

Comparison of Black–Scholes Model and Monte-Carlo Simulation on Stock Price Modeling

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Abstract. Option price and its valuation are crucial issues in finance research. In this research we implement Black-Scholes option pricing model and compare it with stochastic modeling, namely the Monte-Carlo Simulation. These two classical models are implemented on newly emerged technology companies like Google and Apple and traditional industry like Esso. The result shows that both option pricing model and numerical simulation are able to yield prices close to actual stock price

1. Introduction

A very important concept of the mathematical finance field is the option and the pricing of it. Being the most notable type of financial derivative, the call option is a type of contract between the seller and the buyer. Within a certain period of time, the buyer could decide whether to buy and the quantity of a certain commodity or financial instrument from the seller at a certain strike price, however, the seller has the obligation to sell what the buyer has demanded. Being classified into the European option and the American option, the call option varies due to alterations of region. European Call option can be exercised by the buyer only on its expiration date, whereas the American Call option can be exercised by the buyer at any time before the expiration day (including the expiration day), being very complicated. Many researches have been done on call option. A classical application is using the Black-Scholes model to examine stock prices [1]. A more recent research investigates Black-Scholes model in S&P 500 pricing [2]. On the other hand, option price can also be simulated by stochastic processing like Monte-Carlo simulation. Monte-Carlo simulation is used in both European and American option pricing [3,4]. In this research, we examine both Black-Scholes model and Monte-Carlo simulation and compare them with the actual price. Furthermore, we select companies from different industries to find out whether the performance of both models matches the actual prices.

2. Black-Scholes Model

Robert C. Merton commenced the options' pricing models, publishing a paper expanding the mathematical comprehension of them which he named as "Black-Scholes Options Pricing Model" [5]. Merton and Scholes received the 1997 Nobel Memorial Prize in Economic Sciences for their work of the option pricing model.

The Black-Scholes Model interprets the relationship between the prices of options and the underlying stock. The Black-Scholes Model has to follow 3 assumptions.

Initially, there does not exist any arbitrage opportunity (a kind of trade that the profit is more than the risk) in the financial market. Secondly, the underlying stock value must follow a geometric Brownian Motion of

$$dS = \mu S dt + \sigma S dW \quad (1)$$

where μ represents the average rate of growth of the underlying stock, σ denotes the volatility (the range of stocks' potential prices) of the stock, and W is a Brownian Motion. A Brownian Motion $W(t)$ is a continuous stochastic process with $W(0) = 0$ and $W(t) = N(0, t)$ for all $t > 0$, where

$N(\mu, \sigma^2)$ represents the normal distribution with expected value μ and variance σ^2 . In a Brownian motion, the future terms cannot be determined from the known terms. Ultimately, the market could only have a stationary risk-free interest rate. Under the risk neutral measure, $dS = (r - \delta)Sdt + SdW^Q$ where r expresses the risk-free rate, δ is the dividend yield and W^Q is the Brownian motion under the risk neutral measure.

3. Call Option

Being considered as a deposit for future investments, call option is a kind of option which bestows the holder the right to buy underlying assets at a certain strike price K in the future, even if the assets' prices could potentially tremendously increase. Simultaneously, if the assets' prices in the future is lower than the strike price K , owners of the call options could decide to stop utilizing the option.

We denote the price of a European Call Option by $C(S_t, t)$, with strike price K and maturity time T . The option's pay-off, $(S - K)^+ = \max(S - K, 0)$, is its terminal condition at maturity time T .

The pricing model of an American Call Option is very complicated, for it does not have a specific maturity time. Therefore, we could only utilize computer programs to calculate the pricing of American Call Options.

3.1 Analytic Solution of European Call Option using Black Scholes Model

The value of a call option for a dividend-paying underlying stock in terms of the Black-Scholes parameters is:

$$C(S_t, t) = S_t e^{-\delta(T-t)} \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2) \quad (2)$$

where

$$d_1 = \frac{\log(S/K) + (r - \delta + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \quad (3)$$

$$d_2 = \frac{\log(S/K) + (r - \delta - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \quad (4)$$

In this pricing method, Φ is the cumulative density function (CDF) of the standard normal distribution. The price of a corresponding put option based on the put-call parity is

$$P(S_t, t) = K e^{-r(T-t)} - S_t + C(S_t, t) = N(-d_2) K e^{-r(T-t)} - N(-d_1) S_t \quad (5)$$

4. Numerical Result using Monte-Carlo Simulation

We calculated the values of one dimensional European call options, utilizing real market data of Apple Inc., Alphabet Inc., The Coca-Cola Co., Exxon Mobil Co., SP 500, and Dow Jones Industrial Average respectively written on the ticker numbers AAPL, GOOG, KO, XOM, SPY, and DJIA with $r = 0.023$ (3-months U.S. treasury bill rate) and $T - t = 1$ (time frame is 1 year). The technique we used is the Monte-Carlo simulation, being commonly applied in asset pricing. The numerical values generated from Monte-Carlo simulation is close to the theoretical values obtained by the Black-Scholes formula, thereafter our results are fairly similar to the actual options' prices.

Apple Inc. is the major American multinational technology cooperation developing and retailing electronics, phones, computer, software, and online services. Alphabet Inc., being normally known as the parent company of Google and its subsidiaries, is an American multinational conglomerate serving technologies, internet software, video games, technology-based automobiles, and biotechnologies. The Coca-Cola Company is an American multinational corporation, manufacturing and retailing nonalcoholic beverages, such as their flagship product of Coca-Cola. Exxon Mobil Corporation, often known as ExxonMobil, is the largest American multinational oil and gas corporation, merging Exxon, Mobil, Esso, and ExxonMobil Chemical. The S&P 500, or the S&P, is an American stock market index based on the capitals of 500 large companies. The Dow Jones

Industrial Average is a stock market index utilizing the asset values and historical trades of 30 American publicly owned large companies.

Table 1. One-Dimension European Call Option Price with Actual Data with $r = 0.0237$, $T = 1$

Ticker	S	K	σ	q	Simulation	BS
AAPL	156.30	155.0	0.2657	0.0143	17.67758	17.626
GOOG	1070.0	1070.0	0.2578	0.0000	120.7345	122.932
KO	46.76	47.0	0.1483	0.0341	2.34684	2.400
XOM	70.46	72.5	0.1908	0.0400	3.967179	3.873
SPY	262.60	263.0	0.1721	0.0169	18.24806	18.683
DIA	243.77	245.0	0.1696	0.0267	15.06899	15.304

5. Conclusion

This research mainly focuses on the pricing of stocks, particularly ones following the European Call Option, utilizing the Black-Scholes Model and the Monte-Carlo Simulation. Using python as a method of calculating, we established programs to price the stocks of multinational companies such as Apple Inc., Alphabet Inc., the Coca Cola Company, and ExxonMobil Cooperation, in comparison with the S&P 500 and Dow Jones Industrial Average Index.

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