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Modeling of the Dynamics of a Carriage Taking into Account the Geometric Nonlinearity of Displacements and Deformation

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Abstract*—A mathematical model of a carriage is presented, describing movement taking into account the geometric non-linearity of displacements and deformations in terms of the body, carriage frames, bolsters and wheel sets. In the design scheme, symmetric linear dimensions can be taken unequal, which allows evaluating the effect of tolerances on dimensions that occur in the manufacture and repair of the car, on the dynamic performance of the car. For the analysis of the dynamics of the car computational methods of numerical integration were used, as well as finding eigenvalues and eigenvectors. The matrix-vector notation of differential equations allowed us to isolate linear and non-linear components. For the linearized model, the eigenvalue problem and the vector are solved, which gives a priori idea of the resonant states of the car in dynamics. Taking into account large angular displacements the equations of the deformations of the elastic connections of the car are obtained. The linearized model is permissible to apply to the study of the oscillations of the car when driving along straight sections of the road and flat curves. The proposed model, which takes into account the geometric nonlinearity of displacements and deformations, should be used to study the dynamics of the car in steep curves.

***Keywords*— mathematical modeling; railway vehicle dynamics; nonlinear dynamics; carriage; car truck**

I. Introduction

By now, the theory of linear oscillations has acquired a complete form [1, 2]. The theory of nonlinear oscillations has no common approaches for analysis, because it is associated with mathematical problems of studying nonlinear differential equations.

Some problems of the analysis of nonlinear oscillations were studied by domestic and foreign scientists [3, 4, 5, 6, 7, 8]. These methods are mostly based on the linearization of the original problem in order to apply the mathematical apparatus of linear differential equations.

In studies of the dynamics of cars several basic directions can be traced: a full-scale experiment [9], analytical methods of research [10, 11], methods of physical modeling, methods based on numerical integration of differential equations [12, 13].

Modern computer tools allow numerical simulation methods based on the integration of differential equations to solve the problem of non-linear vibrations of the car when driving.

Solving the problems of improving the dynamic performance of a carriage due to structural changes (modernization) of the bogie requires the development of a refined spatial model of the car's oscillations as it moves along straight and curved sections.

II. Simulation Approach

The design scheme of the carriage is shown in Fig. 1. It represents the car in the form of a mechanical system consisting of nine solids which are interconnected by elastic and damping elements. The same figure shows the accepted positive direction of the coordinates.

Fig. 1 shows that:

x, y, z, θ, φ, ψ – body coordinates;

ψ_{B1}, ψ_{B2} – the coordinates of the bolsters of the first and second bogies;

x_{F1}, y_{F1}, z_{F1}, θ_{F1}, φ_{F1}, ψ_{F1} – the frame coordinates of the first bogie;

x_{F2}, y_{F2}, z_{F2}, θ_{F2}, φ_{F2}, ψ_{F2} – second dolly frame coordinates;

y_{w1}, y_{w2}, ψ_{w2}, ψ_{w2} – coordinates of the wheel sets of the first bogie;

y_{w3}, y_{w4}, ψ_{w3}, ψ_{w4} – coordinates of the wheel sets of the second bogie.

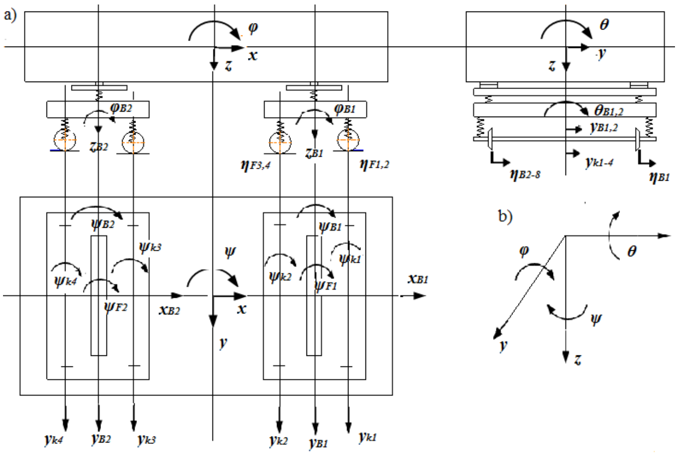

Fig. 1. The design scheme of the carriage (a) and the adopted coordinate system (b).

Based on the adopted figure 1 of the car design scheme, its state at any time is determined by the 28th coordinates. The complete set of these coordinates determines the state vector of the calculated dynamic system, i.e.:

$$\vec{U} = \{x, y, z, \theta, \varphi, \psi, \psi_{B1}, \psi_{B2}, x_{F1}, y_{F1}, z_{F1}, \theta_{F1}, \varphi_{F1}, \psi_{F1}, x_{F2}, y_{F2}, z_{F2}, \theta_{F2}, \varphi_{F2}, \psi_{F2}, y_{w1}, y_{w2}, \psi_{w2}, \psi_{w2}, y_{w3}, y_{w4}, \psi_{w3}, \psi_{w4}\}^{T} \quad (1)$$

where $\vec{U}$ – system state vector; the symbol "t" means the operation of transposing a string.

Similarly (1) it is possible to determine the state vectors by velocities $\dot{\vec{U}}$ and accelerations $\ddot{\vec{U}}$. In this case, the components of the vectors $\dot{\vec{U}}$ и $\ddot{\vec{U}}$ there will be respectively the first and second time derivatives of the coordinates of the design scheme, i.e. speed and acceleration.

Differential equations for a dynamical system are derived according to the d'Alembert principle. All symmetric linear dimensions of the design scheme are taken unequal. This is done so that in the calculations it was possible to take into account dimensional tolerances that take place in the manufacture and repair of the car.

Studies of the rolling stock dynamics as a rule are based on the assumption of small angular displacements. Such an assumption determines the linear dependence of the deformations on the angular coordinates, i.e. at small angles $\sin\psi \approx \mathrm{tg}\psi \approx \psi$, $\cos\psi \approx 1$. At large angles this assumption is not valid. Therefore along with the geometrically linear a mathematical model is proposed that does not use the assumption of small angles of rotation of the car in the plan.

III. CALCULATION METHODS USED IN COMPUTER SIMULATION OF THE CARRIAGE DYNAMICS

The calculation methods used in analyzing of the dynamics of a carriage include: a method of numerical integration and finding eigenvalues and eigenvectors.

The above differential equations describing the oscillations of all bodies of the design scheme of the car can be more compactly presented in a vector-matrix form.

$$[M]\cdot\ddot{\vec{U}} + [B]\cdot\dot{\vec{U}} + [C]\cdot\vec{U} + \vec{F} + \vec{Q} = 0, \quad (2)$$

where $\vec{U}$ – the coordinate vector of the design scheme (Fig. 1), defined by the components by expression (1);

$[M]$ – inertial matrix;

$[B]$ – damping matrix;

$[C]$ – stiffness matrix;

$\vec{F}$ – nonlinear vector defined by accepted physical and geometric hypotheses;

$\vec{Q}$ – cargo vector defined by external disturbances.

The system of differential equations (2) is written using the d'Alembert principle. It highlights the linear part, and all nonlinearities are contained in vectors $\vec{F}$ and $\vec{Q}$.

System (2) is a mathematical model of the dynamic process of moving a carriage along straight and curved sections of a railway track. Integrating this system of differential equations is a procedure for digital simulation of oscillations.

The stiffness matrix $[C]$ in (2) is determined by the stiffnesses of the elastic elements and the linear dimensions of the car. In this case the damping matrix $[B]$ has the same structure as the stiffness matrix $[C]$, in which the stiffness coefficients are replaced by the coefficients of inelastic resistance. Matrices $[C]$ and $[B]$ are always symmetric and positive definite. This property of matrices a priori determines the stability of oscillations of a linearized system. The overall structure of the matrix $[C]$ and vectors $\vec{F}$ and $\vec{Q}$ shown in figure 2, where shaded cells denote nonzero elements and empty cells denote zero.

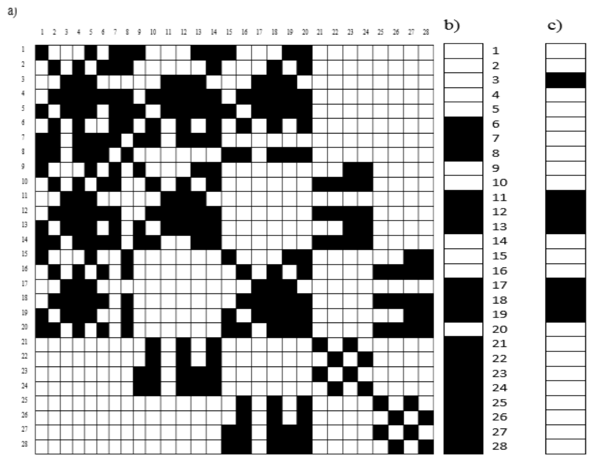

Fig. 2. Structure: a) matrix [C]; b) vector $\vec{F}$; c) vector $\vec{Q}$.

A. Method of numerical integration of differential equations

There were two basic requirements for the integration method. This is the ability to integrate differential equations in which the order of the highest derivative is two (as in the usual d'Alembert equations) and as high as possible which is determined by the number of calculations of the right side of the equation at one integration step.

The difference iterative method of integrating differential equations was used [14]. Its essence is as follows. If we have the vector form of the differential equation (2), reduced to the normal Cauchy form in the form:

$$\ddot{\vec{U}}_i = -[M]^{-1}\cdot\{[B]\cdot\dot{\vec{U}}_i + [C]\cdot\vec{U}_i + \vec{F}_i + \vec{Q}_i\} \qquad (3)$$

where the index i denotes the number of the integration step in time, then the value of the vector $\vec{U}$ the next step is looking for a Taylor expansion:

$$\vec{U}_{i+1} = \vec{U}_i + \dot{\vec{U}}_i\cdot h + \ddot{\vec{U}}_i\cdot\frac{h^2}{2!} + \dddot{\vec{U}}_i\cdot\frac{h^3}{3!}, \qquad (4)$$

where h – integration step.

Differentiating (4) by h, we get for $\dot{\vec{U}}_{i+1}$:

$$\dot{\vec{U}}_{i+1} = \dot{\vec{U}}_i + \ddot{\vec{U}}_i\cdot h + \dddot{\vec{U}}_i\cdot\frac{h^2}{2!}. \qquad (5)$$

Replacing in (4) and (5) $\dddot{\vec{U}}_i$ the difference, $\dddot{\vec{U}}_i = \frac{\ddot{\vec{U}}_{i+1}-\ddot{\vec{U}}_i}{h}$, get iterative integration formulas:

$$\left.\begin{aligned}\vec{U}_{\substack{i+1\\m}} &= \vec{U}_i + \dot{\vec{U}}_i\cdot h + \ddot{\vec{U}}_i\cdot\frac{h^2}{3} + \ddot{\vec{U}}_{\substack{i+1\\m-1}}\cdot\frac{h^2}{6};\\ \dot{\vec{U}}_{\substack{i+1\\m}} &= \dot{\vec{U}}_i + (\ddot{\vec{U}}_i + \ddot{\vec{U}}_{\substack{i+1\\m-1}})\cdot\frac{h}{2},\end{aligned}\right\} \qquad (6)$$

where m – iteration number at one integration step.

Calculations by formulas (6), (3) are repeated cyclically at one step m times until the condition:

$$\left|\vec{U}_{\substack{i+1\\m}} - \vec{U}_{\substack{i+1\\m-1}}\right| \le \vec{\xi} \qquad (7)$$

where $\vec{\xi}$ – accuracy of iterations.

If at carrying out m iterations, condition (7) is not fulfilled, then the integration step decreases (for example, it is divided in half) and the calculations by formulas (6), (3) and (7) are performed with a reduced step. Thus the described integration method does not require lowering the order of the highest derivative to unity (as provided for in most traditional methods); it works with the automatic selection of the step h in depending on the specified accuracy $\vec{\xi}$. It is quite well tested in the studies of many authors [15]. The disadvantage of this method is that when integrating high-order systems, computing time on a computer increases.

Therefore, in order to speed up the computational procedure, a modification of this method was proposed which allows obtaining closed non-iteration formulas for integration. To obtain these formulas the third derivative $\dddot{\vec{U}}_i$ in the Taylor expansions (4) and (5) is represented by the difference not in accelerations but in displacements as follows:

$$\dddot{\vec{U}}_i = -\frac{1}{h^3}\cdot[(\vec{U}_{i-2} - 2\cdot\vec{U}_{i-2} + \vec{U}_i) - (\vec{U}_{i-1} - 2\cdot\vec{U}_i + \vec{U}_{i+1})]. \qquad (8)$$

Substituting (8) into (4) and (5) we get:

$$\left.\begin{aligned}\vec{U}_{i+1} &= \frac{3}{5}\cdot\vec{U}_i + \frac{3}{5}\cdot\vec{U}_{i-1} - \frac{1}{5}\cdot\vec{U}_{i-2} + \frac{6}{5}\cdot\dot{\vec{U}}_i\cdot h + \frac{3}{5}\ddot{\vec{U}}_i\cdot h^2;\\ \dot{\vec{U}}_{i+1} &= \frac{1}{2\cdot h}\cdot(\vec{U}_{i+1} - 3\cdot\vec{U}_i + 3\cdot\vec{U}_{i-1} - \vec{U}_{i-2}) + \dot{\vec{U}}_i + \ddot{\vec{U}}_i\cdot h.\end{aligned}\right\} \qquad (9)$$

Indices i+1, i, i-1, i-2 refer to the step number of the calculation (integration). In the formula (9) $\ddot{\vec{U}}_i$ calculated from equation (3), the remaining vectors are known. The integration procedure is carried out according to the formulas (9) and (3) with the subsequent shift of the arrays i+1 in i; i in i-1; i-1 in i-2.

Thus the use of the proposed integration method using the formula (9) gives a non-iteration process which significantly reduces the computation time on a computer as compared to the iteration (6), (7) but the integration step is not automatically selected depending on the specified accuracy ξ. However the order of the integration step can be reliably determined when solving the eigenvalue problem for the system under study.

Vector differential equation (2) in its formulation represents the Cauchy problem.

To begin the integration process using formulas (9) and (3) it is necessary for this problem to know the initial conditions, i.e. vectors $\vec{U}_{i-2}$, $\vec{U}_{i-1}$, $\vec{U}_i$, $\vec{U}_{i+1}$ and $\dot{\vec{U}}_i$. Essentially vector $\vec{U}_i$ determines at the beginning of integration the coordinates of the static position of the car before driving with the speed v. In this formulation of the problem, it is assumed that before the start of the movement the carriage stood still, therefore, $\vec{U}_{i-2}$ = $\vec{U}_{i-1}$ = $\vec{U}_i$, a $\dot{\vec{U}}_i$ = 0. To define a vector $\vec{U}_i$, (3) is integrated with zero initial conditions until the moment when $\vec{U}_i$ will stop changing in the process of counting, and $\dot{\vec{U}}_i \approx 0$. Such a technique was used in the search for the initial static deformations of the car bodies [16]. At the same time external disturbances (unevenness of rails) in the process of calculating the initial conditions $\vec{U}_i$ do not change in time and depend only on the initial installation of the wheels of cars in a fixed coordinate system.

A. Calculation of the eigenvalues of the system

The solution of the eigenvalue problem is put for the equation:

$$[M]\cdot\ddot{\vec{U}} + [C]\cdot\vec{U} = 0. \qquad (10)$$

Representing the solution of equation (10) in the form $\vec{U}=\vec{U}^{*}\cdot\sin(\lambda\cdot t+\alpha)$, get the eigenvalue problem:

$$[C]\cdot\vec{U}^{*}=\lambda^{2}\cdot[M]\cdot\vec{U}^{*} \quad (11)$$

or

$$[A]\cdot\vec{U}^{*}=\lambda^{2}\cdot\vec{U}^{*} \quad (12)$$

where $[A]=[M]^{-1}\cdot[C]$.

Equation (12) represents the eigenvalue problem in standard form. If the matrix $[A]$ is positive definite and symmetric then all eigenvalues for (12) will be real numbers. The stiffness matrix $[C]$ is always symmetric and positively defined (a consequence of Newton's third law) but when multiplied by the inverse matrix $[M]$ the symmetry property is lost and the matrix $[A]$ is asymmetric.

The method of symmetrization [17] of the matrix in problem (12) was applied. For the particular case when the matrix $[M]$ is diagonal, equation (12) is reduced to the form:

$$[A_u]\cdot\vec{X}_u=\lambda^{2}\cdot\vec{X}_u, \quad (13)$$

where $[A_u]=[M]^{-1/2}\cdot[C]\cdot[M]^{-1/2}$.

Thus we have the standard form of the eigenvalue problem in the form (13) with a symmetric matrix $[A_u]$ which guarantees the validity of all eigenvalues λ^2.

As you can see equation (13) is written for the new eigenvector $\vec{X}_u$.

To return to the original vector of its own $\vec{U}^{*}$:

$$\vec{U}^{*}=[M]^{-1/2}\cdot\vec{X}_u. \quad (14)$$

In solving problem (11) relations (13), (14) were used. Determining the eigenvalues λ^2 the car's own frequency spectrum is determined and a restriction is given for the integration step in the form:

$$h\le T/20=\pi/(10\cdot\lambda), \quad (15)$$

where T – period of the higher frequency spectrum.

B. Consideration of geometric non-linearity of spring deformations at large wagging angles of the body, bolsters, bogies frames and wheel sets in curves

As already mentioned at low angles take $\sin\psi\approx\psi$, $\cos\psi\approx1$ (hypothesis of geometric linear displacements and deformations). If we reject such an assumption then displacements and deformations in the longitudinal and transverse directions will be expressed through the functions sinψ и cosψ.

For example, consider the components of the longitudinal and transverse deformations of the springs of the central suspension when turning the bolster and the frame of the bogie at an angle ψ (Fig. 3). The points at which the axes of symmetry of the springs are located are denoted by 1, 2, 3, and 4.

The longitudinal movement of point 1 (Fig. 3) with the turn of the bolster of the first bogie at an angle ψ_{F1} will be:

$$X_{F1}=E_1-\rho_{F1}\cdot\sin(\psi_{o1}-\psi_{F1}), \quad (16)$$

where E_1 – constructive size (distance from the axis of symmetry of the bolster to the center of the spring);

ρ_{F1}·– radius from the center of the bolster bogie to point 1;

ψ_{o1} – the angle between the longitudinal axis of symmetry of the bolster and the radius ρ_{F1}.

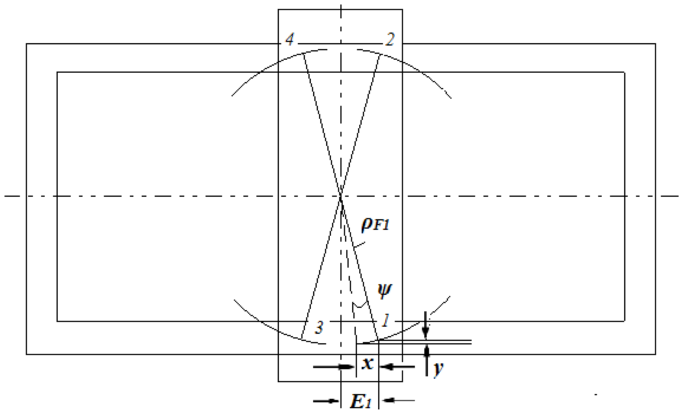

Fig. 3. Longitudinal and transverse deformations of the central suspension springs when turning the bolster of the bogie frame.

The expression (16) can be represented:

$$x_{F1}=E_1-\rho_{F1}\cdot\sin\psi_{o1}\cdot\cos\psi_{F1}-\rho_{F1}\cdot\cos\psi_{o1}\cdot\sin\psi_{F1}). \quad (17)$$

From the diagram in figure 3 it follows: in expression (17) members $\rho_{F1}\cdot\sin\psi_{o1}=E_1$, $\rho_{F1}\cdot\cos\psi_{o1}\cdot=D_1$ then:

$$x_{F1}=E_1\cdot(1-\cos\psi_{F1})+D_1\cdot\sin\psi_{F1}. \quad (18)$$

Using similar reasoning we can write the expression for the transverse displacement of point 1 when the beam is rotated by an angle $\psi_{F1:}$

$$y_{F1}=E_1\cdot\sin\psi_{F1})-D_1\cdot(1-\cos\psi_{F1}). \quad (19)$$

When turning the frame of the cart at an angle ψ_{B1} longitudinal and transverse movements of point 1 will be:

$$\left.\begin{aligned} x_{B1}&=E_1\cdot(1-\cos\psi_{B1})+D_1\cdot\sin\psi_{B1}.\\ y_{B1}&=E_1\cdot\sin\psi_{B1}-D_1\cdot(1-\cos\psi_{B1}).\end{aligned}\right\} \quad (20)$$

In the design of the bogie, the bolster is associated with the upper support surface of the spring and the frame is connected with the lower one; therefore for the rule of signs adopted in the design scheme (Fig. 1), the deformation of the first spring $\bar{x}_1$, determined by the angular displacements will be $\bar{x}_1=x_{T1}-x_{Б1}$ or taking into account (18) and (20) we have:

$$\bar{x}_1 = E_1 \cdot (\cos\psi_{F1} - \cos\psi_{B1}) + D_1 \cdot (\sin\psi_{B1} - \sin\psi_{F1}) \quad (21)$$

The deformation values for other springs of the central suspension are determined similarly.

Then taking into account (21) the expression of the longitudinal deformations of the springs of the central suspension will take the form

$$\left.\begin{aligned}
\Delta_{x1} &= x - x_{b1} - \varphi \cdot H_1 - \varphi_{B1} \cdot h_{p1} + E_1 \cdot (\cos\psi_{F1} - \cos\psi_{B1}) + D_1 \cdot (\sin\psi_{B1} - \sin\psi_{F1});\\
\Delta_{x2} &= x - x_{b1} - \varphi \cdot H_1 - \varphi_{B1} \cdot h_{p1} + E_2 \cdot (\cos\psi_{F1} - \cos\psi_{B1}) + D_2 \cdot (\sin\psi_{F1} - \sin\psi_{B1});\\
\Delta_{x3} &= x - x_{b1} - \varphi \cdot H_1 - \varphi_{B1} \cdot h_{p1} + E_3 \cdot (\cos\psi_{B1} - \cos\psi_{F1}) + D_3 \cdot (\sin\psi_{B1} - \sin\psi_{F1});\\
\Delta_{x4} &= x - x_{b1} - \varphi \cdot H_1 - \varphi_{B1} \cdot h_{p1} + E_4 \cdot (\cos\psi_{B1} - \cos\psi_{F1}) + D_4 \cdot (\sin\psi_{F1} - \sin\psi_{B1});\\
\Delta_{x5} &= x - x_{b2} - \varphi \cdot H_2 - \varphi_{B2} \cdot h_{p2} + E_5 \cdot (\cos\psi_{F2} - \cos\psi_{B2}) + D_5 \cdot (\sin\psi_{B2} - \sin\psi_{F2});\\
\Delta_{x6} &= x - x_{b2} - \varphi \cdot H_2 - \varphi_{B2} \cdot h_{p2} + E_6 \cdot (\cos\psi_{F2} - \cos\psi_{B2}) + D_6 \cdot (\sin\psi_{Б2} - \sin\psi_{T2});\\
\Delta_{x7} &= x - x_{b2} - \varphi \cdot H_2 - \varphi_{B2} \cdot h_{p2} + E_7 \cdot (\cos\psi_{B2} - \cos\psi_{F2}) + D_7 \cdot (\sin\psi_{B2} - \sin\psi_{F2});\\
\Delta_{x8} &= x - x_{b2} - \varphi \cdot H_2 - \varphi_{B2} \cdot h_{p2} + E_8 \cdot (\cos\psi_{B2} - \cos\psi_{F2}) + D_8 \cdot (\sin\psi_{F2} - \sin\psi_{B2}).
\end{aligned}\right\} \quad (22)$$

When describing the lateral deformations of the central suspension springs we use the design scheme (Fig. 3).

The transverse displacement of point 1 when the beam is rotated by an angle ψ_{F1} and bogie frames at an angle ψ_{B1} will be described by expressions (19) and (20).

Then similar to expressions (22) the lateral deformations of the springs of the central suspension taking into account the geometric nonlinearity of displacements will have the form:

$$\left.\begin{aligned}
\Delta_{y1} &= y - (L_1 + E_1) \cdot \sin\psi - \theta \cdot H_1 + D_1 \cdot (\cos\psi_{B1} - \cos\psi_{F1}) + E_1 \cdot (\sin\psi_{F1} - \sin\psi_{B1}) - U_{n1};\\
\Delta_{y2} &= y - (L_1 + E_2) \cdot \sin\psi - \theta \cdot H_1 + D_2 \cdot (\cos\psi_{T1} - \cos\psi_{F1}) + E_2 \cdot (\sin\psi_{F1} - \sin\psi_{B1}) - U_{n2};\\
\Delta_{y3} &= y - (L_1 + E_3) \cdot \sin\psi - \theta \cdot H_1 + D_3 \cdot (\cos\psi_{F1} - \cos\psi_{B1}) + E_3 \cdot (\sin\psi_{B1} - \sin\psi_{F1}) - U_{n1};\\
\Delta_{y4} &= y - (L_1 + E_4) \cdot \sin\psi - \theta \cdot H_1 + D_4 \cdot (\cos\psi_{F1} - \cos\psi_{B1}) + E_4 \cdot (\sin\psi_{B1} - \sin\psi_{F1}) - U_{n2};\\
\Delta_{y5} &= y - (L_2 + E_5) \cdot \sin\psi - \theta \cdot H_2 + D_5 \cdot (\cos\psi_{B2} - \cos\psi_{F2}) + E_5 \cdot (\sin\psi_{F2} - \sin\psi_{B2}) - U_{n3};\\
\Delta_{y6} &= y - (L_2 + E_6) \cdot \sin\psi - \theta \cdot H_2 + D_6 \cdot (\cos\psi_{B2} - \cos\psi_{F2}) + E_6 \cdot (\sin\psi_{F2} - \sin\psi_{B2}) - U_{n4};\\
\Delta_{y7} &= y - (L_2 + E_7) \cdot \sin\psi - \theta \cdot H_2 + D_7 \cdot (\cos\psi_{F2} - \cos\psi_{B2}) + E_7 \cdot (\sin\psi_{B2} - \sin\psi_{F2}) - U_{n3};\\
\Delta_{y8} &= y - (L_2 + E_8) \cdot \sin\psi - \theta \cdot H_2 + D_8 \cdot (\cos\psi_{F2} - \cos\psi_{B2}) + E_8 \cdot (\sin\psi_{B2} - \sin\psi_{F2}) - U_{n4}.
\end{aligned}\right\} \quad (23)$$

where $U_{n1\text{-}4}$ – lateral movements of pallets of cradle suspension.

The longitudinal and transverse deformations of the axle suspension springs are determined based on the movements of the corresponding frame points and wheel pairs. Formulas for their determination are obtained in the same way as the above example.

Thus obtained longitudinal and transverse movements of the points of the frames and wheel sets which are determined by the angles ψ_B and ψ_k included in the expression of the longitudinal and transverse deformations of the springs of the axle suspension stage.

Without cluttering up the presentation of this issue with calculations it can be noted that later in describing the relative displacements and relative velocities in the elements of the design scheme which were written for small angles taking into account the geometric nonlinearity is based on the replacement of the angle ψ on $\sin\psi$.

On the basis of the stated mathematical models two versions of programs for personal computers were developed. One program has been developed for a mathematical model using the assumption of geometric linearity of displacements and deformations. Another version of the program was developed taking into account the features outlined in the article, which take into account the geometric nonlinearity of the problem. Both of these programs were tested so that at small angles of rotation in the plan (movement in straight sections) the results of the calculations almost coincided.

The reliability of the mathematical model and software for PC is confirmed by the convergence of the results of calculation and experiment.

Thus a mathematical model of a carriage has been proposed taking into account geometric non-linearity due to large angles of rotation of the body, bolsters, bogie frames, wheel sets in plan.

IV. CONCLUSIONS

1. To study the dynamics of a carriage a spatial design scheme has been developed and general differential equations of motion in straight and curved sections of the track have been written.

2. In the design scheme symmetric linear dimensions can be taken to be unequal which allows to evaluate the effect of tolerances on the dynamic performance of the car.

3. Matrix-vector notation of differential equations allowed us to isolate linear and non-linear components. For the linearized model the eigenvalue problem and the vector are solved. This gives a priori idea of the resonant states of the car in dynamics. In addition, the value of the frequency spectrum allows to determine the upper limit of the integration step in computer simulation.

4. The equations of the deformations of the elastic connections of the car taking into account large angular displacements are obtained.

5. A mathematical model of a carriage has been developed which describes the movement with regard to the geometric nonlinearity of movements and deformations determined by not small angular displacements in terms of the body, bogie frames, bolsters and wheel sets.

6. To study the oscillations of a car when driving along straight sections of the road and flat curves, it is permissible to use the assumption of smallness of angular oscillations (the hypothesis of geometric linearity). In the study of dynamics in steep curves it seems reasonable to apply the proposed model taking into account the geometric nonlinearity of displacements and deformations.

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