

# Mathematical Modeling of Composite and Inhomogeneous Deformable Materials Based on Scanning and Finite Element Method

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**Abstract** – The paper presents a method of mathematical modeling of composite and heterogeneous deformable materials used, in particular, in modern aircraft structures, such as MS-21. The technique is based on scanning a deformed body by a computer tomograph, with further mathematical transformation of the information obtained for its use in the finite element method analyzing the stress-strain state of parts made of composite and heterogeneous materials. Traditionally, the mechanical characteristics of a real deformable solid body are given as an average value, for example, common for the material of the part and obtained during the test of standard samples. The main purpose of the presented technique is, along with improving the accuracy of calculations, the determining of the level of defects in the material and the degree of its impact on the performance of the product as a whole. The method for determining the real change in the mechanical characteristics of the material of deformable solid bodies is based on the use, on the one hand, of the pixel characteristics of raster images scanned by a computer tomograph; on the other hand, the averaged data on the mechanical characteristics of the material obtained as a result of full-scale tests of standard samples are used. The result of this approach makes it possible to simulate these inhomogeneities in the construction of a finite element model of real deformable solid bodies. The developed approach can be applied to any physical principles of scanning, such as x-ray, ultrasonic, laser and etc., as well as for all types of materials when the information obtained as a result of scanning is formed in the form of a digital (raster) image.

**Key words** — mechanical characteristics of the material; deformable solid body; heterogeneity; finite element method; CT scanning

## I. INTRODUCTION

The composite wing of MS-21 is a complex and innovative product. The most modern Russian fighter Su-57 is a quarter of composite parts, and 70 % of the wing covering of the

aircraft is made of carbon fiber. The researchers face the task of creating objective methods for modeling and analysis of stress-strain states of progressive structures.

In modern research and development works, the practice of mathematical modeling of objects by the finite element method (FEM) is increasingly developing, which allows to solve a wide range of problems and, in particular, to effectively analyze the stress-strain state (SSS) of solids [1-3]. The development of the presented mathematical modeling is aimed at obtaining a finite element (FE) model of a deformable solid body (DSB), corresponding to the real object as much as possible, which directly depends on the task in the model of geometric parameters and mechanical characteristics.



Fig. 1. Examples of materials with non-uniform mechanical characteristics: a – material of natural origin (concrete); b – metal; c – composite materials

The determination of these characteristics in the FE model of the DSB in modern practice involves assumptions caused, for example, by their averaging, which ultimately affects the accuracy of the results of the use (analysis) of the mathematical model. That is, traditionally, the mechanical characteristics of the DSB are determined on the basis of the test results of standard samples, which lead to the use of their averaged values in the FE model. While within the parts and even the sample there is a significant heterogeneity (Fig. 1) of mechanical characteristics of the material. The simplifications presented in the model of real products (DSB) can lead to an inaccurate assessment of their reliability.

II. RESEARCH METHOD

A. The basic principle of modeling the heterogeneity of the mechanical characteristics of the material

To solve the presented problem, the paper proposes to use the technology of computer tomographic scanning of the DSB, which makes it possible to recognize changes in the mechanical characteristics of the material, and most importantly, to use these data to improve the accuracy and realism in the mathematical FE simulation of the real DSB [4, 5].

A real change in the mechanical characteristics of the material of the natural origin, based on the processing of their scanning raster image, is presented in Fig. 2.

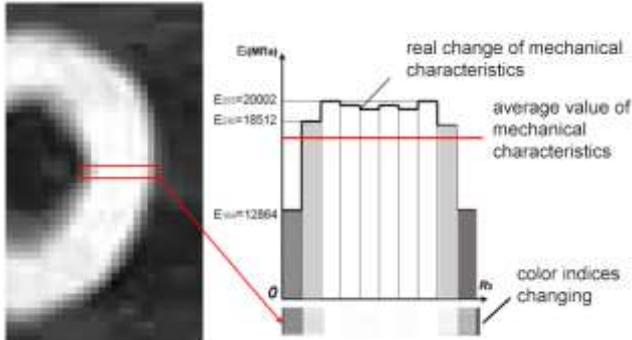


Fig. 2. Real change of mechanical characteristics from the values of color indices

To study the structure and, accordingly, the structure of mechanical characteristics, in [6-10] the authors scanned the samples of composite material using micro- and nano-computer tomography. The results showed the ability to view the location and direction of the fibers, as well as the location of voids within the structure. In addition, in works [6, 10], the places of internal cracks are shown. Thus, scanning of the DSB is an important and the best method of research of heterogeneity of its material and an assessment of production quality.

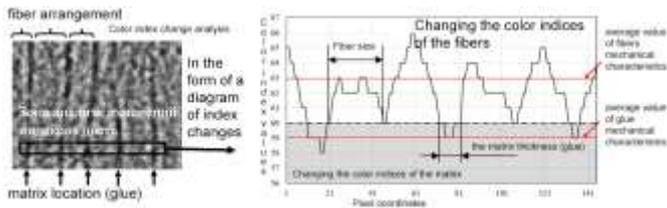


Fig. 3. Analysis of a raster image of a composite material scan

In Fig. 3 the result of scanning of the composite material in the form of a raster image of the sample cross-section of a regular shape (scanning is completed by V. G. Tolstikov) is demonstrated. Except for the designation of a real change in the mechanical characteristics of the fibers and the matrix (glue) in the structure, the scanning result also shows the possibility to assess the quality of composite materials manufacture (fiber arrangement, thickness of the glue layer, internal defects, etc.).

As a result of scanning, a batch of images is formed, each of which is a raster drawing [11, 12]. To simulate the heterogeneity of the material, a principle based on two components is used: – the pixel characteristics of these scanning raster images; and – the data of full-scale tests of standard samples.

The determination of the dependence of mechanical characteristics, in the form of an elastic modulus, on the value of the color indices is carried out using a specific weighting coefficient that establishes a relationship between the average value of the color indices  $n_{cp}$  and the experimental value  $E_{OII}$  [4, 5].

The average value of the modulus of elasticity ( $E_{OII}$ ), obtained in the experiment of the standard samples compression (tension), is used in the interpretation of the changes of the mechanical characteristics according to the pixel characteristic.

Therefore, the weight factor determining the transition from the pixel color index to the modulus of elasticity is defined by the expression of the form:

$$k_E = \frac{E_{OII}}{n_{cp}} \tag{1}$$

Thus, assigning the values of the elastic modulus of the material  $E_i$  to the pixels, each of which has a corresponding color index, will be made using the weighting coefficient  $k_E$ , according to the formula:

$$E_i = n_i k_E, \tag{2}$$

where  $E_i$  – the elastic modulus value corresponding to the value of the color index number  $n_i$ ;  $k_E$  – the weight coefficient of the modulus of elasticity.

The implementation of dependencies (1) and (2) is performed by solving the following tasks:

- obtaining the average and/or typical value of the color indices of a raster image package;
- assessing the dependence of mechanical characteristics (linear or nonlinear) on the values of color indices;
- determining mechanical characteristics of the material in nodes and finite elements of the FE model.

A detailed description is presented in certain works [4, 5].

B. Linear and nonlinear dependence of the elastic modulus on the color index values of scanning raster images

The change in the modulus of elasticity is determined on the basis of linear or nonlinear dependence on the values of the color indices. For the material with a low degree of inhomogeneity, the dependence can be constructed using the linear law (2), for the identification of which the only constant value of the weight coefficient of the elastic modulus is used.

However, in nature and technology such materials are rare. Most often, the DSB material has a certain structure with respect to the mechanical characteristics, in which several

areas can be selected. Each of them has its own, different from the other, range of modulus of elasticity.

In Fig. 4 a raster image obtained under a microscope is presented, where the structure of the material in the form of fibers and matrices is visible. That is, the composite material consists of the fiber and matrix and their mechanical characteristics are very different from each other. Approximately, the data on the elastic modulus of the fibrous composite material, where for glass fiber it is about  $E = 50$  GPa, for the epoxy matrix  $E = 2.5$  GPa [13].

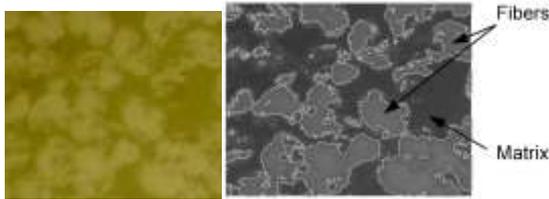


Fig. 4. The structure of the composite material according to mechanical properties

For such materials, in each area of the structure of the DSB it is proposed to use its own weight coefficient. In this case, the dependence of the change in the modulus of elasticity of the DSB on the color indices is represented as a piecewise linear function (Fig. 5).

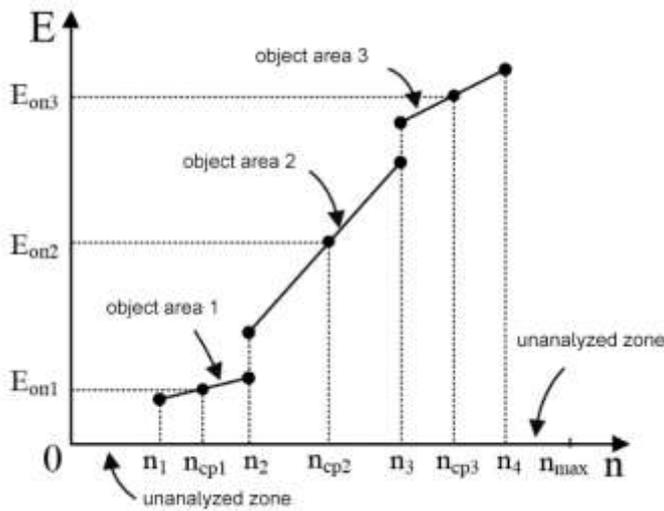


Fig. 5. Piecewise linear function of the dependence of the elastic modulus of the DSB on the values of color indices

The piecewise linear function of the dependence of the elastic modulus change on the value of the color indices  $E_f(n)$  for the DSB as a whole is written as:

$$E_f(n) = \begin{cases} n \frac{E_{on_i}}{n_{cp_i}}, & n \in [n_i, n_{i+1}] \\ 0, & n \notin [n_i, n_{i+1}] \end{cases} \quad (1)$$

The use of a piecewise linear function is difficult in terms of the efficiency of the computational algorithm, for example, for the following reasons:

- the greater the number of the areas of the DSB structure is, the greater the number of weight coefficients and the more difficult the calculation of the elastic modulus for nodes and finite elements is;
- the transition values of the color indices between the DSB areas or objects are not a single value, but the ranges of the color indices values in the raster image (Fig. 6) and others.

To solve the presented problem, it is suggested to construct a nonlinear dependence in the form of a spline, for example, a cubic one, on the basis of a piecewise linear function.

The discussed nonlinear dependence of the modulus of elasticity of the DSB on color indices is universal for combining several zones with different order of mechanical characteristics (Fig. 6).

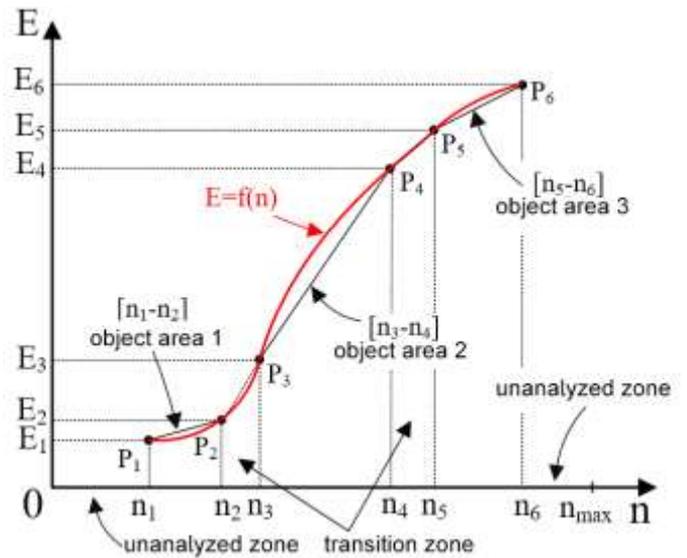


Fig. 6. Continuous function of the elasticity modulus dependence of the DSB on the values of color indices

Thus, the calculation of the value of the elastic modulus of the DSB is determined as a continuous function of the color index value, having the form:

$$E = f(n). \quad (4)$$

To implement this formula, it is necessary to construct an interpolation function through the obtained points of the piecewise continuous function, which are the limits of the color indices denoting the zones of different regions of the DSB structure (Fig. 6). Through a set of points  $P_i(E_i, n_i)$  the construction of the interpolation function occurs. These coordinates are defined by expressions of the form:

$$E_i = \frac{E_{on_i}}{n_{cp_i}} \cdot n_i, \quad (5)$$

$$E_{i+1} = \frac{E_{on_i}}{n_{cp_i}} \cdot n_{i+1}, \quad (6)$$

where  $i = 1, 3, 5 \dots$  – an odd integer indicating the initial color index of the regions of the DSB structure (Fig. 6).

To construct the function (4) in this paper, a cubic spline based on Bezier curves [14] is used, as it provides a smaller error compared to linear interpolation.

Further, the interpretation of scan data is associated with the interpolation and transfer of the color indices values from pixels to nodes in the FE mesh. The most complex and general case occurs when the nodes of the FE grid lie between adjacent planes with a raster image, having significant differences in shape and size, and are characterized by the appearance of additional internal contours and other geometric features. In this case, a number of mathematical dependencies defined with respect to the straight line inclined to the Z axis is used for interpolation. A more detailed description is presented in [15].

### III. ALGORITHM AND SOFTWARE IMPLEMENTATION

In Fig.7 a general structure of the algorithm reflecting the full range of modules is shown: from the DSB scanning to obtaining its FE model and SSS analysis.

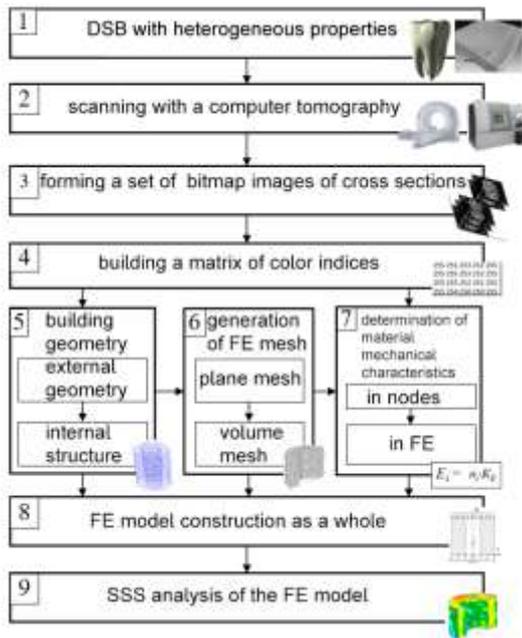


Fig. 7. Mathematical and FE modeling of the real DSB

The block diagram of the module algorithm for determining the mechanical characteristics of the material in the nodes and finite elements of the DSB FE model (Fig. 7, block 7) is shown in Fig. 8.

The main stages of the algorithm (Fig. (7)) are:

1. Defining the coordinates of the finite element nodes that are written in the  $Nodes(n,5)$  matrix. The topology of node numbers in finite elements is written in the matrix  $Elms(m,10)$  – for the elements of HEX type or  $Elms(m,6)$  – for the elements of TET type.

2. Checking the coordinates of nodes for location in the raster image package sections or in the space between sections.

3, 4. Sorting nodes by increasing z-coordinate and the solution of the problem of the package (the rejection) of the nodes within the space between the sections.

5. The assignment of the index number of the pixel color to each node of the FE mesh section. The result of the determination of the color indices of the nodes is populated in the matrix  $Nodes(n,5)$ .

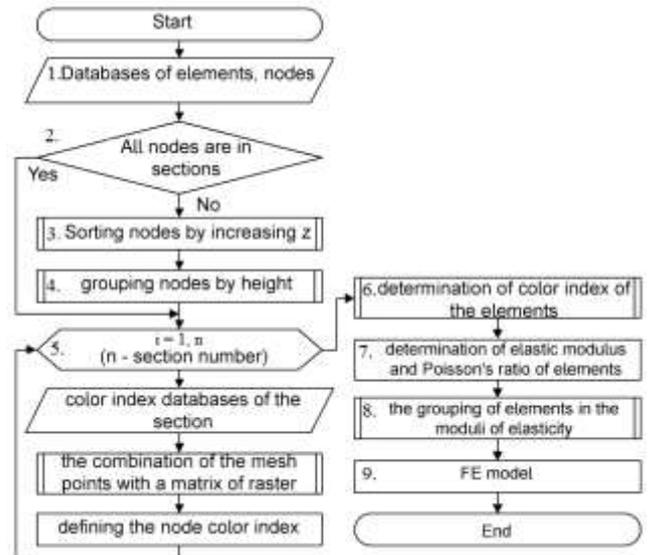


Fig. 8. Block diagram of the data processing algorithm of nodes, finite elements and a scanning raster image package to determine the real change in the mechanical characteristics of the material in the FE model

6. Defining the color indices of each finite element as the arithmetic mean of the color indices of its nodes. The result is filled into the matrix  $Elms(m,10)$  or  $Elms(m,6)$ .

7. The determination of the real change in the mechanical characteristics of the finite elements material in the form of modulus of elasticity.

8. Packing the final items into groups with identical values of the modulus of elasticity or rejection of finite elements within the same range of variation of modulus of elasticity;

9. The result is written as a structure file \*.pcl.

#### Patran Command Language

To be able to use the results of determining the heterogeneity of the mechanical characteristics of the material in the FE model, it is necessary to convert the results in the form of a file \*.pcl, using the Patran Command Language [16,17]. To do this, the functions of the language «material.create» and «elementprops\_create» are used.

#### «Material.create»

The function «material.create» [16, 17] is to present the obtained change in mechanical characteristics (modulus of

elasticity, Poisson's ratio, etc.) in the database of the FE model. The structure of the function «material.create» is shown in Table 1.

TABLE I. STRUCTURE OF THE FUNCTION «MATERIAL.CREATE» PATRAN COMMAND LANGUAGE

Function name	Structure of the function
material.create	(analysis_code, analysis_code_id, analysis_type, analysis_type_id, material_name, material_id, material_descrip, material_cat, material_cat_id, directionality, directionality_id, linearity, linearity_id, mat_type_name, mat_type_id, const_model_name, const_model_id, options, option_names, option_ids, active, active_status, action, material_exists, external, external_status, prop_ids_label, prop_names, prop_ids, prop_values_label, prop_values)

**Code pattern:** `material.create( "Analysis code ID", 1, "Analysis type ID", 1, "Steel", 0, "Date: 20-Apr-17 Time: 0:11:34", "Isotropic", 1, "Directionality", 1, "Linearity", 1, "Homogeneous", 0, "Linear Elastic", 1, "Model Options & IDs", [ "", "", "", "", "" ], [ 0, 0, 0, 0, 0 ], "Active Flag", 1, "Create", 10, "External Flag", FALSE, "Property IDs", [ "Elastic Modulus", "Poisson Ratio" ], [ 2, 5, 0 ], "Property Values", [ "210000", "0.30", "" ])`

«Elementprops\_create»

The «elementprops\_create» function [16, 17] assigns to each finite element its mechanical characteristics of the material in the FE model. Its structure is presented in Table 2.

TABLE II. STRUCTURE OF THE FUNCTION «ELEMENTPROPS\_CREATE» PATRAN COMMAND LANGUAGE

Function name	Structure of the function
elementprops_create	(prop_name, elem_type_id, geo_opt_id, con_opt_id, for_opt_id, lam_opt_id, dof_opt_id, property_ids, property_types, property_data, app_region)

For example, to assign to a finite element of number 235 the property of the material “Steel” obtained, up, by the function «material.create».

**Code pattern:** `elementprops_create( "Telo_Steel", 71, 25, 30, 1, 1, 20, [ 13, 21, 4124, 4126, 4125, 8111, 4401, 4402, 4403, 4404, 4405, 4406, 4407, 4410, 4411 ], [ 5, 4, 4, 4, 4, 4, 4, 1, 1, 1, 1, 1, 1, 6 ], [ "m:Steel", "", "", "", "", "", "", "", "", "", "", "", "" ], [ "m:Steel", "", "", "", "", "", "", "", "", "", "", "", "" ], [ "m:Steel", "", "", "", "", "", "", "", "", "", "", "", "" ], "Element 235")`

In Fig. 9 the interface of the program for determining the inhomogeneity of the mechanical characteristics of the material in the nodes and finite elements is presented.

IV. STUDY OF THE ACCURACY AND CONVERGENCE OF THE USE OF THE PRESENTED TECHNOLOGIES IN THE CONSTRUCTION AND ANALYSIS OF FE MODELS

As an example of the investigated DSB, the materials of natural origin are used in the work. The presented choice of the DSB and its analysis is not fundamental, but they are due to two important circumstances: a high degree of

heterogeneity of the natural origin material and the individuality of its geometry, as well as a high level of technology and quality of CT scanning in medicine and technology. It should also be noted that the natural origin material is well studied in practice that contributes to a high level of control of the correctness (reliability) of its mathematical modeling.

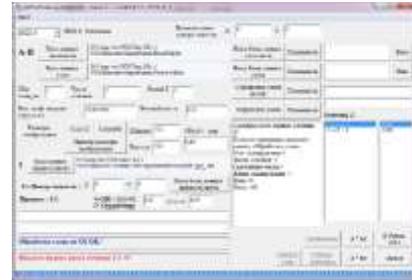


Fig. 9. Program interface for determining the heterogeneity of the mechanical characteristics of the material in the nodes and finite elements

The first study uses samples in the form of parallelepipeds (Fig. 10) having real prototypes [18–21].

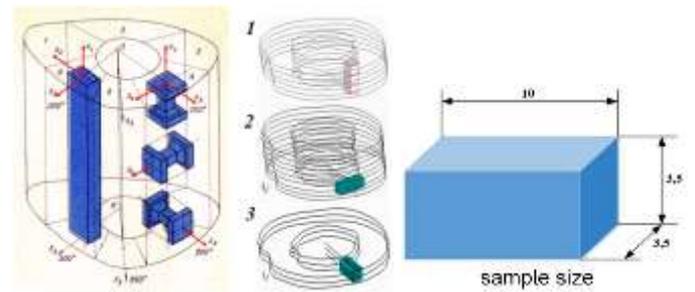


Fig. 10. Dimensions and orientation of real samples: 1 – axial; 2 – circumferential; 3 – radial

The results of the study are shown in Fig. 11, as a proof of the accuracy and convergence of the FEM numerical solution with respect to the data of the full-scale experiment [18–19].

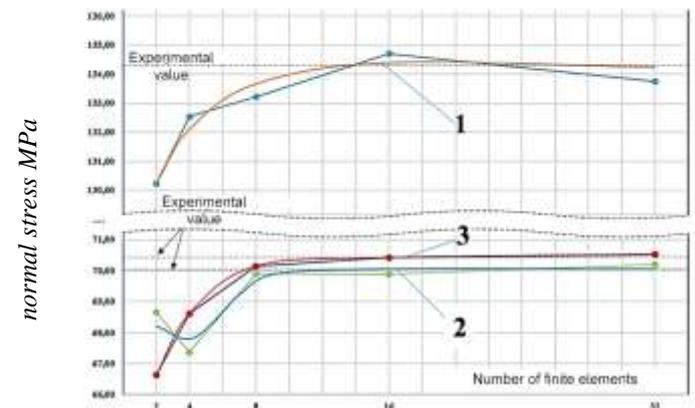


Fig. 11. The graph of convergence of the normal stress in the center of the samples, in the directions: longitudinal (1), circumferential (2) and radial (3)

The results show that, in order to achieve the required accuracy of the FE numerical solution, a FE grid with a density of 3 or more finite elements by 1 mm is required. Furthermore, the result shows that the inhomogeneity property

of the mechanical characteristics of the DSB material can be reflected by a set of finite elements, each with its elastic modulus and isotropic structure of the material.

An additional research was carried out to study the applicability of two different types of the volumetric FE (tetrahedron and hexahedron) using incompatible form functions [20].

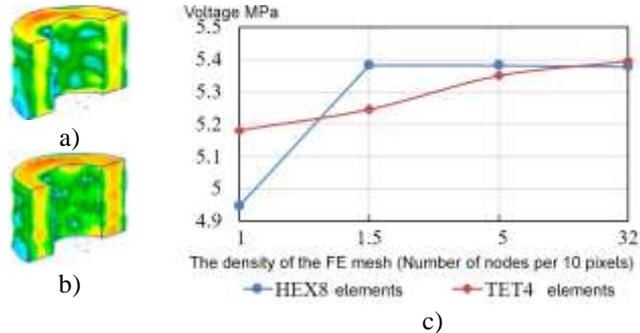


Fig. 12. The result of the SSS analysis with the use of the FE type HEX (a) and TET (b); and a graph of convergence results (c)

The results show that the FE of hexahedron type has the best indicators of accuracy and convergence of the results (Fig. 12), as well as resource costs.

#### V. CONCLUSION

A set of mathematical methods for modeling the interpretation of raster images of CT scanning is presented, which allows studying any complex structures of the real DSBs, the results of which are used in the construction of their FE models, taking into account the heterogeneity of the mechanical characteristics of the material in them.

The results of the study of accuracy and convergence also confirm the accuracy and reliability of the application of the presented technology of simulation based on scanning of the real DSBs taking into account the heterogeneity of the mechanical characteristics of the material.

The developed technology of mathematical modeling can be applied to any physical principles of scanning, such as x-ray, ultrasonic, laser, etc. and for all types of materials, when the information obtained as a result of scanning is formed in the form of a digital (raster) image.

This method is a tool for studying the stress-strain state of real aircraft components and panels made of composite and other materials with heterogeneity of the distribution of mechanical characteristics of the material used for advanced aircraft MS-21.

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