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Statistics of Individual Tests for Market Graph Identification in Market Network

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***Abstract*—The concept of random variables network used to model the complex system of random nature is discussed. The problem of threshold graph identification to network analysis of the complex system is considered as multiple decision statistical procedure. The properties of robustness of different tests for testing individual hypotheses for threshold graph identification are investigated by simulations.**

***Index Terms*—random variables network, network structure, multiple decision statistical procedure, individual test, robustness of significance level, robustness of power function**

I. Introduction

Network analysis is a popular and useful tool of complex systems investigation. Network analysis is based on the network model construction which can be represented as a complete weighted graph with nodes corresponds to the elements of the complex system and weights of edges are given by some measure of similarity between the elements. Such model reflect information on dependence structure between elements. In order to identify key information on the structure different network structures which are subgraphs of the network model are considered.

Most popular subgraphs are threshold graph [1]–[3] and maximum spanning tree [4]. Maximum spanning tree is a spanning tree of the network model with maximum total weight. Kruscal algorithm [5] is well known way to construct maximum spanning tree. Threshold graph is a subgraph of the network model with weights of edges are greater than given threshold. Algorithms for threshold graph identification are proposed in [1]–[3].

In the article the complex systems of random nature where the elements of the system are characterized by a random variables are considered. Available data to network analysis of the complex system is sample of observation. It implies that algorithms of network structures identification has to be considered as a statistical procedures [6]. To model the complex system of random nature the concept of random variables network was introduced in [7]. Random variables network is a pair (X, γ), where $X = (X_1, \dots, X_N)$ is a random vector and γ is a measure of dependence between random variables.

Most popular random variables network is Pearson correlation network with normal distribution or pair (X, γ^P), where vector $X = (X_1, \dots, X_N)$ has multivariate normal distribution $N(\mu, \Sigma)$, where $\mu = (\mu_1, \dots, \mu_N)$ is a vector of expectations, $\Sigma = (\sigma_{ij})$ is a covariance matrix and measure γ^P is Pearson correlation $\rho_{i,j} = \rho(X_i, X_j) = \frac{E(X_i-\mu_i)(X_j-\mu_j)}{\sqrt{\sigma_{ii}\sigma_{jj}}}$. Procedure for threshold graph identification in Pearson correlation network is based on simultaneous testing of the hypotheses $h_{ij} : \rho_{i,j} \le \rho_0 : \forall i, j = 1, \dots, N; i \neq j$. In [8] properties of standard procedure for threshold graph identification based on sample Pearson correlations was investigated. It was shown that the procedure is optimal statistical procedure for threshold graph identification in Pearson correlation network with normal distribution in the class $\mathcal{D}$ of procedures satisfying the following conditions:

- Any statistical procedure $\delta(x) \in \mathcal{D}$ is invariant with respect to the group of shift/scale transformations of the sample space.
- Risk function of any statistical procedure $\delta \in \mathcal{D}$ is continuous with respect to parameter.
- Individual tests $\varphi_{i,j}(x)$ generated by any $\delta \in \mathcal{D}$ depends on observations $x_i(t); x_j(t), t = 1, \dots, n$ only.

However in [9] it was shown by simulations that sample Pearson correlation is not robust under deviations of vector $X = (X_1, \dots, X_N)$ distribution from normal distribution. To overcome the issue in [10] sign measure $\gamma^{Sg}_{i,j} = p^{i,j} = P((X_i - \mu_i)(X_j - \mu_j) > 0)$ was introduced. In [7] procedures for network structures identification in sign similarity network (X, γ^{Sg}) with elliptically contoured distribution $X - ECD(\mu, \Lambda, g)$ [11] with density

$$f(x; \mu, \Lambda) = |\Lambda|^{-\frac{1}{2}} g\{(x-\mu)'\Lambda^{-1}(x-\mu)\}, \qquad (1)$$

where Λ is a positive definite matrix, function $g(x) \ge 0$ and

$$\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} g(y'y) dy_1 \dots dy_p = 1$$

was considered. It was shown for known vector μ that procedure for threshold graph identification based on simultaneous testing of the hypotheses $h_{ij} : p^{i,j} \le p^0 : \forall i, j = 1, \dots, N; i \neq j$ is robust in the class of $ECD(\mu, \Lambda, g)$. Also it was proved that $p^{i,j} = \frac{1}{2} + \frac{1}{\pi} \arcsin \rho_{i,j}$ i.e. inequality $\rho_{i,j} \le \rho_0$ is equivalent to inequality $p^{i.j} \le p^0$. This mean that threshold graph in Pearson correlation network with elliptically contoured distribution with threshold ρ_0 is equivalent to threshold graph in sign similarity network with elliptically contoured distribution with threshold $p^0 = \frac{1}{2} + \frac{1}{\pi} \arcsin \rho_0$.

In real practice it is unrealistic to assume known μ. To deal with unknown μ in [12] it was proved that for the sample $X(1),\dots,X(n)$ from elliptically contoured distribution $ECD(\mu,\Lambda,g)$ one has $P((X_i(t)-\mu_i)(X_j(t)-\mu_j)>0)=P((X_i(t)-\overline{X_i})(X_j(t)-\overline{X_j})>0), \forall t=1,\dots,n$ where $\overline{X_i}=\frac{1}{n}\sum_{t=1}^{n}X_i(t)$. It implies that threshold graph in Pearson correlation network with elliptically contoured distribution with threshold ρ_0 is equivalent to threshold graph in sign similarity network with elliptically contoured distribution with threshold $p^0=\frac{1}{2}+\frac{1}{\pi}\arcsin\rho_0$ under the case of unknown μ too.

Another way to avoid the problem of unknown μ is to consider Kendall correlation network (X,γ^{Kd}) with elliptical distribution where measure γ^{Kd} of dependence given by

$$\gamma^{Kd}=\tau_{i,j}= \\ =P\left[(X_i(t)-X_i(t-1))(X_j(t)-X_j(t-1))>0\right],$$

where $\binom{X_i}{X_j}$ is a random vector with cumulative distribution function $F_{X_iX_j}(x,y)$ and $\binom{X_i(t-1)}{X_j(t-1)}$, $\binom{lX_i(t)}{X_j(t)}$ are the independent copies of the vector $\binom{X_i}{X_j}$. It follows from results of [13] if $X-ECD(\mu,\Lambda,g)$ then $p^{i,j}=\tau_{i,j}$.

Therefore there are three random variables networks (X,γ^P), (X,γ^{Sg}), (X,γ^{Kd}), such that threshold graphs in the networks are equivalent. In [7] robust multiple decision statistical procedure for threshold graph identification in sign similarity network (X,γ^{Sg}) is constructed for the case of known vector μ. Such robustness properties are follows from robustness properties of individual tests. The aim of the present paper is to investigate by simulations the robustness properties of individual tests in Pearson correlation network with elliptical distribution (X,γ^P), in sign similarity network (X,γ^{Sg}) with unknown μ and in Kendall correlation network (X,γ^{Kd}).

The paper is organized as follows: in section II the concepts of true and sample threshold graphs are introduced; in section III the robustness of statistical procedures is discussed; in section IV individual tests of statistical procedures for threshold graph identification in Pearson correlation network are discussed; in section V individual tests of statistical procedures for threshold graph identification in sign similarity network with elliptically contoured distribution with known μ are discussed; in section VI individual tests of statistical procedures for threshold graph identification in Kendall correlation network with elliptically contoured distribution are discussed; in section VII investigation of robustness of individual tests by numerical experiments are presented; in section VIII the conclusion of the article is given.

II. Threshold graph

A. True threshold graph

To formulate the problem of threshold graph identification we will use the concept of adjacency matrix of graph $G=(V,E)$. Matrix

$$S=\begin{pmatrix} 0 & s_{12} & \dots & s_{1N} \\ s_{12} & 0 & \dots & s_{2N} \\ \dots & \dots & \dots & \dots \\ s_{1N} & s_{2N} & \dots & 0 \end{pmatrix}.$$

with elements

$$s_{ij}=\begin{cases} 1, & (i,j)\in E \text{or there is edge (i,j)} \\ & \text{between vertices i and j in graph G,} \\ 0, & \text{otherwise,} \end{cases}$$

is called adjacency matrix of graph G.

True threshold graph TG for threshold γ_0 is defined by adjacency matrix

$$TG=\begin{pmatrix} 0 & tg_{12} & \dots & tg_{1N} \\ tg_{12} & 0 & \dots & tg_{2N} \\ \dots & \dots & \dots & \dots \\ s_{1N} & s_{2N} & \dots & 0 \end{pmatrix} \quad (2)$$

with elements

$$tg_{ij}=\begin{cases} 1, & \gamma_{i,j}>\gamma_0 \\ 0, & \gamma_{i,j}\le\gamma_0. \end{cases}$$

B. Sample Threshold Graph

In real practice available data for analysis is sample of observations

$$\begin{pmatrix} x_1(1) \\ x_2(1) \\ \dots \\ x_N(1) \end{pmatrix},\dots,\begin{pmatrix} x_1(n) \\ x_2(n) \\ \dots \\ x_N(n) \end{pmatrix}.$$

The problem of the threshold graph identification can be considered as multiple hypotheses testing problem of the following individual hypotheses:

$$h_{ij}:\gamma_{i,j}\le\gamma_0 \quad (tg_{i,j}=0) \\ \text{versus } k_{ij}:\gamma_{i,j}>\gamma_0 \quad (tg_{i,j}=1). \quad (3)$$

Let

$$\varphi_{ij}(x)=\begin{cases} 1, & \text{edge } (i,j) \text{ is added to} \\ & \text{the sample network structure,} \\ 0, & \text{otherwise,} \end{cases} \quad (4)$$

be the tests for individual hypotheses (3) testing.

Any statistical procedure for the threshold graph identification is based on individual tests $\varphi_{ij}(x)$ (4) of testing the individual hypotheses $h_{ij}:\gamma_{i,j}\le\gamma_0$ versus $k_{ij}:\gamma_{i,j}>\gamma_0$ (3).

Let $\mathcal{G}$ be the set of $N\times N$ adjacency matrices. The procedure $\delta(x)$ for the threshold graph identification accept the decision d_Q that threshold graph has adjacency matrix $Q, Q\in\mathcal{G}$ if and only if $\Phi(x)=Q$, where

$$\Phi(x)=\begin{pmatrix} 0 & \varphi_{12}(x) & \dots & \varphi_{1N}(x) \\ \varphi_{12}(x) & 0 & \dots & \varphi_{2N}(x) \\ \dots & \dots & \dots & \dots \\ \varphi_{1N}(x) & \varphi_{2N}(x) & \dots & 0 \end{pmatrix}. \quad (5)$$

Threshold graph defined by the matrix (5) is called *sample* threshold graph.

III. Robustness of Statistical Procedures

Let $w(S,Q)$ be the loss from decision d_Q that threshold graph has adjacency matrix Q for the case when true threshold graph has adjacency matrix S. According to [14] the quality of any statistical procedure is measured by risk function

$$R(S,\theta,\delta) = \sum_{Q\in\mathcal{G}} w(S,Q)P_\theta(\delta=d_Q), \theta=(\mu,\Lambda,g), \theta\in\Omega_S, \quad (6)$$

where Ω_S is the parametric set such that threshold graph has adjacency matrix S and $P_\theta(\delta=d_Q)$ is the probability of decision d_Q for given θ.

In the article we are interested in statistical procedures satisfying the following definition

Let vector X has elliptically contoured distribution $ECD(\mu,\Lambda,g)$ (1). Statistical procedure δ is robust if $R(S,\theta,\delta)$ does not depend from function g.

It is natural to connect the loss function $w(S,Q)$ with difference of two graphs defined by matrices (2) and (5). This difference is defined by the numbers of erroneously included edges and erroneously non included edges. Then for the problems of network structures identification it is natural to consider the loss function which takes into account these numbers. Therefore we apply the concept of additive loss function, introduced in [15]. In the case loss function $w(S,Q)$ can be written in the following form:

$$w(S,Q) = \sum_{\substack{i,j:s_{i,j}=0\\ q_{i,j}=1}} a_{ij} + \sum_{\substack{i,j:s_{i,j}=1\\ q_{i,j}=0}} b_{ij}, \quad (7)$$

where a_{ij}, b_{ij} are the losses from the errors of the first, second kinds respectively.

In [16] it is shown that under additive loss function (7) risk function (6) of statistical procedure for threshold graph identification has the form:

$$R(S,\theta,\delta) = \sum_{i=1}^{N}\sum_{j=1}^{N} r(s_{i,j},\theta,\varphi_{ij}), \quad (8)$$

where $r(s_{i,j},\theta,\varphi_{ij})$ is risk function of test φ_{ij}.

Then from (8) it follows that robust individual tests φ_{ij} lead to robust statistical procedure δ for threshold graph identification.

IV. Pearson Correlation Network

For Pearson correlation network individual hypotheses have the form: $h_{i,j}:\gamma^P_{i,j}\le\gamma^P_0$.

For Pearson correlation network with normal distribution individual test is [11]:

$$\varphi^{PN}_{ij}(x) = \begin{cases}1, & p^{PN}_{i,j} < \alpha_{i,j},\\ 0, & p^{PN}_{i,j} \ge \alpha_{i,j},\end{cases} \quad (9)$$

where p-value $p^{PN}_{i,j}$ is calculated from

$$p^{PN}_{i,j} = 1-\Phi\left(\sqrt{n-1}\frac{r_{i,j}-\gamma^P_0}{\sqrt{1-r^2_{i,j}}}\right).$$

Here $\Phi(x)$ is distribution function of $N(0,1)$ and

$$r_{i,j} = \frac{\sum_{t=1}^n (x_i(t)-\overline{x_i})(x_j(t)-\overline{x_j})}{\sqrt{\sum_{t=1}^n (x_i(t)-\overline{x_i})^2 \sum_{t=1}^n (x_j(t)-\overline{x_j})^2}}$$

is the sample Pearson correlation.

For Pearson correlation network with elliptical distribution individual test is [11]:

$$\varphi^P_{ij}(x) = \begin{cases}1, & p^P_{i,j} < \alpha_{i,j},\\ 0, & p^P_{i,j} \ge \alpha_{i,j},\end{cases} \quad (10)$$

where p-value $p^P_{i,j}$ is calculated from

$$p^P_{i,j} = 1-\Phi\left(\sqrt{\frac{n-1}{1+\overline{\kappa}}}\frac{r_{i,j}-\gamma^P_0}{\sqrt{1-r^2_{i,j}}}\right).$$

Here $\overline{\kappa} = \frac{\sum_{t=1}^n (x(t)-\overline{x})'S^{-1}(x(t)-\overline{x})}{(n-1)N(N+2)}$ is estimation of kurtosis parameter, $x(t)=(x_1(t),\dots,x_N(t))$ and $S=\sum_{t=1}^n (x(t)-\overline{x})'(x(t)-\overline{x})$ is estimation of covariance matrix.

V. Sign Similarity Network

For sign similarity network (X,γ^{Sg}) individual hypotheses have the form: $h_{i,j}:\gamma^{Sg}_{i,j}\le\gamma^{Sg}_0$. Individual tests have the form [7]:

$$\varphi^{Sg}_{ij} = \begin{cases}1, & p^{Sg}_{i,j} < \alpha_{i,j},\\ 0, & p^{Sg}_{i,j} \ge \alpha_{i,j},\end{cases} \quad (11)$$

where p-value $p^{Sg}_{i,j}$ is defined from

$$p^{Sg}_{i,j} = 1 - F_{\gamma^{Sg}_0}\left(T^{sg}_{i,j}\right).$$

Here $F_{\gamma^{Sg}_0}(x)$ is the distribution function of the binomial distribution $b(n,\gamma^{Sg}_0)$ and

$$T^{sg}_{i,j} = \sum_{t=1}^n I_{i,j}(t),$$

where

$$I_{i,j}(t) = \begin{cases}1, & (x_i(t)-\mu_i)(x_j(t)-\mu_j)\ge 0,\\ 0, & (x_i(t)-\mu_i)(x_j(t)-\mu_j)<0.\end{cases}$$

VI. Kendall Correlation Network

For Kendall correlation network (X,γ^{Kd}) individual hypotheses have the form: $h_{i,j}:\gamma^{Kd}_{i,j}\le\gamma^{Kd}_0$. Individual tests are [17]:

$$\varphi^{Kd}_{ij} = \begin{cases}1, & p^{Kd}_{i,j} < \alpha_{i,j},\\ 0, & p^{Kd}_{i,j} \ge \alpha_{i,j},\end{cases} \quad (12)$$

where p-value $p^{Kd}_{i,j}$ are defined from

$$p^{Kd}_{i,j} = 1-\Phi\left(\sqrt{\frac{9n(n-1)}{2(2n+5)}}\left(T^{Kd}_{ij}-\gamma^{Kd}_0\right)\right).$$

Here

$$T^{Kd}_{ij} = \frac{1}{n(n-1)}\sum_{t\ne s} sign\left((x_i(t)-x_i(s))(x_j(t)-x_j(s))\right).$$

VII. Experimental Results

To obtain experimental results let us consider the class $\mathcal{K}(\Lambda)$ of elliptically contoured distribution with fixed matrix Λ. It follows that all considered random variables networks generate network models with equivalent true threshold graphs.

Since the quality of statistical procedure is measured by (8) then it is sufficient to investigate the properties of individual tests for testing individual hypotheses $h_{i,j} : \gamma_{i,j} \leq \gamma_0$ vs $k_{i,j} : \gamma_{i,j} > \gamma_0$.

Let us consider the two-dimensional elliptically contoured distribution

$$\binom{X_i}{X_j} - ECD\left(\binom{\mu_i}{\mu_j}, \begin{pmatrix} \lambda_{ii} & \lambda_{ij} \\ \lambda_{ij} & \lambda_{jj} \end{pmatrix}, g\right).$$

In the present paper the hypothesis $\lambda_{ij} \leq 0$ is tested.

We are interested in the following questions:

- robustness of significance level of individual tests with respect to g;
- robustness of power function of individual tests with respect to g.

To investigate these questions we simulate a certain number of observation (n) using the mixture distribution. The mixture distribution is constructed as follows: vector $X = (X_i, X_j)$ takes value from normal distribution $N(0, \Lambda)$ with probability ϵ and from Student distribution with 3 degree of freedoms $t_3(0, \Lambda)$ with probability $1 - \epsilon$. Then density of simulations has the form:

$$f_{sim}(x) = \epsilon f_{Normal}(x) + (1 - \epsilon) f_{Student,3}(x).$$

Below the obtained experimental results concerning the robustness of significance level and robustness of power function of the individual tests are discussed.

A. Robustness of Significance Level

Obtained experimental results which concern the robustness of significance level of individual tests (9), (10), (11), (12) allows to make the following conclusions.

- For $\alpha = 0.1$ and $\lambda_{ij} = 0$ test $\varphi_{ij}^{PN}(x)$ does not robust to deviation from normality. Namely under $n = 50, \epsilon = 1$ one has 104 rejections from 1000 experiments. But for decreasing of ϵ the number of rejections is increased. For $\epsilon = 0$ one has 255 rejections.
- For $\alpha = 0.1$ and $\lambda_{ij} = 0$ test $\varphi_{ij}^{P}(x)$ does not robust to deviation from normality. Namely under $n = 50, \epsilon = 1$ one has 108 rejection from 1000 experiments. But for decreasing of ϵ the number of rejections is increased. For $\epsilon = 0$ one has 177 rejections.
- For $\alpha = 0.05$ and $\lambda_{ij} = 0$ test $\varphi_{ij}^{Kd}(x)$ does not robust to deviation from normality. Namely under $n = 50, \epsilon = 1$ one has 52 rejection from 1000 experiments. But for decreasing of ϵ the number of rejections is increased. For $\epsilon = 0$ one has 94 rejections.
- For all α and $\lambda_{ij} = 0$ test $\varphi_{ij}^{Sg}(x)$ is robust to deviation from normality [7].

Common point with respect to robustness of significance level for all considered tests is that for $\alpha = 0.5$ the probability of first kind error is equal to 0.5 for any ϵ.

B. Robustness of Power Function

Obtained experimental results which concern the robustness of power function of individual tests (9), (10), (11), (12) allows to make the following conclusions.

- For
  $$\alpha = 0.05,\ n = 100,\ \epsilon = 1,\ \lambda_{ij} = 0.3$$
  power function of test $\varphi_{ij}^{PN}(x)$ is 0.927 ($\hat{\alpha} = 0.046$). But for
  $$\alpha = 0.05,\ n = 100,\ \epsilon = 0,\ \lambda_{ij} = 0.3$$
  power function of test $\varphi_{ij}^{PN}(x)$ is 0.771 ($\hat{\alpha} = 0.21$).
- For
  $$\alpha = 0.05,\ n = 100,\ \epsilon = 1,\ \lambda_{ij} = 0.3$$
  power function of test $\varphi_{ij}^{P}(x)$ is 0.933 ($\hat{\alpha} = 0.046$). But for $\alpha = 0.05$, $n = 100$, $\epsilon = 0$ and $\lambda_{ij} = 0.3$ power function of test $\varphi_{ij}^{P}(x)$ is 0.611 ($\hat{\alpha} = 0.111$).
- For
  $$\alpha = 0.1,\ n = 25,\ \epsilon = 1,\ \lambda_{ij} = 0.45$$
  power function of test $\varphi_{ij}^{Kd}(x)$ is 0.828 ($\hat{\alpha} = 0.103$). But for
  $$\alpha = 0.1,\ n = 25,\ \epsilon = 0,\ \lambda_{ij} = 0.45$$
  power function of test $\varphi_{ij}^{Kd}(x)$ is 0.780 ($\hat{\alpha} = 0.125$).
- Power function of the test $\varphi_{ij}^{Sg}(x)$ is robust to deviation from normality [7].
- For power function of the tests $\varphi_{ij}^{PN}(x), \varphi_{ij}^{P}(x)$ the result of the robustness does not valid. Namely for $\varphi_{ij}^{PN}(x), \varphi_{ij}^{P}(x)$ power function is 0.94 for $\epsilon = 1, \lambda_{ij} = 0.15, n = 100$. But power function is 0.95 for $\epsilon = 0, \lambda_{ij} = 0.35, n = 100$. Therefore these tests have close power functions for different alternative points $\lambda_{ij} = 0.15$ and $\lambda_{ij} = 0.35$ for the same number of observations $n = 100$. It implies that the power functions of the tests $\varphi_{ij}^{PN}(x), \varphi_{ij}^{P}(x)$ are not robust.
- For $\alpha = 0.5$ significance levels and power functions of the tests $\varphi_{ij}^{Sg}(x)$ and $\varphi_{ij}^{Kd}(x)$ are robust to deviation from normality.
- For $\alpha = 0.5$ power function of $\varphi_{ij}^{Kd}(x)$ is uniformly better than $\varphi_{ij}^{Sg}(x)$. Namely for $\epsilon = 0, \lambda_{ij} = 0.3, n = 50$ power function of $\varphi_{ij}^{Kd}(x)$ is 0.97. For $\varphi_{ij}^{Sg}(x)$ power function is 0.97 for $\lambda_{ij} = 0.4$ or power function of $\varphi_{ij}^{Sg}(x)$ is 0.97 for $n = 100$.

Moreover for illustrative purpose the comparison of power functions of individual tests (9), (10), (11), (12) for different ϵ are presented in the Fig. 1, 2, 3.

VIII. Conclusion

The properties of robustness of different tests for testing individual hypotheses for threshold graph identification are investigated. One interesting result is that inclusion of the estimation of kurtosis parameter in individual test (10) proposed in [11] does not lead to the robustness of significance level for considered sample sizes. Another interesting result concerns the properties of individual Kendall test (12) which

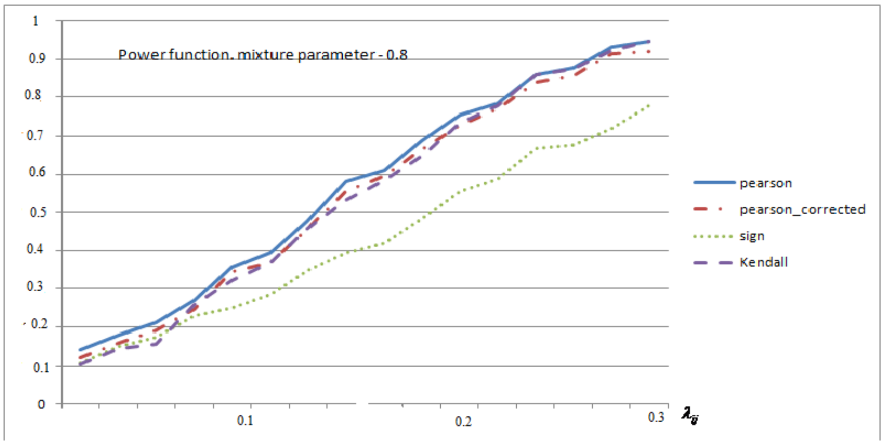

Fig. 1. Power functions of tests (9), (10), (11), (12) for $\alpha = 0.1$. Mixture parameter $\epsilon = 0.8$. $n = 100$

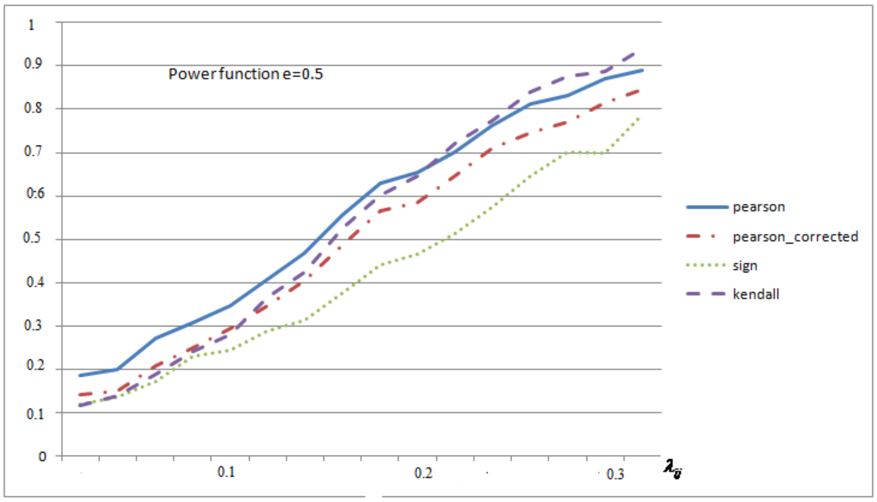

Fig. 2. Power functions of tests (9), (10), (11), (12) for $\alpha = 0.1$. Mixture parameter $\epsilon = 0.5$. $n = 100$

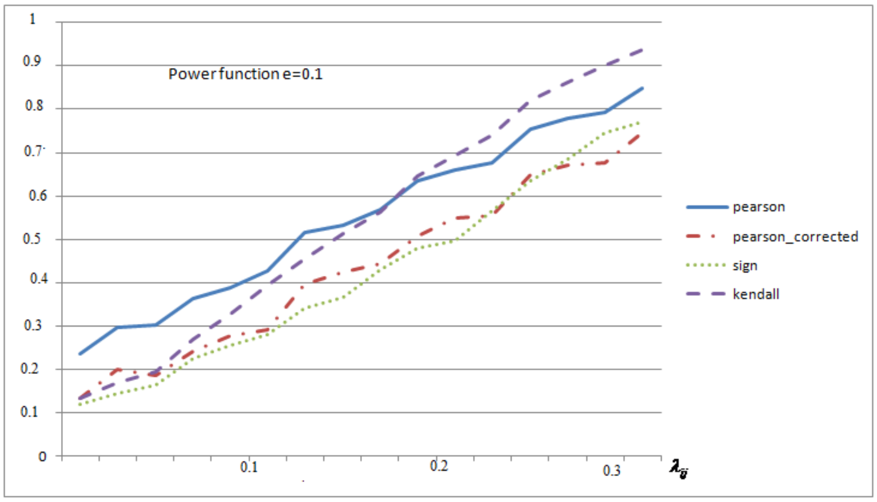

Fig. 3. Power functions of tests (9), (10), (11), (12) for $\alpha = 0.1$. Mixture parameter $\epsilon = 0.1$. $n = 100$

is better among the considered level α tests (see Fig. 2, 3) with respect to power. However individual Kendall test (12) does not robust to deviation from normality unlike sign test (11) which is robust to deviation from normality in the class of elliptically contoured distributions $ECD(\mu, \Lambda, g)$ for the case of known μ (see [7]) .

From the other side all results are obtained by simulations. The problem of theoretical justification of obtained result remains the open problem.

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