

# *The use of virtual distance learning technologies in conducting pre-study courses in mathematics to prepare for the unified state examination at a specialized level*

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**Abstract** — The article explores the use of virtual distance technologies to solve complex math problems in preparation for the unified state examination.

The purpose of the article is to create a learning functional tool that makes it possible to remotely conduct pre-study courses in mathematics using the IDRo interactive board.

The study was conducted on the basis of modeling methods and system analysis, methods of decision theory, methods for solving inverse problems, as well as methods of induction and deduction.

The features and advantages of using the IDro interactive board are considered, graphical methods for solving mathematical problems are updated. It is proved that the IDro interactive board is a universal tool for collaboration of several users with a common virtual graphic space.

**Keywords** — *interactive board, distance educational technologies.*

## I. INTRODUCTION

The work is devoted to the use of virtual distance technologies for solving atypical mathematical problems and increasing the efficiency and quality of the educational process.

Pre-study courses are organized with the aim of preparing for the unified state exam at a specialized level, conducting a final repetition, identifying problem areas in the students' knowledge and correcting them.

The tasks of the second part of the profile exam in mathematics are of high complexity, and also, often, the need to use a graphical solution method. Moreover, for some tasks, the graphical method is the only possible and requiring complex constructions.

The best option for conducting pre-study courses is the situation when high school students study with a teacher face-to-face. In this case, the teacher has the opportunity to draw

geometric figures, problem solving areas, areas of feasible solutions, etc. on a blackboard or a squared sheet step by step.

Unfortunately, face-to-face classes are not always possible. In some cases, students cannot attend classes due to health status, in some cases because of living in a remote area, etc. In such cases, the most appropriate training method is the distance learning method using various online services for collective or individual work, such as interactive boards.

It should be noted that in Russia, distance and electronic educational technologies are mainly used in the educational process of higher education institutions. And, specifically, innovative directions for the development of new educational technologies are being developed [1], [2]; issues of achieving the quality and effectiveness of training are studied [4], [6]; features of teaching people with disabilities are considered [3], [5].

## II. RESEARCH METHODOLOGY

### A. *Interactive educational platforms*

Interactive board is an educational tool that makes it possible to combine text, image, video, audio in an interactive format. These services appeared in the mid-2000s and were becoming increasingly popular.

The main issues of this subject are discussed in detail in scientific articles [1-17].

A feature of an interactive board is that it can be used to hold video conferences. Moreover, you can print text on it, draw graphics, add pictures, chat, add and remove users who have access to the board online. We also note that other users who have access to the board can also draw, write text and add files to this board. These tools make it possible to successfully use interactive boards when conducting distance pre-study courses.

If the mathematical problem is not very difficult, then a teacher of pre-study courses can show a presentation on the board with its phased solution.

However, the situation is complicated when the example contains complex geometric constructions. In this case, a teacher should not only represent a phased solution to the problem but also provide a student with the opportunity to make changes to the drawing. Thus, there is a joint solution to the problem by a student and a teacher, and not copying according to an already decided pattern.

One example of an interactive board with all the features discussed above is the IDroo service.

The IDroo Skype app is an online educational platform for conducting online classes. The program is quite popular among teachers and tutors, because it makes it possible to significantly simplify the subject mastering by students in their usual computer environment.

IDroo is an interactive board with many additional tools, which is essentially a universal workspace. The service is positioned by the developers as educational, but it has been successfully used for conducting online meetings and other events in the business sphere.

The main function of IDroo is an interactive board with support for many formats and options. Working with geometric shapes, inscriptions of various prints, tracings and sizes, random drawings and even mathematical formulas can literally blur the boundaries between a real workspace and a virtual one.

The ability to work with multiple boards at once can significantly simplify the task, eliminating the need to erase the existing note to create a new one. All final and intermediate results can be saved to disk or sent as images to participants, as well as sent directly to print.

Another key feature of IDroo is its focus on the involvement of each participant in the conference. The basis of collaboration is the ability to edit the notes on the board in turn by all participants, as well as leave your own comments about this or that situation.

It is important to note that the results of the work can be saved not only by a teacher who opened the virtual graphic space but also by any participant. Everyone can edit notes by any available means: both by a mouse and a graphic tablet.

The main plus of IDroo lies in the functionality. The ability to work with various types of graphics, text, and even formulas makes the scope of the application incredibly wide [16, 17].

Simultaneous editing of boards by several users is another undoubted advantage that makes the implementation of joint projects incredibly simple. It is impossible not to note the presence of the Russian interface, which increases the availability of this resource.

Thus, IDroo is a universal tool for collaboration of several users with a common virtual graphic space.

To begin work, a teacher must go to [www.idroo.com](http://www.idroo.com), register and send a participant a link to the created interactive

board. A participant, clicking on the link, goes to the interactive board. To begin work, you must register by leaving your email address. After this, collaboration begins.

A teacher writes an example that needs to be analyzed. The IDroo application has a formula editor, so complex mathematical formulas can be written pretty quickly. A teacher can also pre-prepare mock-ups of graphs on the interactive board, with the help of which in the future it will be necessary to solve the problem. It is possible to change the background color, choosing between white, yellow, squared on the interactive board. The squared background color is especially relevant in the case of geometric constructions.

It is also possible to insert pictures cut from electronic textbooks on the interactive board space. This is used, for example, when a teacher gives homework to applicants.

### III. RESULTS OF THE RESEARCH. DISCUSSION OF RESULTS

Let us consider in more detail the solution of some problems of the second part of the unified state examination at a specialized level.

Statement of the problem is as follows:

For what value of  $a$  the system has at least one solution on the interval  $[5, 6]$ .

$$\begin{cases} ax \geq 3 \\ a < \sqrt{x+4} \\ 2x \geq 3a+6 \end{cases} \quad (1)$$

A teacher writes down the statement of the problem and gradually builds graphs of the studied functions, indicating the scope of each inequality. A student also has access to the interactive board and can build function graphs on it and edit them in real time (Figure 1).

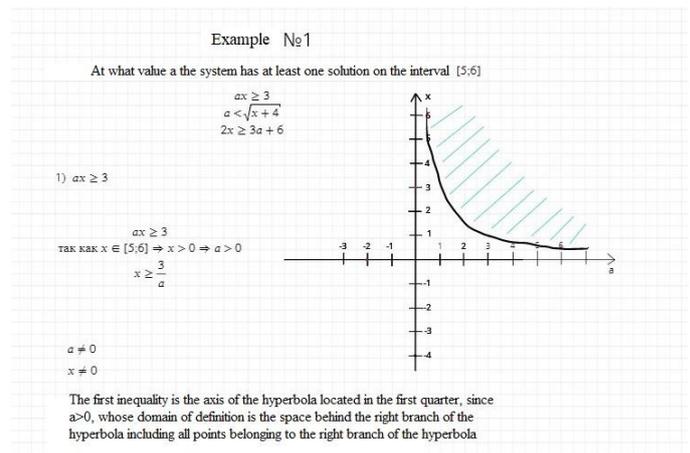


Figure 1 - Graphic representation of a solution to the first inequality of a set of constraints

The first inequality is the axis of the hyperbola located in the first quarter (since  $a > 0$ ), whose domain is the space to the right and above the right half-hyperbola, including all points belonging to the right half-hyperbola.

After analysis, a graphic image of the first inequality of the system with a student, a teacher proceeds to build a graphic image of the second inequality of the system. If necessary,

applicants can reflect on the interactive board constraints, acceptable region, comments (Figure 2).

2) the Second inequality is a parabola whose branches are directed upwards with a vertex at a point with coordinates (0;-4). The scope of the definition of this inequality is, enclosed between the branches of the parabola

$$a < \sqrt{x+4} \quad \text{since } a > 0$$

$$a^2 < x+4$$

$$x > a^2 - 4$$

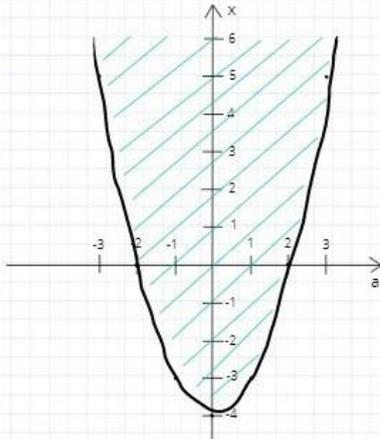


Figure 2 - Graphic representation of a solution to the second inequality of a set of constraints

The second inequality is a parabola, the halves of which are directed up with the vertex at a point with coordinates (0; -4). The domain of this inequality is the area enclosed between the halves of the parabola.

Using the elements of the interactive board, a teacher and applicants can make constructions, as well as hatch the domain and the range of inequality values. On the interactive board, you can build almost any shape, for example an ellipse, a hyperbola, a parabola, a circle. It all depends on the skills and experience of professionalism in the field of these constructions.

The third inequality is a straight line, the domain of which is the area located to the left of this straight line (Figure 3).

It is worth adding again that applicants, along with a teacher, can edit text and drawings in real time. That is, there is an active joint work of a teacher and applicants.

3) the Third inequality is a straight line, the area of definition of which is the area located to the left of the given straight line, including all points belonging to the given straight line

$$2x \geq 3a + 6$$

$$x \geq \frac{3}{2}a + 3$$

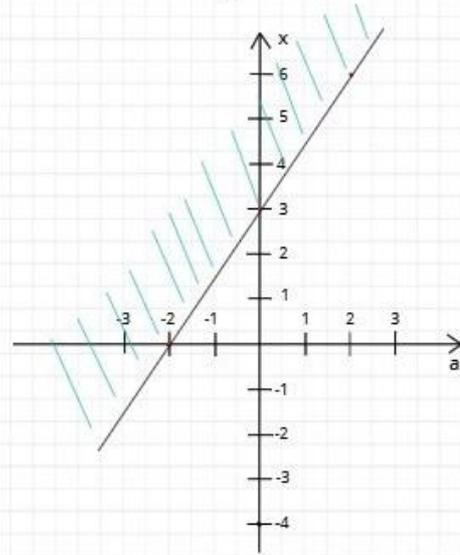
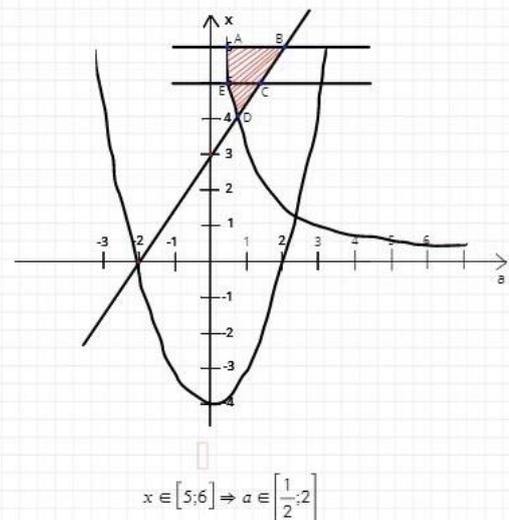


Figure 3 - Graphic representation of a solution to a third inequality of a set of constraints

After we consider the solution of each inequality of the system separately, we can superimpose the solutions of all inequalities in one coordinate system. There is a region *ABCDE* hatched in red. By the statement of the problem, the system of inequalities must have at least one solution on the interval [5; 6]. Thus, the hatched region is limited to the region *ABCE* (Figure 4).

4) when overlaying the definition areas of each of the three inequalities, the area *ABCDE* shaded in red is obtained color. By the condition of the problem, the system of inequalities must have at least one solution on the interval [5;5]. Thus the shaded area is limited to the *ABCE*.



$$x \in [5;6] \Rightarrow a \in \left[\frac{1}{2}; 2\right]$$

Figure 4 - Graphic representation of a solution of a system of inequalities

If we combine phased steps of solving the system of inequalities on one interactive board, then we can get the following interactive board (Figure 5).

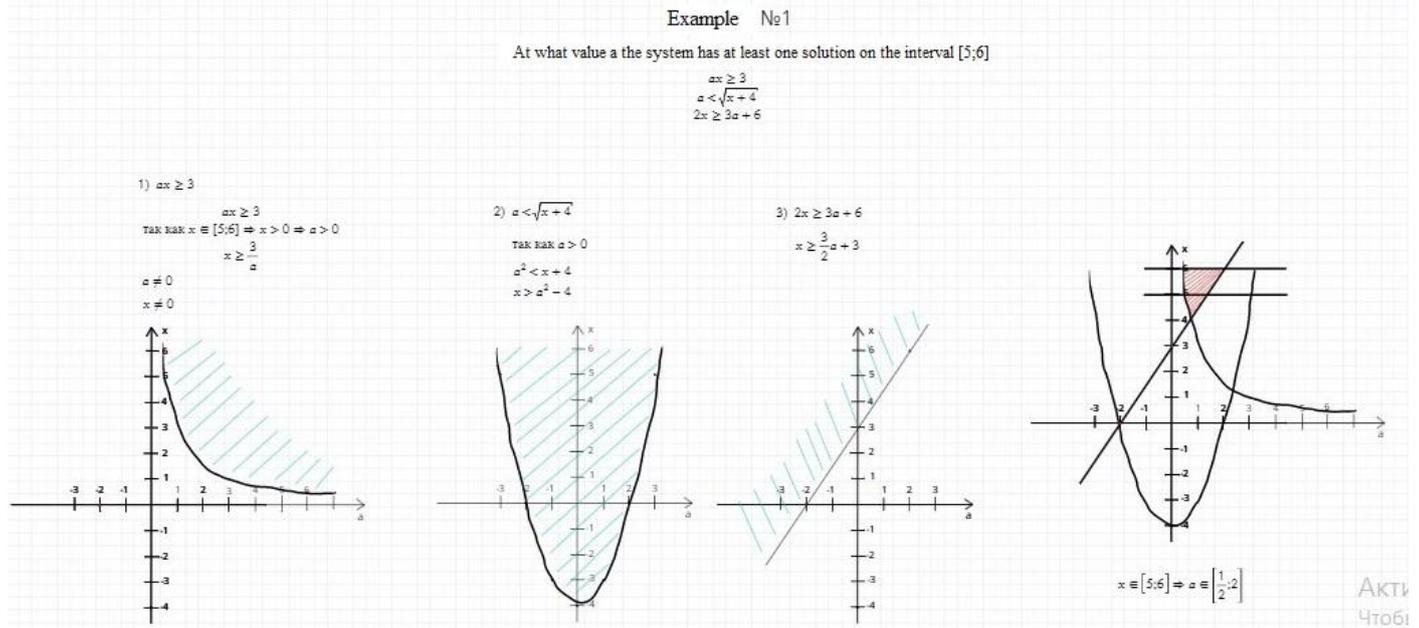


Figure 5 - Graphic representation of a phased solution of a system of inequalities

The interactive board is saved automatically even when you close the IDRoо tab. Thus, applicants can re-enter the interactive board at any time convenient for them, repeat the examples they have completed, and identify points in the solution of the problem that still have questions.

Consider the following example. At what value does a system of equations have three or five solutions?

$$\begin{cases} (y-1)(y-x-2)(y+x-4) = 0 \\ (x-1)^2 + (y-2a)^2 = 3a^2 - 7a + 5 \end{cases} \quad (2)$$

A teacher can write an example using a graphical editor or cut it out of an electronic textbook and paste it on the interactive board field. We used a graphic editor.

Similarly, as a teacher explains the example on the board in the classroom, the example is explained on the interactive board. What a teacher most often speaks out loud can be recorded on the interactive board. As mentioned above, applicants can re-enter the interactive board at any time convenient for them and review.

So, a teacher explains that the first equation of the system characterizes the following combination of graphs (Figure 6).

$$\begin{cases} y = 1 \\ y = x + 2 \\ y = -x + 4 \end{cases} \quad (3)$$

If necessary, applicants can reflect on the interactive board intersection points of function graphs, constraints, acceptable region, comments, etc.

After analysis a graphic image of the first equation of the system with applicants, a teacher proceeds to build a graphic image of the second equation of the system. (Figure 7).

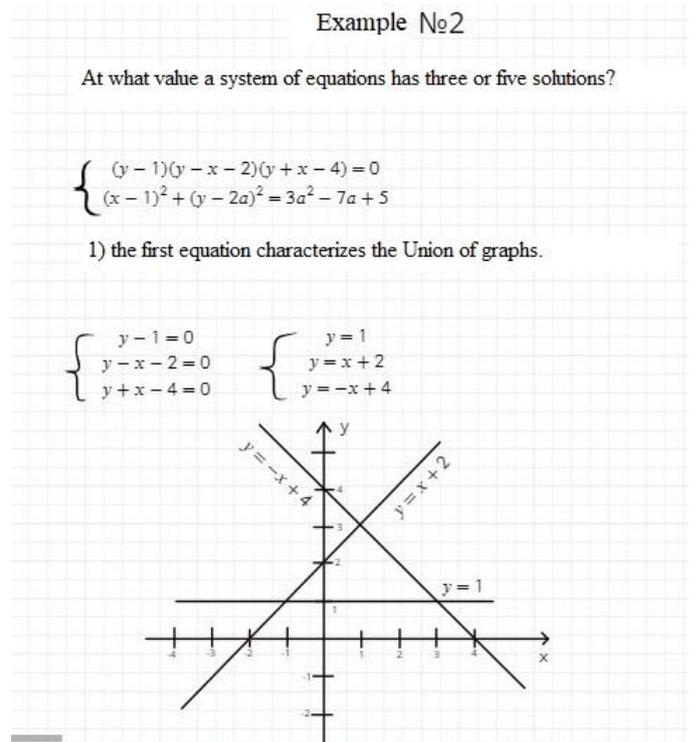


Figure 6 - Graphic representation of a solution to a first equation of an equation system

The second equation of the system is a circle centered at a point  $O(1; 2a)$  and radius  $R = \sqrt{3a^2 - 7a + 5}$  if  $R > 0$ , a point with coordinates  $(1; 2a)$  if  $R = 0$ . The second equation has no solutions if  $R < 0$  (Figure 7).

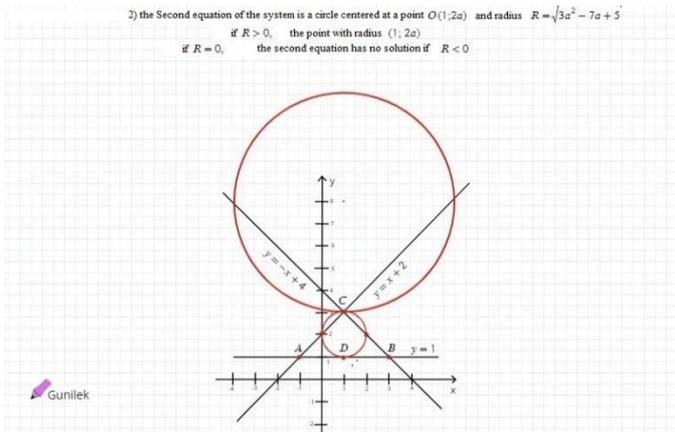


Figure 7 - Graphic representation of a solution to the second equation of an equation system

The circle is a reflection in a line  $x = 1$ . An odd number of intersection points will be if the coordinates of the points  $C(1; 3)$  and  $D(1; 1)$  are the solution to the system:  $A(-1; 1), B(3; 1), C(1; 3), D(1; 1)$ .

It is worth adding again that applicants, along with a teacher, can edit text and drawings in real time. That is, there is an active joint work of a teacher and applicants.

A teacher offers applicants to consider cases when the circle passes through the points  $C(1; 3)$  and  $D(1; 1)$  separately (Figure 8).

Unlike a parabola, it is easiest to draw a circle, since circle, ellipse, rectangle are already predefined in the toolbar.

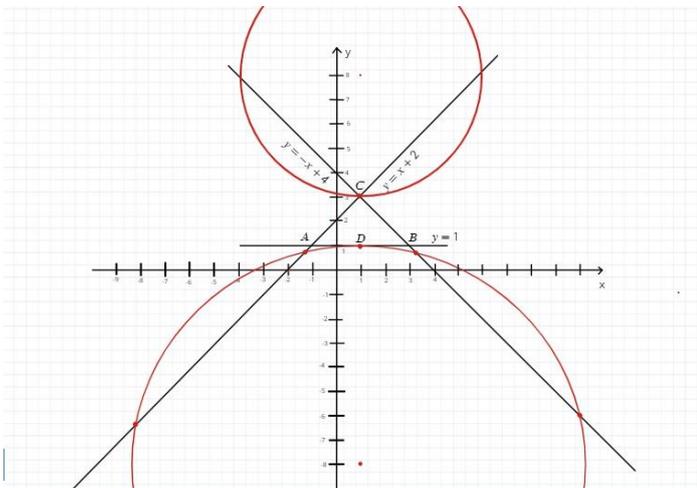


Figure 8 - Consideration of cases of a circle passage through points  $C(1; 3)$  and  $D(1; 1)$

Consider the case when a circle passes through a point  $C(1; 3)$ . We have

$$(3 - 2a)^2 = 3a^2 - 7a + 5. \tag{4}$$

It follows then that

$$a^2 - 5a + 4 = 0. \tag{5}$$

And then,  $a_1 = 4; a_2 = 1$ .

If  $a_1 = 4$ , then  $(x - 1)^2 + (y - 2)^2 = 1$  in this case there are 4 solutions.

If  $a_2 = 1$ , then  $(x - 1)^2 + (y - 8)^2 = 25$  in this case, there are 3 solutions.

Consider the case when a circle passes through a point  $D(1; 1)$ . We have

$$(1 - 2a)^2 = 3a^2 - 7a + 5. \tag{6}$$

It follows then that

$$a^2 + 3a - 4 = 0. \tag{7}$$

And then,  $a_1 = -4; a_2 = 1$ .

If  $a_1 = -4$ , then  $(x - 1)^2 + (y + 8)^2 = 81$  in this case, there are 5 solutions.

If  $a_2 = 1$ , then  $(x - 1)^2 + (y - 8)^2 = 25$  in this case, there are 3 solutions.

A teacher can record this solution using a graphical editor or insert a photograph of the solution "by hand".

Similarly, as a teacher explains the example on the board in the classroom, the example is explained on the interactive board. What a teacher most often speaks out loud can be recorded on the interactive board (Figure 9).

$$A(-1;1), B(3;1), C(1;3), D(1;1)$$

A circle is symmetric with respect to a straight line  $x = 1$

Not an even number of points if the coordinates of the points  $C(1;3)$  or  $D(1;1)$  are the solution of the system

a) The circle passes through the point  $C(1;3)$

$$(3 - 2a)^2 = 3a^2 - 7a + 5$$

$$a^2 - 5a + 4 = 0$$

$$a_1 = 4, a_2 = 1$$

If  $a = 1$ , то  $(x - 1)^2 + (y - 2)^2 = 1$  - 4 point

If  $a = 4$ , то  $(x - 1)^2 + (y - 8)^2 = 25$  three decisions

b) The circle passes through the point  $D(1;1)$

$$(1 - 2a)^2 = 3a^2 - 7a + 5$$

$$a^2 + 3a - 4 = 0$$

$$a_1 = 1, a_2 = -4.$$

If  $a = -4$ , то  $(x - 1)^2 + (y + 8)^2 = 81$  - 5 decision

Answer :  $a = 4, a = -4.$

Figure 9 - Consideration of various solutions to the system of equations

From figures 6-9, you can see that all the calculations that can be put down a squared sheet or on a standard board can be done on an interactive board. It is possible to build graphs, write formulas, text, highlight certain areas on an interactive board.

Thus, if there is no opportunity to conduct face-to-face classes, and the level of complexity of the solved examples is high enough, especially in the case of graphic methods of solution, a worthy alternative is an interactive board.

With this method of training, applicants, while away from a teacher, can simultaneously change a drawing, indicate on a graph these changes and supplement it. Thus, there is a joint solution to the problem by applicants and a teacher, and not copying according to an already decided pattern. And as mentioned above, applicants can re-enter the interactive board at any time convenient for them and review.

#### IV. CONCLUSION

Information technology in teaching mathematics performs two tasks. Firstly, they act as a tool for mastering mathematical knowledge. With the help of programs, a teacher will be able to more quickly and clearly demonstrate various calculations, and an applicant will be better able to master the program material, as well as verify the correctness of the fulfillment of the proposed tasks. Secondly, mathematical knowledge helps to understand the methodology of working with a particular software product. These are various mathematical packages, text editors, table processors, with the help of which an applicant can draw up the results of practical work, perform or verify some calculations. The use of

information technology in the process of teaching mathematics is associated not only with an increase in the percentage of independent work of applicants, but also with the transition of the learning process to a new stage of development, where a teacher acts not only as a source of information but also as an organizer of independent activity of applicants.

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