

# Mathematical Modelling of the Power Regulator by the Example of an A11VO Axial Piston Pump (Bosch Rexroth)

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**Abstract** – The purpose of the article is to develop a mathematical model of the power regulator using an A11VO axial-piston pump (Bosch Rexroth) as an example and determine the effect of pressure on the control process parameters. A mathematical model of the regulator was developed; a scheme for its solution in the Matlab-Simulink environment was suggested; dependencies of the spool movement ( $x_1$ ), piston ( $x_2$ ), transition time ( $t$ ) and other parameters on the pressure in the pressure line ( $p$ ) have been determined. The mathematical model of the regulator establishes the effect of pressure on control process parameters. When the pressure increases from 30 to 45 MPa, the regulator spool moves at a distance varying from 0.4 - 2.8 mm. The spool movement is proportional to the pressure. The piston movement is more sensitive to the change in pressure. It changes within the range of 3–19 mm, while the relationship is non-linear. The cylinder pressure and the ratio of rocker arms are proportional to the pressure and increase 1.5 times; the transition time of is expressed by a non-linear dependence and decreases two-fold.

**Keywords** – volumetric hydraulic drive, adjustable axial piston pump, regulator, pressure in the pressure line.

## I. INTRODUCTION

The adjustable hydraulic power drive is used in machines, including construction and road, forestry and agricultural ones. Large hydraulic machines have small dimensions and weight, high power density and torque output, reliable overload protection, and an independent layout. This makes a large hydraulic drive compact, manageable, and convenient [1-4].

Currently, the development of hydraulic drives is aimed at increasing its energy efficiency through the use of volumetric adjustable machines with hydraulic regulators. These are high-tech systems that include the power section and the pump control system. The control functions are performed by regulators which can maintain constant pressure, change the pump flow, and limit power.

## II. METHODS AND MATERIALS

The purpose of the article is to develop a mathematical model of the power regulator using an A11VO axial-piston pump (Bosch Rexroth) as an example and determine the effect of pressure on control process parameters.

Figure 1 shows a regulator's design scheme.

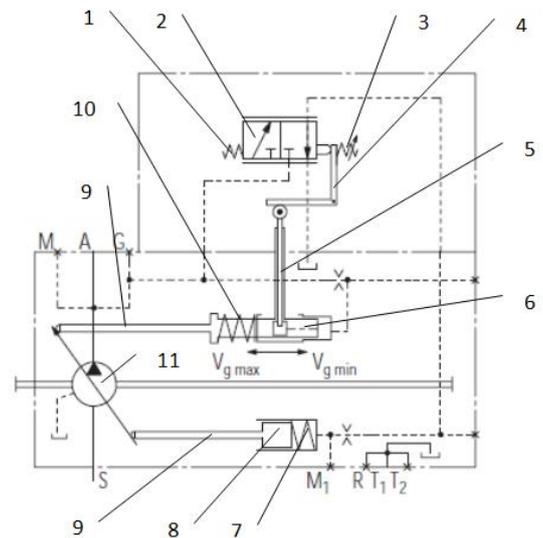


Fig. 1. Regulator design scheme:

- 1 - spring; 2 - 2 position spool;
- 3 - adjustable spring; 4 - rocker;
- 5 - plunger; 6 - the piston; 7 - spring
- 8 - cylinder; 9 –burden system; 10 - spring;
- 11 - adjustable pump

The regulator consists of two-position spool 2, spring 1, rocker arms 4 with adjustable spring 3, piston 6 with plunger 5

and spring 10, cylinder 8 with spring 7, rod system 9 connecting piston 6 and cylinder 8 with inclined pump washer 11. The outlet of pump 11 is connected to one of the lines of spool 2 and piston 6; cylinder 8 is connected to the drain through the second line of spool 2.

At the initial moment, the pump outlet pressure is  $p = 0$ ; under the action of adjustable spring 3 and rocker 4, spool 2 moves to the extreme left position connecting cylinder 8 with the drain. Under the action of springs 7 and 10 and rod system 9, piston 6 occupies an extreme right position where the inclined washer of pump 11 is in the minimum flow position.

The piston outlet pressure is  $0 < p < p_{nom}$  is not sufficient to turn rocker 4 and release spool 2 from the action of adjustable spring 3; piston 6 remains in the maximum flow position.

When the pump outlet pressure is  $p > p_{nom}$ , plunger 5 overcomes the force of adjustable spring 3; the force which tends to move it to the left begins acting. Since the cavity of cylinder 8 is connected to the tank, piston 6 overcoming the force of springs 7 and 10 moves to the extreme left position, and through the system of rods 9 transfers the inclined washer of pump 11 to the maximum flow position. The effort of plunger 5 is not enough to rotate rocker 4 and release valve 2 from the action of adjustable spring 3. Therefore, piston 6 remains in the maximum flow position.

At the pump outlet pressure  $p > p_{nom}$ , plunger 5 overcomes the force of adjustable spring 3; under the action of spring 1, spool 2 begins to move to the right, connecting the outlet of pump 11 with cylinder 8.

Through rod system 9, the pressure in cylinder 8 and the force of springs 7 and 10 move piston 6 with plunger 5 to the right reducing the length of lower rocker arm 4. Due to the difference in moments created by plunger 5 and adjustable spring 3, rocker 4 turns counterclockwise and, overcoming the force of spring 1, moves spool 2 to its original position. Cylinder 8 is connected with the drain; piston 6 occupies a new equilibrium position corresponding to the reduced pump flow. Thus, when the pressure increases, the regulator reduces the pump flow rate not changing the pump power rate.

### III. RESULTS

The works [7-11] are devoted to the mathematical description of elements of the volumetric hydraulic drive. Based on these studies, the operation of the regulator can be described by the following system of differential equations:

$$m_1 \frac{d^2 x_1}{dt^2} = p S_1 - c_1(x) \left( x_1 \frac{l_2}{l_1 - x_2} \right) - k \frac{dx_1}{dt} \quad (1)$$

$$m_2 \frac{d^2 x_2}{dt^2} = p_1 S_3 - p S_2 - x_2 (c_2 + c_3) \quad (2)$$

$$k_1 \frac{dp_1}{dt} = \mu b \left( x_1 \frac{l_2}{l_1 - x_2} \right) \sqrt{\frac{2(p - p_1)}{\rho}} \quad (3)$$

where  $m_1$  - reduced mass of plunger 5 and rocker 4, kg;  $x_1$  - movement of plunger 5, m;  $p$  - pump outlet pressure, Pa;  $S_1$  - cross-sectional area of plunger 5,  $m^2$ ;  $c_1(x)$  - stiffness of adjustable spring 3, N/m;  $l_1$  - length of lower rocker arm 4, m;  $l_2$  - length of upper rocker arm 4, m;  $x_2$  - movement of piston 6, m;  $k$  - coefficient of "fluid" friction, Ns/m;  $c_0$  - stiffness of

spring 1, N/m;  $x_0$  - preloading of spring 1, m;  $m_2$  - reduced mass of piston 6, cylinder 8 and traction system 9, kg;  $p_1$  - cylinder pressure 8, Pa;  $S_2$  - cross-sectional area of piston 6,  $m^2$ ;  $S_3$  - cross-sectional area of cylinder 8,  $m^2$ ;  $c_2$  and  $c_3$  - stiffness of springs 7 and 10, N/m;  $k_1$  - coefficient of elasticity of the working fluid and the r wall material;  $\mu$  - coefficient dependent on the shape of the flow area of the spool;  $b$  - height of the flow area of spool 2, m;  $\rho$  - working fluid density,  $kg/m^3$ ;  $t$  - regulatory process time, sec.

When developing the mathematical model, the following assumptions were made:

- due to their small size, movements of the rocker are linear;
- stiffness of spring 1 is sufficient for the joint indissoluble movement of the spool and the rocker;
- adjustable spring 3 is non-linear.
- resistance of moving parts is taken into account by the coefficient of "fluid" friction.

Equation (1) characterizes the joint movement of plunger 5 and rocker 4; equation (2) describes the joint movement of piston 6, cylinder 8 and rod system 9; equation (3) determines the pressure in cylinder 8.

To solve the system of equations in the Matlab-Simulink environment, a scheme shown in Figure 2 has been developed [12].

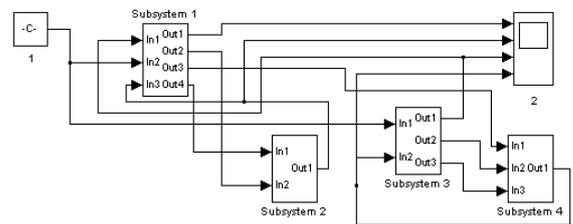


Fig. 2. Matlab-Simulink solution scheme

The scheme contains 4 subsystems having their own inputs and outputs connecting the subsystem with other objects. The system input is the pressure whose values are specified in block 1. The output is the ratio of lengths of rocker arms 4, the movement of spool 2 and piston 6, the pressure in cylinder 8 and other parameters reflected in block 2 and portable oscillators.

Subsystem 1 (Fig. 3) forms the ratio of the arms and the movement of the upper arm in the regulation process.

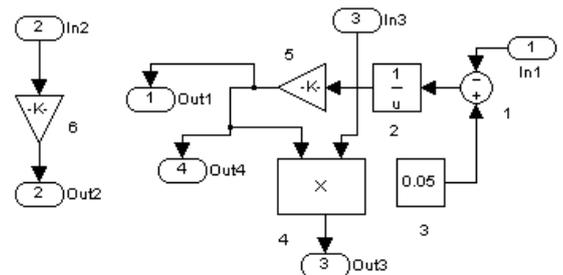


Fig. 3. Structure of subsystem 1

The subsystem inputs are as follows: 1 – movement of piston 6; 2 - pressure; 3 - movement of spool 2. The output are as follows: 1 and 4 - the ratio of lengths of rocker arms; 2 - plunger force; 3 - movement of the upper rocker arm.

From the output of block 1, the difference between the initial length of the lower rocker arm and the movement of piston 6 is removed and fed to block 2, where the value reciprocal of the specified difference is formed. In block 5, multiplication by the length of the upper rocker arm makes it possible to determine the ratio of the arms; in block 4, it makes it possible to move the upper rocker arm.

Subsystem 2 determines the vertical movement of plunger 5. The input is the ratio of lengths of the rocker arms and the plunger force; the output is the plunger movement.

In blocks 1-3, a nonlinear characteristic of adjustable spring 3 is formed. Multiplying it by the movement of the upper rocker arm in block 4, we can determine the resistance force of the latter. In block 5, the difference between the force of the plunger, the resistance of the adjustable spring and the “liquid” resistance to the movement of the plunger is formed (block 7). Taking into account the reduced masses of the plunger and the rocker (block 6), double integration (blocks 8 and 9) and initial conditions (block 10 and 11), the output is the movement of the plunger.

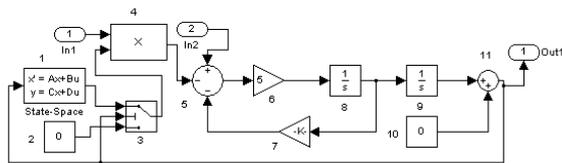


Fig. 4. Structure of subsystem 2

Subsystem 3 (Fig. 5) makes it possible to move piston 6, cylinder 8 and rod system 9. The subsystem inputs are pressure in the pressure line and cylinder 8; the outputs are the speed and movement of the piston, the pressure difference in the pressure line and cylinder 8.

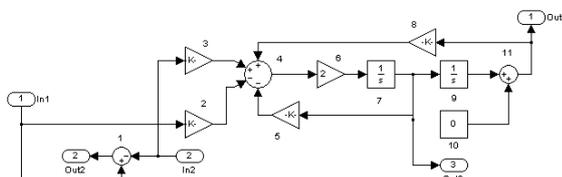


Fig. 5. Structure of subsystem 3

In block 4, the difference between the force of cylinder 8, the force of springs 7 and 10 (block 9), the force acting on piston 6 and its resistance to "fluid" friction (block 5) is formed. Taking into account the reduced masses (block 6), double integration (blocks 7 and 9), initial conditions (block 10 and 11), the piston can be displaced.

Subsystem 4 (Fig. 6) generates pressure in cylinder 8 whose inputs are: 1 - movement of the spool 2; 2 - pressure

difference in the pressure line and cylinder 8; 3 - the speed of movement of the piston.

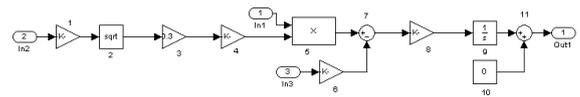


Fig. 6. Structure of subsystem 4

The chain of blocks 1-5 determines the flow of working fluid into cylinder 8 through the throttling slot of spool 2; blocks 6 and 7 determine the difference between the flow and consumption in cylinder 8; block 9 takes into account the coefficient of elasticity of the working fluid and the material of the regulator walls. After integration (block 9), taking into account the initial conditions (block 10 and 11), the pressure in cylinder 8 is formed at the output of the subsystem.

Figure 7 presents a simulation fragment.

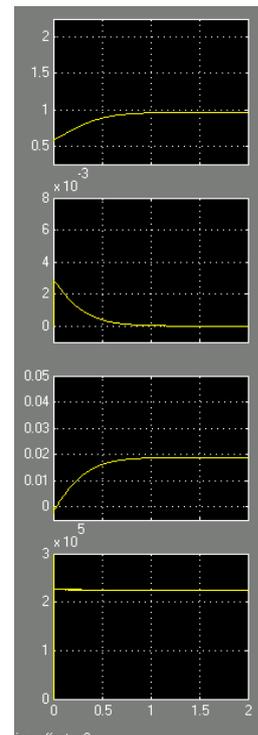


Fig. 7. The fragment of simulation results: a - the ratio of length of arms of the rocker 4; b - movement of the spool 2; c - movement of the piston 6; d - pressure in the cylinder 8

At the initial ratio of the rocker arms  $k = 0.6$  and the pressure  $p = 40$  MPa, the regulator spool moves to the right (Figure 7-b) and opens the flow area through which the working fluid from the pressure line enters cylinder 8. The pressure rises to  $2.25 \times 10^5$  Pa (Fig.7-g) and is transmitted through the system to piston 6 which moves at a distance of 20 mm (Fig.7-c), reducing the pump flow and increasing the ratio

of the rocker arms to 0.9 (Fig. 7-a). In 0.8 seconds, the moments of forces on the rocker arms become balanced, and the spool returns to its original position connecting cylinder 8 to the drain line and preventing further reduction of the pump flow.

Pressure dependences of control process parameters (Fig. 8) were determined using the mathematical model of the regulator.

When the pressure increases from 30 to 45 MPa, the spool moves in the range of 0.4 - 2.8 mm. The displacement of the spool is proportional to the pressure, since characteristics of the adjustable spring are linear.

The piston movement is more sensitive to pressure changes. When pressure changes in the specified interval, the piston moves at a distance of 3 - 19 mm. In this case, the dependence is nonlinear.

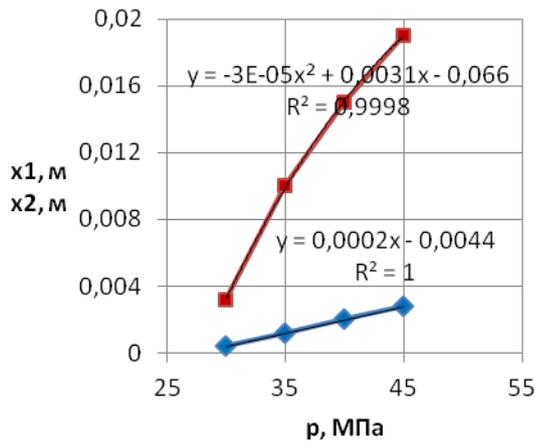


Fig. 8. The dependence of spool movement  $x_1$  and piston  $x_2$  on the pressure in the pressure line  $p$

Figure 9 shows the dependences of other parameters of the control process on the pressure in the pressure line. The pressure in cylinder 8 and the ratio of rocker arms 4 are proportional to the pressure in the pressure line. They increase 1.5 times in the specified range, the transition time is expressed by a non-linear relationship and decreases two-fold.

#### IV. CONCLUSION

When the pressure increases from 30 to 45 MPa, the regulator spool moves in the range of 0.4 - 2.8 mm. The spool movement is proportional to the pressure.

The piston movement is more sensitive to the change in pressure. It changes within the range of 3–19 mm, while the relationship is non-linear. The cylinder pressure and the ratio of arms of the rocker are proportional to the pressure in the

pressure line and increase 1.5 times; the transition is expressed by a non-linear dependence and decreases two-fold.

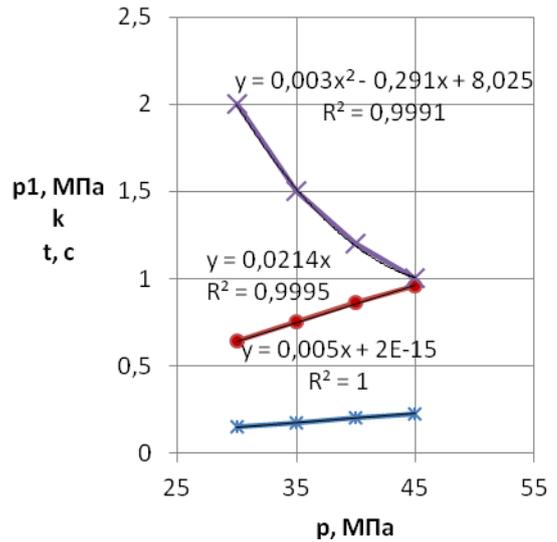


Fig. 9. The dependence of pressure  $p$  in cylinder 1, the ratio of arms of rocker  $k$  and transition time  $t$  on pressure  $p$  in the pressure line

- [1] G.M. Ivanov, V.K. Sveshnikov, The main directions of development of a modern hydraulic drive, Equipment and tools for professionals. Metalworking, 2013, Vol. 4, pp. 32–35.
- [2] Vasilchenko V.A., Sobolev V.O. Regulators working fluid consumption for hydraulic drives of mobile machines // Construction and road machines. 2007. No. 12. P. 26–28.
- [3] V.A. Vasilchenko, A.K. Shekunov, Modern advances in the design of a hydraulic drive and their application in mechanical engineering, Bulletin of mechanical engineering, 2008, Vol. 7, pp. 86–95
- [4] Sveshnikov V.K. Machine hydraulic actuators. Moscow: Mashinostroenie, 2008. 640 p.
- [5] Retrieved from: <https://studfiles.net/preview/2873524/page:35/>
- [6] N.V. Lipchii, Research Methodology: a textbook. Krasnodar: KubSAU, 2013, 290 p
- [7] V.G. Zedgenizov, A.N. Strelnikov, D.S. Biryukov. Mathematical model of the axial piston pump controller with hydraulic control, Aircraft machine building and transport of Siberia: Collection of articles, The XI All-Russian scientific and technical conference "Aircraft and machine building and transport of Siberia." ISTU, 2018, pp. 20-24.
- [8] Regulators of hydraulic pumps with variable displacement. Retrieved from: [http://cdmteh.ru/regulyatori\\_gidronasosov.html](http://cdmteh.ru/regulyatori_gidronasosov.html)
- [9] V.D. Boyko, M.A. Andreev, S.E. Semenov, Peculiarities of verification of the mathematical model of an axial piston pump regulator with electrohydraulic control, Science and education, MSTU n.a. N.E. Bauman. Engineering Bulletin, Vol. 12, 2014
- [10] T.M. Bashta, Hydraulic and hydropneumatic automation. Moscow: "Mashinostroenie", 1972, 320 p
- [11] V.N. Prokofyev, Yu.A. Danilov, Axial-piston adjustable hydraulic drive, Moscow: Mashinostroenie, 1969, 496 p
- [12] I.V. Chernykh, Simulation of electrical devices in MATLAB, SimPowerSystems and Simulink Moscow: DMK Press, St. Petersburg: Peter, 2008, 288 p