

# Flow-Based Upscaling for Voronoi Grid near a Hydraulic Fracture

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**Abstract**—An important task in reservoir simulation is to take into account the hydraulic fracture effect on well performances. Flow-based upscaling procedures are used to capture the nonlinear pressure variations near the fracture. A numerical near-well upscaling procedure for finite conductivity hydraulic fractures is presented. Spatial discretization is performed using consistent fine and coarse unstructured Voronoi grids. An algorithm for constructing such grids is described. Solution of a simple steady-state flow equation near the fracture is obtained on the fine grid and upscaled to the coarse grid. The flow inside the fracture is directly modeled in fracture cells on a coarse grid, providing the ability to resolve transient effects inside the fracture. As an example, a problem of the two-phase flow in the heterogeneous isotropic reservoir with two hydraulic fractures is reviewed. Simulation results obtained by the upscaling procedure are shown to be significantly more accurate than the ones obtained by the classical Embedded Discrete Fracture Model (EDFM) method.

**Keywords**— *reservoir simulation, upscaling, discrete fracture model, well performance (key words)*

## I. INTRODUCTION

Modeling the flow of hydrocarbons in subsurface formations generally involves a numerical solution of large systems of nonlinear mass conservation equations. One of the challenge is to take into account the highly nonlinear pressure distribution in the near-well region. Since the size of typical reservoir is much greater than the wellbore size and the fracture width, a full-scale numerical simulation within an acceptable time can be carried out only on a coarse grid with cells several orders of magnitude larger than the wellbore diameter.

The Discrete Fracture Model (DFM) [1] and Embedded Discrete Fracture Model (EDFM) models [2, 3] are normally used for modeling hydraulic fractures in full-field simulations. The pressure inside a coarse cell with a fracture in this case is assumed to vary linearly in the normal direction to the fracture. In fact, the pressure variation near a hydraulic fracture is strongly non-linear, and as will be shown later in the example, linear assumption introduces a significant error in the well performance.

The numerical transmissibility upscaling is a way of capturing the nonlinear pressure variations in the near wellbore region. Here all the heterogeneities are resolved by numerical solution of the steady-state flow equation on a fine grid. Mascarenhas [4] described the procedure of numerical upscaling for a horizontal well on a Cartesian grid. Ding et al. [5] described the upscaling for hydraulic fractures.

In this paper, we propose a numerical near-well upscaling procedure for finite conductivity hydraulic fractures on the unstructured Voronoi grid [6]. The flow inside the fracture is directly modeled in fracture cells on a coarse grid. This approach resolves all the transient effects inside the fracture. A radial grid near fracture tips is used, as in [7]. Local upscaling is considered for simplicity, although the generalization of the procedure to global and locally global [8] cases is straightforward. This approach can be regarded as an alternative method of constructing the expanded well region in the Karimi-Fard's technique [9].

## II. SIMULATION MODEL

The equations for multiphase flow in a reservoir and a fracture are:

$$\frac{\partial}{\partial t}(\phi \rho_{\alpha} S_{\alpha}) + \nabla \cdot (\rho_{\alpha} \vec{v}_{\alpha}) = \rho_{\alpha} q_{\alpha}, \quad \vec{v}_{\alpha} = -\frac{k k_{r\alpha}}{\mu_{\alpha}} \nabla p_{\alpha},$$

$$\sum_{\alpha} S_{\alpha} = 1, \quad \left. \frac{\partial p_{\alpha}}{\partial n} \right|_{\partial \Omega} = 0,$$

where  $\alpha$  – phase index (water, oil or gas),  $\phi$  – rock porosity,  $\rho_{\alpha}$  – density,  $S_{\alpha}$  – saturation,  $v_{\alpha}$  – flow rate,  $k$  – isotropic permeability,  $k_{r\alpha}$  – phase relative permeability,  $\mu_{\alpha}$  – viscosity,  $p_{\alpha}$  – pressure,  $q_{\alpha}$  – fluid sources and sinks,  $\partial \Omega$  – outer boundary of the domain  $\Omega$  under consideration. Domain  $\Omega$  includes hydraulic fractures, i.e. the flow in the reservoir and inside the fractures is modeled by the same equations.

The finite volume discretization of the flow equations with two-point flux approximation and replacement of the time derivative by the backward difference lead to a system of nonlinear equations, which is usually solved by the Newton's method. In this case, the phase flow between two cells is

$$q_{\alpha,ij} = -T_{ij} \frac{k_{r\alpha}}{\mu_{\alpha}} (p_{\alpha,i} - p_{\alpha,j}),$$

where single index  $i$  is a value in  $i$ -th cell and double index  $ij$  is a value on the face between cells  $i$  and  $j$ . If the pressure is assumed to vary linearly between the cell nodes, a linear formula can be used to calculate the transmissibility:

$$T_{ij} = k_{ij} \frac{A_{ij}}{d_{ij}},$$

where  $A_{ij}$  is an area of the face between the cells and  $d_{ij}$  is distance between the cell nodes. The harmonic average of  $k_i$  and  $k_j$  is used for the calculation of  $k_{ij}$ . The cells  $i$  and  $j$  can be both reservoir control volumes and fracture control volumes.

In fact, even in a single-phase incompressible case, the pressure field near a well varies strongly nonlinearly in the direction  $r$  toward the well:

$$p(r) \propto \ln(r).$$

The use of the linear formula for transmissibility calculation near the well introduces some error in the calculation of fluxes. The upscaling method captures the pressure nonlinearities between cells through special treatment of coefficient  $T_{ij}$ .

### III. NEAR-WELL UPSCALING

The local problem to be solved numerically for each well on a fine grid is:

$$\nabla \cdot (k \nabla p) = 0, \quad p|_{\partial \Psi} = 0, \quad p|_w = 1,$$

where  $\Psi \subset \Omega$  is a near-well domain covering hydraulic fracture,  $\partial \Psi$  is an outer boundary of  $\Psi$ ,  $w \in \Psi \subset \Omega$  is a wellbore position. The solution  $p$  contains detailed information about all the steady-state pressure nonlinearities in the region. Note that, in some homogeneous cases, it can be expressed analytically, but in this article we consider reservoir with general isotropic heterogeneity, so a numerical solution is used. Transmissibility between the coarse cells for the global problem is calculated through the solution  $p$  as follows:

$$T_{ij} = -\frac{q_{ij}}{\langle p \rangle_i - \langle p \rangle_j} = -\frac{\int_{V_i \cap V_j} k \nabla p \cdot d\vec{s}}{\frac{1}{|V_i|} \int_{V_i} p dv - \frac{1}{|V_j|} \int_{V_j} p dv},$$

where  $q_{ij}$  – single phase flow between cells  $i$  and  $j$ ,  $\langle p \rangle_i$  – average value of pressure  $p$  in cell  $i$ ,  $V_i$  –  $i$ -th cell,  $|V_i|$  –  $i$ -th cell volume,  $V_i \cap V_j$  – face between  $i$ -th and  $j$ -th cells.

### IV. GRID GENERATION

We propose to construct two consistent grids: a coarse grid for a global problem and a fine grid for local problems. Coarse and fine grids are considered consistent if:

1. Each cell of the coarse grid can be represented as a union of a set of cells of the fine grid;
2. Each face of the coarse grid can be represented as a union of a set of faces of the fine grid.

The consistency property allows one to obtain accurate values of the integrals over cells and faces of a coarse grid for calculating transmissibilities. We use the following algorithm to construct consistent unstructured grids:

Construct a grid for the domain  $\Omega$  in which the near-well regions are fully resolved using fine cells, while the other reservoir regions are represented by coarse cells. This grid is regarded as a fine grid and will be used to solve the local problems.

Construct another grid for the domain  $\Omega$  by combining some sets of fine cells in the fine grid into coarse cells. This grid is regarded as a coarse grid and will be used to solve the global problem.

We use the two-dimensional unstructured Voronoi grid [8]. Nodal points near the well for the fine grid are arranged linearly along the fracture normal and radially at the tips of the fracture, as shown in Fig. 1. Hydraulic fractures in the fine grid are represented by the DFM method, i.e. the fracture control volumes are built on faces between the reservoir control volumes (Fig. 2). These fracture cells are geometrically represented as one-dimensional lines, but have a nonzero volume depending on fracture width. The volume of neighboring reservoir cells adjacent to the fracture cell is appropriately reduced to ensure that the total volume of the region remains unchanged. Each fracture cell has two faces connecting it to two adjacent reservoir cells. The nodal points is arranged in such a way that the reservoir cells adjacent to the fracture have the shape of a rectangle (or trapezium shape at the tips of the fracture) with a height of 10 cm and a length equal to the typical diameter of the coarse reservoir cell (about 100 m).

The fine grid cells near the fracture are combined into coarse cells to build coarse grid, as shown in Fig. 3. The fracture cells remain unchanged and appear in the resulting coarse grid. Each fracture cell becomes linked to only one coarse reservoir cell – just like in the EDFM model. The number of fine cells to merge into single coarse cell is chosen according to the desired size of the coarse cell.



Fig. 1. The nodal points used for building the fine Voronoi grid near the hydraulic fracture. The blue points denote the nodes. The black line denotes the hydraulic fracture.

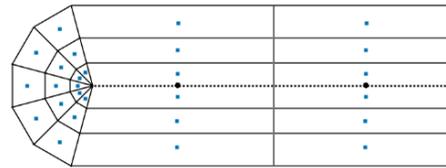


Fig.2. Part of the fine Voronoi grid near the hydraulic fracture. The blue points denote the reservoir cells nodes. The black points denote the fracture cell nodes. The black lines denote the reservoir cell faces. The dashed lines represent the fracture cells.

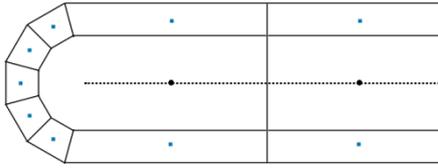


Fig.3. Part of the coarse Voronoi grid near the hydraulic fracture. All the designations are the same as in the Fig. 2. The fracture cell nodes geometrically coincide with the nodes of corresponding coarse cell.

Thus, the fracture control volumes are included in the coarse grid and used to solve the global problem. These fracture cells will certainly reduce the maximum allowable time step in Newton's method due to their small volumes, but this will occur only in the moments when the saturation front inside the fracture changes rapidly. Such moments arise, for example, when well controls change, water breaks into producers or gas evolves from the oil. However, they are quite short compared to the entire simulation period. The fracture cell transmissibilities is likely to be the same order of magnitude as the reservoir cell transmissibilities due to their high permeability, hence the fracture cells must not have a significant negative impact on the condition number of the linearized equations arising in Newton's method.

#### V. NUMERICAL EXAMPLE

As an example, we reviewed a problem of the two-phase flow in the heterogeneous isotropic reservoir. Producer and injector with hydraulic fractures operate at a constant bottomhole pressure. The width of the both fractures is 1 mm. The producer's fracture permeability is set to  $500000 \cdot 10^{-12} m^2$  to represent infinite conductivity, and injector's fracture permeability is set to  $500 \cdot 10^{-12} m^2$ . The reservoir permeability field is shown in Fig. 4. The consistent fine and coarse grids are shown in Fig. 5.

Fig. 6 shows the pressure obtained by solving two local problems on the fine grid. It is clear that the pressure distribution in the two local regions is different since the fractures have different permeability.

The solution of the global problem is carried out in three different cases. In the first case, the simulation is performed entirely on the fine grid using the classical linear formula to calculate all the fine transmissibilities between the cells. This case is considered as the reference one.

The second simulation is performed on the coarse grid using the upscaling procedure. Near-well coarse-scale transmissibilities are calculated from the pressure field depicted on Fig. 6.

The third simulation is performed on the coarse grid using the linear formula to calculate all the coarse-scale transmissibilities. This case is equivalent to using the EDFM model on the coarse grid.

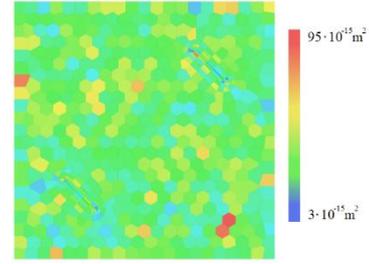


Fig.4. Reservoir permeability in the fine grid cells

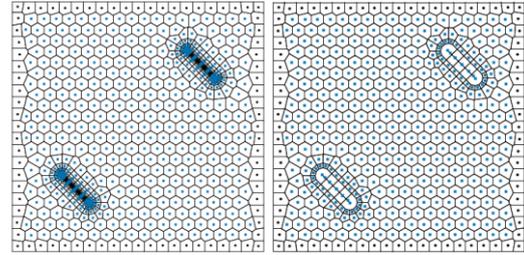


Fig.5. The fine grid (left) and the coarse grid (right). Producer is located at the bottom left corner, injector – at the top right corner

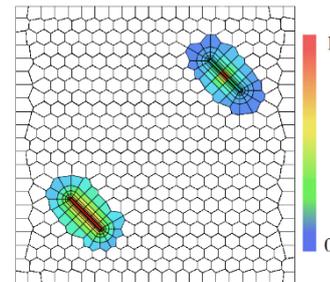


Fig.6. Pressure from solving two local problems on the fine grid.

In the third case the transmissibility between the fracture cell  $j$  and the coarse reservoir cell  $i$  containing this fracture cell is calculated as follows (see Fig.7):

$$T_{ij} = k_i \frac{A_i}{\langle d \rangle_i} = k_i \frac{2dl_i h_i}{\langle d \rangle_i}, \quad \langle d \rangle_i = \frac{1}{|V_i|} \int_{V_i} x_n dv = \frac{dy_i}{4},$$

where  $k_i$  – reservoir cell permeability, which is equal to the average weighted permeability of the fine reservoir cells forming this coarse cell;  $A_i$  – fracture surface area in the cell  $i$ ;  $dl_i$  – fracture cell length in the cell  $i$ ;  $h_i$  – fracture cell height in the cell  $i$ ;  $\langle d \rangle_i$  – average normal distance to the fracture in the cell  $i$ ;  $dv$  – elementary volume of the reservoir cell;  $x_n$  – normal distance from the elementary volume to the fracture.

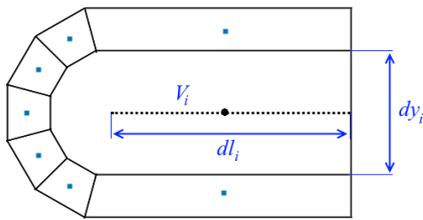


Fig.7. Calculation of the transmissibility between the fracture cell and the coarse cell with the linear assumption.

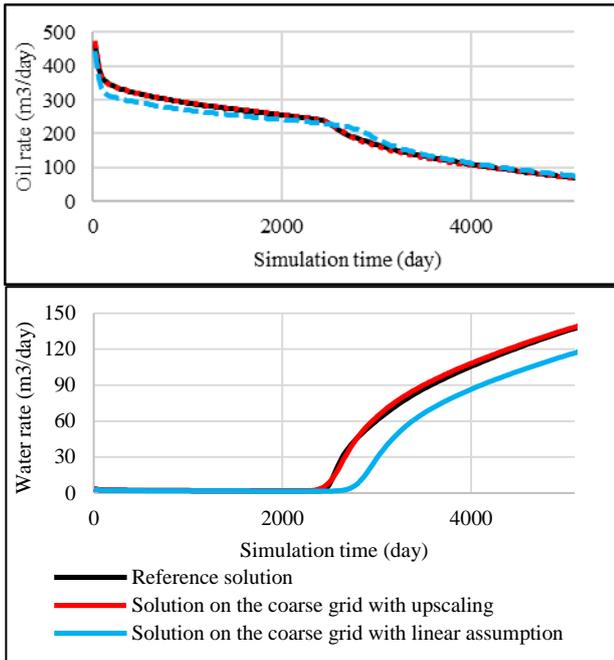


Fig. 8. Oil and water rates, calculated for three cases.

Fig. 8 shows the performance of the production well in three simulation cases. In the period from 0 to 2000 days, the oil rate obtained by the coarse grid with linear assumption is lower than the reference oil rate by 25 m<sup>3</sup>/day on average. Therefore, the linear assumption in this case introduces up to 8% error in the well inflow calculation. This error finally leads to the water breakthrough time increase by 300 days. On the other hand, the oil rate obtained by the coarse grid with upscaling procedure in the period from 0 to 2000 days deviates from the reference solution by no more than 1%, and the corresponding water breakthrough time deviates by no more than 30 days.

## VI. CONCLUSIONS

A numerical near-well upscaling procedure for finite conductivity hydraulic fractures is described. The proposed approach uses the consistent fine and coarse grids containing fracture cells and directly models the flow inside fractures on a coarse grid, hence takes into account the transient and bubble point effects inside the fracture in full-field cases.

It was shown that the linear assumption in the EDFM model can introduce a significant error in the solution of the global problem, whereas the upscaling procedure can resolve the pressure nonlinearities by appropriate treatment of the coarse-scale transmissibilities and substantially improve the solution accuracy.

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