

Research on Equity Refinancing, Debt Restructuring and Firm Pricing Based on Financial and Economic Distress

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Abstract. This paper distinguishes financial and economic distress, and gives a comprehensive solution to firm's distress by means of integrative equity refinancing and debt restructuring. It constructs a model to price the firm under different financial conditions, especially when the liquidation value is less than 0. In the model, the relationships among financial distress, economic distress, equity refinancing, debt restructuring and renegotiation are clarified. The optimal restructuring boundary is also determined endogenously by maximizing the interests of both shareholders and creditors with the viewpoint of harmony.

1. Introduction

If the firm is short of liquidity and can't pay the debt coupon, it will fall into financial distress. If the firm's business prospects deteriorate and its commercial value decreases, it will fall into economic distress [1]. Kim [2] and Asquith et al. [3] used liquidity indicators as triggers for default, and concluded that when liquidity is insufficient to pay coupons, the firm is in financial distress and goes into bankruptcy. Taking the asset value as the default trigger, Leland [4], Black and Cox [5] et al. assumed that when the liquidation value touches the contract debt obligation and the equity value is 0, the firm falls into financial distress and goes into bankruptcy. Davydenko [1] separated economic distress from financial distress, classified the models proposed by Leland et al. [4-5] as default models induced by economic distress, and empirically studied the relationship between firm's distress and default. Michael et al. [6] had proved that debt restructuring is one of the effective ways for firm to solve financial and economic distress.

Strategic debt service (SDS, hereafter), as an important way of debt restructuring, has been introduced by Hart and Moore [7]. SDS involves a simple idea: when liquidation is costly, it may be possible for equity-holders to pay a reduced coupon without triggering liquidation, since rejecting the offer and liquidating the firm may leave debt holders even worse off. On the relationship between SDS and financial distress, Anderson [8] and Acharya et al. [9] assumed that when SDS occurs, as long as liquidity is insufficient to pay contract coupons or reduced coupons, the firm will be in financial distress and go into bankruptcy. Fan and Sundaresan [10] had tried to combine the debt restructuring with the cash flow clause. If asset value touches the cash flow clause before hitting the debt restructuring threshold, the firm can not pay the coupon, which leads to liquidity default and bankruptcy instead of debt restructuring. Goto and Suzuki [11] distinguished liquidity default from strategic default, and argued that financial distress leads to liquidity default, while SDS is purely a speculative behavior of shareholders. In summary, the existing literature does not study the causal relationship between financial distress and SDS, and SDS has obvious characteristics of speculation and deliberate default.

With respect to debt restructuring threshold of SDS model, Anderson [8] and Acharya et al. [9] assumed that debt restructuring occurs when firm's liquidation value touches the value of the debt principal and coupon. Fan and Sundaresan [10] endogenously determined the debt restructuring threshold with the goal of maximizing shareholders's interests, which might be greater than or less

than the financial distress threshold. However, debt restructuring with liquidation value less than 0 has hardly been studied in present SDS models.

In addition, equity refinancing is also one of the important ways to solve the firm's financial distress. Asvanunt et al. [12] considered equity refinancing to solve the problem of liquidity shortage. In present literatures on debt pricing, models on liquidity default caused by financial distress (such as Kim et al. [2]) mostly assume that equity financing costs are huge to prohibit equity refinancing. However, models on endogenous default caused by economic distress (such as Leland [4]) mostly assume that the cost of equity refinancing is zero, and firms will issue stocks to raise coupon funds as long as the value of equity is greater than zero, so the issue of equity refinancing is not considered in the models. In terms of strategic default, Anderson et al. [8] also assumed huge equity refinancing costs to prohibit equity refinancing. Fan and Sundaresan [10] proposed that equity refinancing could be used to raise funds for SDS, but they haven't embedded equity refinancing in their model.

In this paper, we set up financial and economic distress indicators to limit the occurrence of SDS after the firm is in financial and economic distress, making SDS as a tool to solve financial and economic distress. Particularly, we construct a comprehensive scheme compatible with equity refinancing and SDS to solve the firm's distress. Besides, we study and price the firm under different financial conditions, especially when the liquidation value is less than 0.

2. Basic setup

The following assumptions underlie the model:

(1) We denote the asset value of the firm by V , the firm's value by ν , the equity value by E , and the debt value by D . The asset value V is described by a geometric Brownian motion:

$$dV = (\mu - q)Vdt + \sigma VB_t \quad (1)$$

Where μ is the instantaneous expected rate of return on the firm's asset value, σ^2 is the instantaneous variance of the return on the firm's asset value, and B_t is a standard Brownian motion. The firm's cash return from firm's operation at any time is qV , where q is the firm's instantaneous cash return rate and $q \leq \mu$. The firm's cash return qV is all used to pay coupons and dividends. In this case, q is also the cash payout ratio. Besides, firm is not allowed to sell assets to distribute dividends.

(2) The firm's debt is a perpetual bond with a permanent instantaneous coupon c , and the value of the tax benefit of debt is τc ($0 \leq \tau < 1$). This paper does not consider the role of tax benefit of SDS, nor does it consider the situation of limited-term bonds and multiple bonds.

(3) We mainly focus on the risk of strategic default and liquidity default, and assume that the risk-free interest rate is a constant r without considering interest rate risk.

(4) When the firm goes bankrupt and liquidates, the variable bankruptcy cost is a certain proportion of the value of the firm's assets, that is αV ($0 \leq \alpha \leq 1$), and the fixed bankruptcy cost is denoted as K ($0 \leq K$). The threshold for firm's bankruptcy is denoted as V_B . The debt holders have strict absolute priority upon bankruptcy: when the liquidation happens, a cost of $\min(V_B, \alpha V_B + K)$ is taken away by outsiders according to the principle of limited repayment; debt holders receive the remaining $\max[0, (1 - \alpha)V_B - K]$, and equity holders receive nothing.

(5) Referring to the liquidity default threshold setting method adopted by Kim [1], the financial distress threshold is set as $V_F = \frac{c}{q}$. At this threshold, the firm's cash return can just pay the coupon c .

(6) The threshold of economic distress is set as $V_E = \frac{K}{1 - \alpha}$, at which the liquidation value of the firm's assets after deducting bankruptcy costs is 0, so as to discuss the situation that the liquidation value is less than 0.

(7) Assuming that no consideration is given to the interaction between financial distress and economic distress, as long as the firm is in financial distress (Whenever $V < V_F$), shareholders and

creditors renegotiate and agree to pay the contract coupon by equity refinancing in advance. When economic distress occurs at the same time (when $V < V_F$ and $V < V_E$) and the value of equity after equity refinancing is lower than that under debt restructuring, debt restructuring is carried out and reduced coupon $S(V)$ is paid. On this basis, the threshold of debt restructuring V_S is determined endogenously, and the threshold V_S is set to satisfy the conditions $V_S < V_F$ and $V_S < V_E$.

(8) Based on the Nash equilibrium game model adopted by Sarkar[13] and Liu Gan[14], we denote η as the equity holders' bargaining power, $(1-\eta)$ ($0 \leq \eta \leq 1$) as the debt holders' bargaining power, and θ as a parameter that reflects the sharing rule for the firm's value in debt restructuring stage. The value allocated by shareholders is $E(V) = \theta v(V)$, and the value allocated by debt-holders is $D(V) = (1-\theta)v(V)$. The Nash equilibrium game model and the Nash solution can be characterized as

$$\begin{aligned} \theta^* &= \arg \max \{ \theta v(V) - 0 \}^\eta \{ (1-\theta)v(V) - \max[(1-\alpha)V - K, 0] \}^{(1-\eta)} \\ &= \min \left[\eta - \eta \frac{(1-\alpha)V - K}{v(V)}, \eta \right] \end{aligned} \quad (2)$$

Based on the previous assumption, $V_S < V_E$, the sharing rule is $\theta = \eta$ for any $V < V_S$.

(9) This paper cites the equity dilution method proposed by Asvanunt et al. [12]. The ratio of financing cost to financing amount is written as γ ($0 \leq \gamma < 1$), and the ratio of shares issued for equity refinancing to shares held by original shareholders before refinancing is written as ω . Assuming that the amount of funds raised through refinancing is \bar{C} , then the original shareholder equity value after equity refinancing is $E' = \frac{1}{1+\omega}E = E - \frac{\bar{C}}{1-\gamma}$. If we denote $\beta = \frac{1}{1-\gamma}$, the reduction of the original shareholder's equity value after dilution is similar to the dividend $-\beta\bar{C}$ earned by the original shareholder.

3. Pricing model

According to the derivative product pricing formula proposed by Duffie[15], the firm value is regarded as the call option held by the firm, and the equity value can be seen as the call option held by the shareholders, with the firm's assets as the underlying asset of both option. And in the perpetuity case, the security becomes time-independent. We determine firm's strategy in different financial situations as follows:

(1) When $V \geq V_F$, the firm has sufficient cash to pay coupon c and receives the value of the tax benefit τc and the cash payout qV regardless of economic distress. So, the firm receives the payout $(qV + \tau c)$ as the holder of call option v , and the shareholders receive the payout $(qV - c(1-\tau))$ as the holder of call option E .

(2) When $V_S \leq V < V_F$, regardless of economic distress, the firm has insufficient cash to pay coupon c , and needs to refinance its coupons before hitting the debt restructuring threshold. According to the assumptions, the reduction of original shareholder's value after equity refinancing is equivalent to dividend $\beta(qV - c)$. Meanwhile the firm receives the value of the tax benefit τc . Therefore the firm and shareholders receive the same payout $(\beta(qV - c) + \tau c)$ as the holder of call option.

(3) When $V < V_S$, the firm's financial and economic distress exist at the same time, the firm triggers debt restructuring threshold and pay the reduced coupon $S(V)$. In order to simplify the calculation, this paper follows the approach of Fan and Undaresan [10], without considering the tax benefit value generated by SDS. Therefore, in the stage of debt restructuring, the firm receives payout qV as the holder of call option v , and shareholders receive payout $(qV - S(V))$ as the holder of call option E .

The firm's strategy in different asset value range is shown in Table 1.

According to firm's strategies, the call option v satisfies the following ordinary differential equations:

$$\frac{1}{2}\sigma^2V^2v_{VV}+(r-q)Vv_V-rv+qV+\tau c=0, \quad V \geq V_F \quad (3)$$

$$\frac{1}{2}\sigma^2V^2v_{VV}+(r-q)Vv_V-rv+\beta(qV-c)+\tau c=0, \quad V_S \leq V < V_F \quad (4)$$

$$\frac{1}{2}\sigma^2V^2v_{VV}+(r-q)Vv_V-rv+qV=0, \quad V < V_S \quad (5)$$

with boundary conditions : when V tends to be infinite, $v(V)$ tends to be $\left(V + \frac{\tau c}{r}\right)$,

$$\lim_{V \downarrow V_F} v(V) = \lim_{V \uparrow V_F} v(V), \quad \lim_{V \downarrow V_S} v(V) = \lim_{V \uparrow V_S} v(V), \quad \lim_{V \downarrow 0} v(V) = 0, \quad \lim_{V \downarrow V_F} v_V(V) = \lim_{V \uparrow V_F} v_V(V), \quad \lim_{V \downarrow V_S} v_V(V) = \lim_{V \uparrow V_S} v_V(V).$$

From the differential Equations (3)-(5) and boundary conditions, we have

$$v(V) = \begin{cases} \frac{\tau c}{r} + V + A_{13}V^{-X}, & \text{when } V \geq V_F; \\ \frac{\tau c - \beta c}{r} + \beta V + A_{22}V^{-Y} + A_{23}V^{-X}, & \text{when } V_S \leq V < V_F; \\ V + A_{32}V^{-Y}, & \text{when } V < V_S. \end{cases} \quad (6)$$

where

$$A_{13} = \frac{\frac{c}{q}(Y+1)(1-\beta) + \frac{\beta c}{r}Y}{(X-Y)\left(\frac{c}{q}\right)^{-X}} + \frac{\frac{\tau c - \beta c}{r}Y - V_S(Y+1)(1-\beta)}{(X-Y)V_S^{-X}}, \quad A_{22} = \frac{\frac{c}{q}(X+1)(1-\beta) + \frac{\beta c}{r}X}{(X-Y)\left(\frac{c}{q}\right)^{-Y}},$$

$$A_{23} = \frac{\frac{\tau c - \beta c}{r}Y - V_S(Y+1)(1-\beta)}{(X-Y)V_S^{-X}}, \quad A_{32} = \frac{V_S(X+1)(\beta-1) + \frac{\tau c - \beta c}{r}X}{(X-Y)V_S^{-Y}} + \frac{\frac{c}{q}(X+1)(1-\beta) + \frac{\beta c}{r}X}{(X-Y)\left(\frac{c}{q}\right)^{-Y}}.$$

The call option E satisfies the following ordinary differential equations:

$$\frac{1}{2}\sigma^2V^2E_{VV}+(r-q)VE_V-rE+qV-c(1-\tau)=0, \quad V \geq V_F \quad (7)$$

$$\frac{1}{2}\sigma^2V^2E_{VV}+(r-q)VE_V-rE+\beta(qV-c)+\tau c=0, \quad V_S \leq V < V_F \quad (8)$$

$$\frac{1}{2}\sigma^2V^2E_{VV}+(r-q)VE_V-rE+qV-S(V)=0, \quad V < V_S \quad (9)$$

with boundary conditions: when V tends to be infinite, $E(V)$ tends to be $\left(V - \frac{c}{r}(1-\tau)\right)$,

$\lim_{V \downarrow V_F} E(V) = \lim_{V \uparrow V_F} E(V)$, $\lim_{V \downarrow V_S} E(V) = \lim_{V \uparrow V_S} E(V)$, $\lim_{V \downarrow V_F} E_V(V) = \lim_{V \uparrow V_F} E_V(V)$, $\lim_{V \downarrow V_S} E_V(V) = \lim_{V \uparrow V_S} E_V(V)$. When $V < V_S$, according to the game model (see Equation (1)), we have $E(V) = \theta v(V) = \eta v(V)$.

By solving the above equations, the equity value is given by:

$$E(V) = \begin{cases} -\frac{c(1-\tau)}{r} + V + B_{13}V^{-X}, & \text{when } V \geq V_F; \\ \frac{\tau c - \beta c}{r} + \beta V + B_{22}V^{-Y} + B_{23}V^{-X}, & \text{when } V_S \leq V < V_F; \\ \eta(V + A_{32}V^{-Y}), & \text{when } V < V_S. \end{cases} \quad (10)$$

where

$$B_{13} = \frac{\frac{c}{q}(Y+1)(1-\beta) - \frac{c-\beta c}{r}Y}{(X-Y)\left(\frac{c}{q}\right)^{-X}} + \frac{\frac{\tau c - \beta c}{r}Y - V_S(Y+1)(\eta-\beta)}{(X-Y)V_S^{-X}}, B_{22} = \frac{\frac{c}{q}(X+1)(1-\beta) + \frac{\beta c - c}{r}X}{(X-Y)\left(\frac{c}{q}\right)^{-Y}},$$

$$B_{23} = \frac{\frac{\tau c - \beta c}{r}Y - V_S(Y+1)(\eta-\beta)}{(X-Y)V_S^{-X}}.$$

The reduced coupon when $V < V_S$ is given by $S(V) = (1-\eta)qV$, and the debt value can be obtained as $D(V) = v(V) - E(V)$. The debt value is

$$D(V) = \begin{cases} \frac{c}{r} + (A_{13} - B_{13})V^{-X}, & \text{when } V \geq V_F; \\ (A_{22} - B_{22})V^{-Y} + (A_{23} - B_{23})V^{-X}, & \text{when } V_S \leq V < V_F; \\ (1-\eta)V + (1-\eta)A_{32}V^{-Y}, & \text{when } V < V_S. \end{cases} \quad (11)$$

where

$$A_{13} - B_{13} = \frac{\frac{c}{r}Y}{(X-Y)\left(\frac{c}{q}\right)^{-X}} + \frac{V_S(Y+1)(\eta-1)}{(X-Y)V_S^{-X}}, A_{22} - B_{22} = \frac{\frac{c}{r}X}{(X-Y)\left(\frac{c}{q}\right)^{-Y}}, A_{23} - B_{23} = \frac{V_S(Y+1)(\eta-1)}{(X-Y)V_S^{-X}}.$$

Table 1. Firm's strategy in different asset value range

Conditions	Asset value range	Firm's strategy
$V_S < V_F \leq V_E$	$V \geq V_E$	The firm does not suffer from economic or financial distress and has sufficient cash to pay coupons directly.
	$V_F \leq V < V_E$	Although the firm suffers from economic distress, it does not suffer from financial distress and has sufficient cash to pay coupons directly.
	$V_S \leq V < V_F$	The firm suffers from economic and financial distress and needs to raise funds to pay coupons by equity refinancing.
	$V < V_S$	The firm suffers from economic and financial distress and needs to pay reduced coupons.
$V_S < V_E \leq V_F$	$V \geq V_F$	The firm does not suffer from economic or financial distress and has sufficient cash to pay coupons directly.
	$V_E \leq V < V_F$	The firm suffers from financial distress and does not suffer from economic distress, and needs to raise funds to pay coupons by equity refinancing.
	$V_S \leq V < V_E$	The firm suffers from economic and financial distress and needs to raise funds to pay coupons by equity refinancing.
	$V < V_S$	The firm suffers from economic and financial distress and needs to pay reduced coupons.

4. Debt restructuring threshold

This paper endogenously determines debt restructuring threshold V_S . When financial distress occurs, shareholders and creditors renegotiate, agree to carry out debt restructuring on the basis of equity refinancing, and determine the optimal proportion θ . Meanwhile, according to the call option theory,

the first derivative of equity value on asset value at refinancing stage, is denoted as ϕ with $0 < \phi < 1$. Furthermore, it is assumed that as long as the equity value after refinancing is not less than that under debt restructuring and the condition $0 < \phi < 1$ is met, the firm can raise funds by equity refinancing to pay coupons. When both financial distress and economic distress occur and the equity value after equity refinancing is lower than that under debt restructuring, debt restructuring is implemented. In this way, on the basis of ensuring that creditors get coupons, the shareholders are not seriously damaged by dilution of equity. In addition, in order to increase the attraction of debt restructuring to creditors, to expand the feasible scope of equity refinancing and to protect the interests of creditors as much as possible, the minimum equilibrium point is set as the threshold V_s if there are multiple equilibrium points where the value of equity in refinancing touches the value of equity in debt restructuring. The threshold V_s is obtained as follows:

- (1) Through renegotiation, the optimal allocation ratio θ is determined as shown in Equation (1).
- (2) According to the smoothing conditions when the equity value in the refinancing stage touches the equity value in the debt restructuring stage, the asset value points satisfying the smoothing conditions are solved and written as V_1, V_2, \dots, V_n ($n \geq 1$). The process is as follows:

Based on smoothing conditions: $\lim_{V \downarrow V_s} E(V) = \lim_{V \uparrow V_s} E(V) = \theta v(V_s) = \eta v(V_s)$, $\lim_{V \downarrow V_s} E_v(V) = \lim_{V \uparrow V_s} E_v(V)$, we obtain

$$\left[\frac{c}{q}(X+1)(1-\beta)(1-\eta) + \frac{\beta c}{r}X(1-\eta) - \frac{c}{r}X \right] \left(\frac{c}{q} \right)^Y V_s^{-Y} + V_s(X+1)(\beta - \eta\beta) + \frac{\tau c - \beta c}{r}(1-\eta)X = 0 \quad (12)$$

In the interval $(0, c/q)$, the numerical solutions of Equation (12) are obtained which are denoted as V_1, V_2, \dots, V_n .

- (3) $V_{s1}, V_{s2}, \dots, V_{st}$ are selected from V_1, V_2, \dots, V_n to satisfy the following conditions:
 - 1) Satisfy basic assumptions: $V_n < V_F$ and $V_n < V_E$.
 - 2) In the interval $[V_n, c/q)$, $\phi(V)$ can be obtained from equation $\phi(V) = \beta - B_{22}YV^{-Y-1} - B_{23}XV^{-X-1}$, and should satisfy condition $0 < \phi(V) < 1$.
- (4) Take the minimum value of $V_{s1}, V_{s2}, \dots, V_{st}$ as the threshold V_s .

5. Summary

This paper distinguishes firm's financial and economic distress, clarifies the relationship between SDS and firm's distress, and gives a comprehensive solution to firm's distress by means of integrative equity refinancing and SDS. It also gives pricing formula of the firm under different financial conditions, especially when the liquidation value is less than 0. At the same time, this paper describes the appropriate boundary conditions for paying coupons, debt renegotiation, equity refinancing and SDS. Besides, the feasibility basis of equity refinancing under financial distress is given. Based on harmonizing and maximizing the interests of both shareholders and creditors, the optimal restructuring boundary is determined endogenously.

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