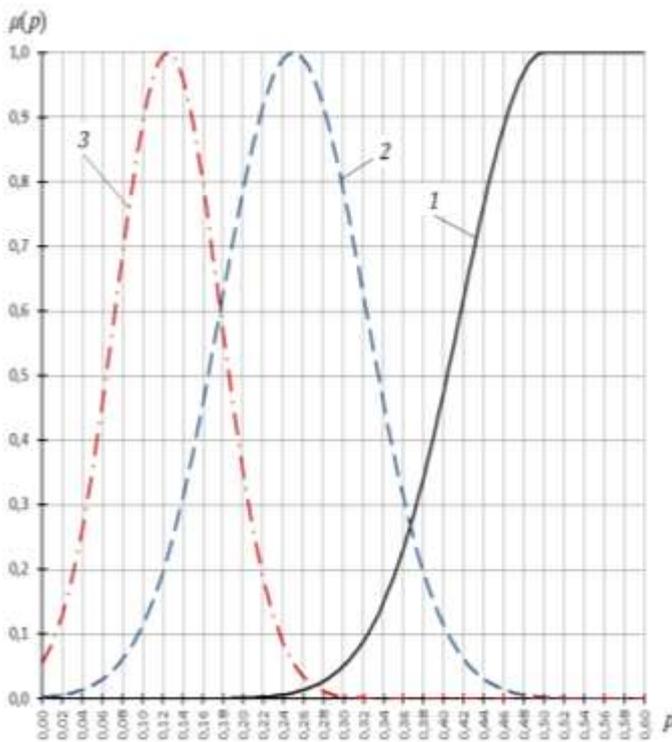


Fig. 1. The graphs of the membership functions of the terms “probably” (1), “probably enough” (2) and “quite probably” (3)

Similarly, to assess the significance of the defect consequences, it is proposed to use a linguistic variable “consequences” (S) with a term set:

- $T(S) = catastrophic - quite catastrophic -$
- $- extremely heavy - heavy - heavy enough - weak -$
- $- very weak - minor - very minor - no,$

where the terms "catastrophic", "heavy", "weak" and "minor" are primary.

As a universal set of the basic variable S of the linguistic variable "consequences", we also use the $[0, 1]$ interval. The membership functions of the primary terms of a S variable are given by the expressions:

$$\mu_{\text{catastrophic}}(S) = \exp\left(-\frac{(s-1)^2}{0.0098}\right);$$

$$\mu_{\text{heav}}(S) = \exp\left(-\frac{(s-0.7)^2}{0.0098}\right);$$

$$\mu_{\text{weak}}(S) = \exp\left(-\frac{(s-0.5)^2}{0.0098}\right);$$

$$\mu_{\text{minor}}(S) = \exp\left(-\frac{(s-0.3)^2}{0.0098}\right).$$

To build membership functions of compound terms, it is proposed to use the semantic rules given in Table. 3. At the

same time for the compound term “absent” the membership function is determined by the ratio:

$$\mu_{\text{no}}(S) = \mu_{\text{catastrophic}}(1 - S)$$

TABLE III. SEMANTIC RULES FOR THE LINGUISTIC VARIABLE "CONSEQUENCES"

Quantifier	b_j parameter with respect to the b parameter of the primary term membership function
Extremely A	$b_j = b + 0.1$
Enough A Very A	$b_j = b - 0.1$

To assess the detection of potential causes and the subsequent type of defect during the control, experts should specify the membership function of the primary terms of the linguistic variable “probability of detection” (Q). In this case, as in the two previous cases, the base variable Q takes values from a single $[0, 1]$ interval.

As a term-set of this linguistic variable, in the first approximation, it is proposed to use the following:

- $T(Q) = \text{reliably} - \text{more or less reliably} - \text{probably} -$
- $- \text{more or less probably} - \text{improbably} -$
- $- \text{very imprbably} - \text{impossible},$

where the terms "reliably" and "probably" are the primary terms.

The membership functions of primary terms can be given by the expressions like:

$$\mu_{\text{reliably}}(q) = \exp\left(-\frac{(q-1)^2}{0.02}\right);$$

$$\mu_{\text{probably}}(q) = \exp\left(-\frac{(q-0.5)^2}{0.045}\right).$$

As semantic rules for obtaining membership functions of compound terms, it is proposed to use the rules given in Table 4. In addition, the membership function of the term “impossible” is associated with the membership function of the term “reliably” by the relation:

$$\mu_{\text{impossible}}(q) = \mu_{\text{reliably}}(1 - q).$$

However, due to the fact that, in accordance with the FMEA methodology, the estimation of the defect detection probability should take large values for small probabilities and small for large ones, for further use it is advisable to switch to the linguistic variable “non detection probability” D with the base variable $d \in [0, 1]$. In this case, since the event consisting in the detection of a defect and the event consisting in its non-

detection are incompatible and the only possible, i.e. they form a complete group of events, the base variables of the corresponding linguistic variables must be related by:

$$d = 1 - q. \tag{3}$$

TABLE IV. SEMANTIC RULES FOR THE LINGUISTIC VARIABLE "PROBABILITY OF DETECTION"

Quantifier	b_j and c_j parameters of membership functions of a compound term in comparison with b and c parameters of the membership function of a primary term
More or less A	$b_j = b - 0.1; c_j = 1.5c$
Little A	$b_j = 0.5b; c_j = 0.7c$
Very A	$b_j = 1.4b; c_j = 0.7c$

As terms of a linguistic variable D , it is proposed to use terms that are opposite in meaning to the terms of a variable Q (Table 5). Then, taking into account the relation (3), the membership functions of the terms of the linguistic variable D can be found from the relation:

$$\mu_D(d) = \mu_Q(1 - q).$$

Thus, when applying the theory of fuzzy sets, the algorithm for conducting FMEA analysis remains unchanged, only the severity of the consequences of a failure (defect), the probability of its appearance and detection is estimated using the terms of the corresponding linguistic variables. After that, a priority risk number is determined based on the application of the α -level generalization principle [24].

TABLE V. RELATIONSHIPS BETWEEN TERMAS OF LINGUISTIC VARIABLES “DETECTION PROBABILITY” AND “NON DETECTION PROBABILITY”

Term of linguistic variable Q	Term of linguistic variable D
impossible	absolute
very improbably	very high
improbably	high
more or less probably	more or less high
probably	moderate
quite probably	low
more or less reliably	very low
reliably	zero

Each term should be considered as a fuzzy number corresponding to the maximum of its membership function. Each such fuzzy number is given by a set of α -sections: $\tilde{x} = \bigcup_{\alpha \in [0,1]} (\underline{x}_\alpha, \bar{x}_\alpha)$, where $\underline{x}_\alpha, \bar{x}_\alpha$ - are the minimum and maximum values of a fuzzy number \tilde{x} at the α -level, respectively. The number of α -levels is selected depending on the required accuracy of calculations.

It should be borne in mind that the universal set for all the linguistic variables and fuzzy sets associated with them is a unit interval, i.e. $U = [0, 1]$ therefore, restrictions are imposed:

$$\underline{x}_\alpha \geq 0; \quad \bar{x}_\alpha \leq 1. \quad (4)$$

If these conditions are violated, $\underline{x}_\alpha = 0$ and $\bar{x}_\alpha = 1$ should be accepted. In this case, the priority risk number will be the result of calculating a clear function of fuzzy arguments, i.e.

$PRN = y = f(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) = \tilde{x}_1 \tilde{x}_2 \tilde{x}_3$, where $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3$ are fuzzy numbers corresponding to the estimated severity of the consequences, the probability of occurrence and the probability of finding a defect.

The result of the function will be a fuzzy number $\tilde{y} = \bigcup_{\alpha \in [0,1]} (\underline{y}_\alpha, \bar{y}_\alpha)$, where \underline{y}_α and \bar{y}_α in accordance with the rules for performing arithmetic operations for positive fuzzy numbers will be determined from the relations:

$$\underline{y}_\alpha = \underline{x}_{1\alpha} \underline{x}_{2\alpha} \underline{x}_{3\alpha}; \quad \bar{y}_\alpha = \bar{x}_{1\alpha} \bar{x}_{2\alpha} \bar{x}_{3\alpha}.$$

As a finite *RPN* value, the result of defuzzification of the obtained fuzzy set \tilde{y} can be used (for example, by the center of gravity method). Since, by definition of the α -level set $\mu_{\tilde{y}}(u) \geq \alpha, \forall u \in [\underline{y}_\alpha, \bar{y}_\alpha]$ and \tilde{y} is a discrete set (the discrete number is determined by the number of α -levels), defuzzification can be performed using the formula:

$$RPN = \frac{\sum_{i=1}^k u_i \mu_{\tilde{y}}(u_i)}{\sum_{i=1}^k \mu_{\tilde{y}}(u_i)}, \text{ where } k \text{ is the selected number of } \alpha\text{-levels.}$$

The found value *PRN* should be compared with the boundary value from the range from 0.0047 to 0.0059. These values are selected by analogy with the traditional FMEA method - as 1/100 and 1/80 parts of the maximum possible value *PRN*, which is 0.472 respectively.

IV. CONCLUSION

Experimental verification of the proposed refined method of FMEA-analysis was carried out by the authors on the example of improving the design of the steering hydraulic car booster. Despite the apparent complexity of the calculations, the proposed approach may well be implemented in practice using simple software tools. At the same time, the introduction of elements of the theory of fuzzy sets into the FMEA procedure makes it possible to proceed to such a powerful decision-making tool as a fuzzy logical conclusion, which is an approximation of the "inputs-outputs" dependence based on linguistic statements "if-then" and logical operations on fuzzy sets.

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