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A Bi-level Programming Model on the Pricing Method for the Air-Rail Intermodal Transport

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***Abstract*—In recent years, China's society has developed rapidly, the high-speed railway network has become more and more perfect, and the competition between high-speed rail and civil aviation has become increasingly fierce. In the country's newly introduced national transportation development plan, the development of the comprehensive transportation system is placed in an important position. The operation of the empty railway is an important part of comprehensive transportation, and it has attracted more and more attention. In the development of air-rail transportation, the decisive role is still in the formulation of fares. A cheaper fare can attract huge passenger traffic, so it is necessary to explore a reasonable method of fare development. This paper chooses to establish a bi-level programming model, and set the upper-level planning objective function to maximize the ticket benefit of the relevant departments of the empty-rail intermodal transportation. The objective function of the lower-level planning is to minimize the passenger's general travel cost and solve the model, so as to obtain a relatively complete The method for formulating fares for air and rail transportation provides a reference for the formulation of air and rail freight fares in China. Finally, the paper validates the model based on the air-rail combined transport line from Beijing to Sanya.**

Keywords—Air-Rail intermodal transport; bi-level programming model; transport service; pricing method

I. INTRODUCTION

Si Bingfeng, Gao Ziyou (2001) [1] solved the railway fare optimization strategy by establishing a bi-level programming model and using a heuristic algorithm based on sensitivity analysis. Katherine Siggerud (2005) [2] describes the narrow and broad distinction between air and rail transport. Raleigh, Peng Jihua (2007) [3] in order to find out how to formulate civil aviation fares to maximize the efficiency of airlines, by establishing a two-level price dynamic pricing and time relationship model for two flights, combined with game theory and random Control theory, the conditions that the optimal solution is satisfied are obtained, and the algorithm and other properties for solving the model are given. Nicole, Eric, and Chris (2010) [4] studied the competitive relationship between high-speed rail and aviation, and used the principle of game theory to compare the passenger flow of European civil aviation coexisting with high-speed railways and aviation, and established a passenger flow equilibrium model. Yang and Zhang (2012) [5] studied the impact of total social welfare on high-speed rail fares. By establishing a bi-level programming model, it was found that if the social benefits of both air and iron increase, the existing fares will decrease. Daniel Albalate (2015) [6] and others believe that high-speed rail and aviation can not put the competition in the first place, but should consider the relationship between the two.

II. AIR RAIL SERVICE PRICING MODEL

A. Bi-level Programming Model

The specific mathematical model of the bi-level programming model is as follows:

$$(U1.1)\ \max F(x,y(x))$$

$$\text{S.t.}\ \ G(x,y(x))\leq 0$$

y(x) is given by the underlying model:

$$(L1.1)\ \min f(x,y)$$

$$\text{S.t.}\ \ g(x,y)\leq 0$$

F(x, y(x)) is the objective function determined by the upper-level plan; x is the decision variable of the upper-level plan; F(x, y) is the objective function determined by the lower-level plan; y is the decision-making variable of the lower-level plan; y is a function of x, that is, y=y(x) is called a reaction function.

In terms of the issue of air and rail fare pricing, the optimization objective of the upper model is to maximize the passenger ticket revenue of the aviation and railway companies = total revenue - total cost (fixed cost + variable cost), and the decision variable is the ticket for the air and rail transport. Price; the lower level is to minimize the travel expenses of passengers. The optimal solution of the model should maximize the revenue of the railway and civil aviation companies and minimize the passenger's general travel expenses.

B. Pricing Model for Air and Rail Transport Services

Usually, when a certain OD market is selected, when passengers have travel demand in two cities, they usually choose the mode of travel with the lowest general travel cost. Traveler's general travel expenses are always positively correlated with passenger demand, while passenger traffic is negatively correlated with general travel expenses. In the end, in the competition of different modes of transportation, the passenger flow between the modes of transportation will reach an equilibrium state. This state can be described as:

$$c_i^w \begin{cases} =u^w & \text{when } q_i^w>0 \\ \geq u^w & \text{when } q_i^w=0 \end{cases} \tag{1}$$

$$\omega \in W, i \in I \tag{1a}$$

$$Q^w = D^w(u^w) \tag{1b}$$

Where c_i^w represents the generalized travel cost of the i-th type of transportation between OD and ω; u^w represents the generalized travel cost between ω in equilibrium; q_i^w represents the i-th traffic between ω Passenger flow in transport mode; Q^w is the passenger demand between ω; $D^w(u^w)$ is the passenger demand function between ω;

So we can get the model:

$$\text{(L)} \quad \min Z(q,Q)$$

$$= \sum_{\omega \in W} \sum_{i \in I} \int_0^{q_i^w} f_i^w(x)dx - \sum_{\omega \in W} \int_0^{Q^w} D_w^{-1}(x)dx \tag{2}$$

$$\text{S. t.} \quad \sum_{i \in I} q_i^w = Q^w, \omega \in W \tag{2a}$$

$$q_i^w \geq 0, \omega \in W, i \in I \tag{2b}$$

Among them, it can be equivalent to $c_i^w = f(q_i^w); D_w^{-1}(x)$ is the inverse function of the demand function; and through the above (L) model, the model has a unique solution.

For (2), it can be transformed by adding a network map that is more than the required road segment, that is, the equation can be transformed into:

$$\min Z(q,Q)$$

$$= \sum_{\omega \in W} \sum_{i \in I} \int_0^{q_i^w} f_i^w(x)dx + \sum_{\omega \in W} \int_0^{e^w} H_w(y)dy \tag{3}$$

Because $H_w(y)$ is similar to the ordinary road segment impedance function, the network distribution problem corresponding to equation (3) has only one fixed requirement.

$$\text{(L1)} \quad \sum_{\omega \in W} \sum_{i \in I} \int_0^{q_i^w} f_i^w(x)dx \tag{4}$$

$$\text{S. t.} \quad \sum_{i \in I} q_i^w = Q_1^w, \omega \in W \tag{4a}$$

$$q_i^w \geq 0, \omega \in W, i \in I \tag{4b}$$

The above is from the perspective of the traveler. Below, we continue to get the most suitable air-rail combined fare from the operator's point of view, so as to maximize the revenue generated by the operating department.

We assume that the revenue of the relevant departments of the air and rail transport is the total ticket revenue minus the fixed cost and variable cost, which can be expressed as:

$$F = \sum_{\omega \in W} q_i^w (u_i^w - c_i^w) - \pi \tag{5}$$

Where q_i^w represents the passenger flow of air-to-rail combined transport between w; c_i^w represents the average transport cost of air-rail combined transport between w; π is a fixed cost;

The fare range of the air and rail transport should be between the railway transportation cost and the sum of the highest fare of the railway and the highest fare of the airline.

$$u_i^{w\,(\min)} \leq u_i^w \leq u_i^{w(\max)}, \omega \in W \tag{6}$$

Where $u_i^{w\,(\min)}$ and $u_i^{w(\max)}$ represent the lowest fare and the highest fare, respectively.

And because the value of its passenger flow cannot be greater than the line capacity of the transportation mode with a smaller capacity, there are constraints:

$$Q_r^w \leq K_I \tag{7}$$

$$K_i = \min(K_1, K_2) \tag{8}$$

Among them, K_1, K_2 are the line capacity of the railway and air transportation in the OD section respectively.

C. Air-rail combined transport service pricing model

Therefore, through the above analysis, from the formulas (4), (5), (6) and (7), the bi-level programming model for air and rail freight fares is:

$$\text{(U)} \quad F = \sum_{\omega \in W} q_r^w (u_r^w - c_r^w) - \pi$$

$$\text{S. t.} \quad u_r^{w(\min)} \leq u_r^w \leq u_r^{w(\max)}, \omega \in W$$

$$q_r^w \leq K_i$$

$$K_i = \min(K_1, K_2)$$

Where q(u) is derived from the underlying model:

$$\text{(L)} \quad \sum_{\omega \in W} \sum_{i \in I} \int_0^{q_i^w} f_i^w(x)\,dx$$

$$\text{S. t.} \quad \sum_{i \in I} q_i^w = Q_1^w, \omega \in W$$

$$q_i^w \geq 0, \omega \in W, i \in I$$

Therefore, we only need to solve this bi-level programming model to get an optimal fare, which can maximize the revenue of the relevant departments of the air and rail transportation, and minimize the travel expenses of the travelers.

III. Case Study: Taking The Air-To-Rail Intermodal Transportation From Beijing To Sanya As An Example

This paper proves the feasibility of the bi-level programming model by studying the air-rail transportation from Beijing to Sanya and by investigating the design related data.

A. Determination of Relevant Data

First of all, through the 12306, Ctrip online to find the air and rail transport fare from Beijing to Sanya is basically stable at around 780 yuan, the hard sleeper price from Beijing to Sanya is 771 yuan, and the airline ticket is generally around 1100 yuan (not considering holidays). The route we chose is shown in Fig. 1.

By inquiring 12306 official website, Beijing to Shijiazhuang high-speed rail ticket is 128.5 yuan, we temporarily assume that its cost is 80 yuan; Shijiazhuang Zhengding Airport to Sanya Phoenix International Airport ticket is generally stable at around 1,000 yuan, we assume its cost is 600 yuan, Therefore, in this paper, the value of $u_r^{w\,(min)}$ is 680, and the value of $u_r^{w(max)}$ is 1000 (cannot exceed the price directly from Beijing to Sanya). which is

$$\text{(U)} \quad F=\sum_{\omega\in W} q_r^w(u_r^w-680) \tag{9}$$

$$\text{S. t.} \quad 680\le u_r^w\le 1000, \omega\in W \tag{10}$$

$$q_1\le K_i$$

$$K_i=\min(K_1,K_2)$$

Note: The assumed fixed cost of air-rail combined transport π has been included in the average cost of rail transport air transport, so we take π to 0 here.

In this case, we intend to take an average calculation. Through telephone consultations with Beijing Capital Airport, Beijing Railway Bureau and Sanya Phoenix International Airport, and random surveys of passengers, the number of passengers flying from Beijing to Sanya is 2,500 passengers (equivalent to the line). The capacity of the air transport mode is 2,500), the number of passengers traveling by rail is about 800, and the number of passengers who choose the transfer plan is about 700. Therefore, in this example, we select the upper limit of passenger demand between Beijing and Sanya. It is 4,000 people.

In order to set the weight. By issuing questionnaires and counting the considerations for the choice of travel modes for nearly 150 passengers, the weights of each influencing factor are drawn up as Table I:

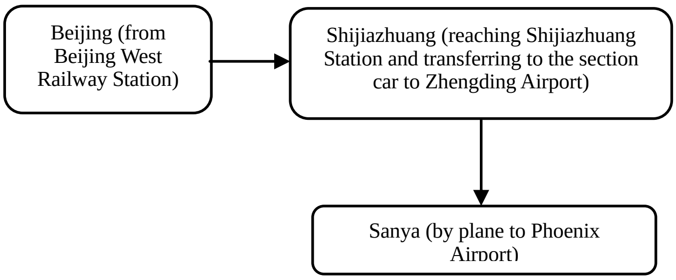

Fig. 1. Road map of Beijing-Sanya air and rail transport.

In the generalized cost function model cited in this paper, we assume a power function form. Take α as 2 and take β as 0.7, where θ is the utility of different transportation, and the expression of θ is:

$$\theta i=-0.25ti-0.35ui-0.4zi \tag{11}$$

Among them, z_i represents the sum of convenience, service quality and comfort; its weight is obtained from the previous survey data.

Here, we assume that the transportation time and convenience, service quality and comfort of each transportation mode are fixed values, and the values of each weighting factor can be obtained. Secondly, since the fare for railway transportation is almost stable, we use the hard sleeper price as the fare for the railway, which is u_3; and the price of civil aviation often changes with the change of market demand (time), which is unstable, so we use the civil ticket. The average value in one month is a stable fare, which is u_2.

By taking the above two tables into (11), you can get:

$$-\theta_1=0.25t_1+0.35u_1+0.4z_1=3+0.35u_1+4=7+0.35u_1$$

$$-\theta_i=0.25t_2+0.35u_2+0.4z_2=1.5+385+8=393.5$$

$$-\theta_i=0.25t_3+0.35u_3+0.4z_3=10.5+269.85+2=282.35$$

Among them, the relationship between passenger flow and fare is the key to solving the model, which is the reaction function. In this paper, in order to simplify the algorithm for solving the model, we obtain the functional relationship between the two by reference[7]:

$$q_i(u_i)=q_i-k(u_i-680) \tag{12}$$

In order to simplify the algorithm, we set k to a fixed value of 160.

$$q_i(u_i)=q_i-160(u_i-680) \tag{13}$$

Then, the model has been obtained as:

$$\text{(U1)} \quad \max F=q_1(u_1)(u_1-680)$$

$$\text{S. t.} \quad 680\le u_1\le 1000$$

$$q_i(u_i)=q_i-160(u_i-680)$$

TABLE I. WEIGHTS OF VARIOUS FACTORS

travel time	0.25
travel expenses	0.35
Convenience	0.2
service quality	0.15
Comfort	0.05

$$(L1)\min Z\left(q,q_1(u_1)\right)=\int_0^{q_1}2q_1^{0.7}+7+0.35u_1 dq_1$$

$$+\int_0^{q_2}2q_2^{0.7}+393.5dq_2+\int_0^{q_3}2q_3^{0.7}+282.35dq_3$$

$$\text{S. t.}\quad q_1+q_2+q_3=4000$$

$$q_i\geq 0, i=1,2,3$$

B. Model Solving Process

First, we use u_0=680 as the initial fare to iterate. The iterative process is as follows:

When u_0=680, we bring it into the lower layer model (L1), and use Matlab to solve it, and get the values of q_1, q_2, q_3, respectively q_1=1715; q_2=813; q_3=1472;

We then substitute the value of q_1 into equation (13) to get the relation of $q_1(u_1)$, and bring it into the upper-level planning model to find the value of u_1 when the benefit is the greatest, u_1=685; F_{m1} = 4596; Here u_1 is substituted into the lower model to find the values of q_1, q_2, q_3 respectively; by analogy, until the optimal value of the upper objective function no longer fluctuates significantly, the u_i obtained at this time Become the best fare.

The iterative results are shown in Table II:

According to the above table, when the fare is iterated 5 times, it becomes 793 and no change occurs. Similarly, we use the value of u_0 to be 780, 880, 980 and iterate again, and the final stable result is 793. According to the previous survey, the existing air and rail freight rate under normal conditions is 780, which is slightly lower than the calculation result of the model. This shows that the fare calculated by the model can enable the airlines and railway departments to obtain greater ticket revenues and promote the more normal development of air and rail transport related transport products.

TABLE II. THE ITERATIVE RESULTS

Initial value	Number of iterations	Passenger flow	Maximum return	Next iteration fare
	1	1715	0	820
	2	1508	162896	801
680	3	1543	167106	794
	4	1550	171052	793
	5	1550	172952	793

C. Result Analysis

The different initial ticket prices obtained from the above converge to the same optimal solution. It shows that although the bi-level programming model is non-convex, it is difficult to obtain a comparatively optimal solution under normal circumstances, but use the model to develop a more reasonable price of air and rail transport tickets is achievable. It can be seen from the above results that the fare of air-rail combined transport is similar to that of railways and air fares. Instead of getting more revenue as the ticket price rises, you can get a larger return within a certain fare range.

IV. CONCLUSION

In recent years, the mileage of high-speed railway operations in China has increased year by year, and high-speed rail transportation continues to affect the living environment of airlines. However, with the rapid spread of air and rail transportation, the relationship between the two has also changed. In China, the air and rail transport is still in its infancy, and many airlines are trying to attract passengers to choose the air and rail transport by increasing their operating costs. In the long run, it will not work.

The air-rail intermodal pricing model established in this paper illustrates the example from Beijing to Sanya. As long as the appropriate parameters can be determined, the bi-level programming model can be used to solve the optimal fare, which maximizes the total revenue of the airline and railway departments. Thereby, the gradual maturity of the mode of transport of air and rail transportation and the gradual improvement of operational efficiency are promoted.

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