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Computer Modeling of Orthogonal in the Amplified Sense Signal

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***Abstract*—The article solved the problem of finding analytic dependency showing the influence coefficients of the second diagonal Hermitian matrix by correlation and spectral properties defined by it in the amplified sense of orthogonal signals. The analytical dependences between the coefficients of the second diagonal of the Hermitian matrix and the correlation and spectral characteristics of ensembles of discrete orthogonal signals in the amplified sense are determined. The use of formulas allows for specific selection of ensembles orthogonal in the amplified sense signals, which reduces the time of synthesis. When the values of the diagonal coefficients modules are constant, the arguments of these coefficients do not affect the values of the modules of the unit elements of sets of discrete orthogonal signals in the amplified sense. The value of the eigenvector arguments of a two-diagonal Hermitian matrix is determined by the arguments of its diagonal coefficients. Correlation functions of signals with equal values of modules of unit elements are determined only by the values of their arguments. For certain ratios of modules and arguments of coefficients of second diagonal Hermitian matrix and corresponding minors can be obtained the required levels of lateral peaks of the correlation functions and the values of the relative effective width of the spectrum of the synthesized ensemble of discrete orthogonal in the amplified sense signals.**

Keywords—Signals sets, orthogonality, correlation, spectrum, Hermitian matrix, two-diagonal matrix.

I. INTRODUCTION

In [1], [2] is shown that for any orthogonal basis finite complex space exists Hermitian self-adjoint operator matrix, the eigenvectors which constitute mentioned basis. The coordinates of the eigenvectors of a Hermitian matrix are generally complex numbers, and the orthogonality condition in a complex space coincides with the condition of orthogonality in the amplified sense submitted to analytical discrete complex-conjugate signal $\dot{y}(t)$ and $\dot{z}^*(t)$ [1]:

$$\int_0^T \dot{y}(t)\cdot \dot{z}^*(t)dt = 0 \qquad (1)$$

where $\dot{y}(t)=a(t)\cdot e^{j\psi_a(t)}$, $\dot{z}^* = b(t)\cdot e^{j\psi_b^*(t)}$, $a(t)=b(t)$ - the signal amplitude, $\psi_a(t)$ and $\psi_b^*(t)$ - phase of signals, with a second complex conjugate.

II. STATEMENT OF THE PROBLEM

However, the proposed in [1] the method has drawbacks related above all to the lack of analytical relationships between the complex Hermitian matrix coefficients and correlation and spectral characteristics ensembles discrete orthogonal in the amplified sense (EDOSSS) represented by its eigenvectors [3].

The purpose of the article is to determine the analytical relationships between the coefficients of the second diagonal of a Hermitian matrix and the correlation and spectral characteristics of EDOSSS described by its own vectors [4].

III. EIGENVECTORS HERMITIAN MATRICES

Consider two-diagonal Hermitian matrix Q self-adjoint operator, presented in exponential form:

$$Q = \begin{vmatrix} 0 & q_{1,2}e^{j\varphi_{1,2}} & 0 & . & 0 & 0 \\ q_{2,1}e^{j\varphi_{2,1}} & 0 & q_{2,3}e^{j\varphi_{2,3}} & . & 0 & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & 0 & q_{k-1,k}e^{j\varphi_{k-1,k}} \\ 0 & 0 & 0 & . & q_{k,k-1}e^{j\varphi_{k,k-1}} & 0 \end{vmatrix}, \qquad (2)$$

where the values of the coefficients $q_{k-1,k} = q_{k,k-1}$, and arguments $\varphi_{k-1,k} = \varphi^*_{k,k-1}$ - complex conjugate.

To find the coordinates of the eigenvectors $\{\dot{x}_k\}$. We solve the following algebraic equation:

$$|Q - \lambda E| = 0, \qquad (3)$$

where λ - eigenvalues; E - identity matrix whose main diagonal in the unit, and the rest zeros.

Expression (3) can be kind of system:

$$\begin{cases} -\lambda\dot{x}_1 + q_{1,2}e^{j\varphi_{1,2}}\dot{x}_2 = 0, \\ q_{2,1}e^{j\varphi_{2,1}}\dot{x}_1 - \lambda\dot{x}_2 + q_{2,3}e^{j\varphi_{2,3}}\dot{x}_3 = 0, \\ \dots\dots\dots\dots, \\ q_{n,m-1}e^{j\varphi_{n,m-1}}\dot{x}_{m-1} - \lambda\dot{x}_m = 0. \end{cases} \quad (4)$$

Solving the system of equations (4) we obtain the coordinates of the eigenvectors of a Hermitian matrix Q that when $\dot{x}_1 = \alpha$ will be determined by the expression of the form [2]

$$\dot{x}_2 = \alpha \cdot \frac{\lambda}{q_{1,2}} \cdot e^{-j\varphi_{1,2}}; \quad (5)$$

$$\dot{x}_3 = \alpha \cdot \frac{\lambda - q_{1,2}^2}{q_{1,2} \cdot q_{2,3}} \cdot e^{-j(\varphi_{1,2}+\varphi_{2,3})}; \quad (6)$$

$$\dot{x}_k = (-1)^{k-1} \cdot \alpha \cdot \frac{\Delta_{k-1}}{q_{12} \cdot q_{23} \cdot \ldots \cdot q_{k-1,k}} \cdot e^{-j(\varphi_{1,2}+\varphi_{2,3}+\ldots+\varphi_{k-1,k})}, \quad (7)$$

where

$$\Delta_{k-1} = \lambda^{k-1} - \lambda^{k-3} \cdot (q_{1,2}^2 + q_{2,3}^2 + q_{3,4}^2 + \ldots + q_{k-2,k-1}^2) +$$

$$+ \lambda^{k-5} \cdot (q_{1,2}^2 \cdot q_{3,4}^2 \cdot \ldots \cdot q_{k-4,k-3}^2 \cdot q_{k-2,k-1}^2) + \ldots + \lambda(q_{1,2}^2 \cdot q_{3,4}^2 \times \ldots$$

$$\ldots \times q_{k/2+1,k/2+2}^2 + \ldots + q_{2,3}^2 \cdot q_{4,5}^2 \cdot q_{k-2,k-1}^2)$$

- minor, composed of elements of the first $k - 1$ rows and columns of the matrix $|Q - \lambda_k E|$, $\alpha = a_{n1}e^{j\psi_{n,1}}$ - free coordinate, the value of which is chosen arbitrarily.

For a description of coordinate eigenvectors two-diagonal Hermitian matrix in polar coordinates, we introduce variables a_k and ψ_k represented by expressions of the form:

$$a_k = (-1)^{k-1} \cdot \alpha \cdot \frac{\Delta_{k-1}}{q_{1,2} \cdot q_{2,3} \cdot \ldots \cdot q_{k-1,k}}$$

$$\psi_k = (\varphi_{1,2} + \varphi_{2,3} + \ldots + \varphi_{k-1,k}) \quad (8)$$

From the analysis of expressions (8) that the value for a_k influence coefficients modules $q_{k-1,k}$ matrix (2), and ψ_k - their arguments $\varphi_{k-1,k} = \varphi_{k,k-1}^*$, With all of them depend on the value of the initial phase α.

Considering expressions (5-8) system eigenvectors of a Hermitian matrix two-diagonal Q you can submit a matrix X type:

$$X = \begin{vmatrix} a_{11}e^{j\psi_{1,1}} & a_{12}e^{j\psi_{1,2}} & \ldots & a_{1m}e^{j\psi_{1,m}} \\ a_{21}e^{j\psi_{2,1}} & a_{22}e^{j\psi_{2,2}} & \ldots & a_{2m}e^{j\psi_{2,m}} \\ . & . & . & . \\ a_{n1}e^{j\psi_{n,1}} & a_{n2}e^{j\psi_{n,2}} & \ldots & a_{nm}e^{j\psi_{n,m}} \end{vmatrix} \quad (9)$$

According to [1] the matrix X EDOSSS is a model and each of its eigenvector is one signal of the ensemble, and can be described as a combination of unit cells as follows:

$$\dot{x}_y(t) = \{a_1e^{j\psi_1},\ a_2e^{j\psi_2},\ a_3e^{j\psi_3},\ \ldots a_{m/2-1}e^{j\psi_{m/2-1}},$$

$$a_{m/2}e^{j\psi_{m/2}},\ a_{m/2+1}e^{j\psi_{m/2+1}},\ \ldots,\ a_me^{j\psi_m}\}, \quad (10)$$

where a_k - the amplitude of a single signal element, ψ_k - the phase of a single signal element.

In accordance with [2] with the proviso that the Hermitian matrix coefficients two-diagonal Q $q_{1,2},\ q_{3,4},\ \ldots q_{k-1,k} \to \lambda$, but $q_{2,3},\ q_{5,6},\ \ldots, q_{k-2,k-1} \to 0$,

Equal amplitudes conditions $a_1 = a_2 = a_3 = \ldots = a_k$ can be achieved for all signals in the ensemble. It should be noted that this condition is fulfilled in the presence of different argument values $\varphi_{k-1,k}$ coefficients of the original two-diagonal Hermitian matrix Q form (2), as evidenced by the expression (8).

The case of equality of the amplitudes of signals of the unit cells $a_1 = a_2 = a_3 = \ldots = a_k$ It has a value in the practice of the binary signals in digital data transmission systems [5, 6].

IV. Effect of coefficients of the second diagonal Hermitian matrix by correlation and spectral characteristics EDOSSS

It is known [7] that the correlation function (CF) signals can be calculated according to the formula of the form:

$$R_{yz}(\rho) = \frac{1}{m} \sum_{k=\rho+1}^{m} x_{y,k} x_{z,k-\rho}^*, \quad (11)$$

where y and z signals in the ensemble represented by their respective unit cells x_y and x_z, ρ - the number of the side CF peak [8].

$$R_{y,z}(\rho) = \frac{1}{m} \cdot \sum_{k=\rho+1}^{m} (-1)^{k-1} \cdot \frac{\alpha \cdot \Delta_{y,k-1}}{q_{1,2} \cdot q_{2,3} \cdot \ldots \cdot q_{k-1,k}} \times$$

$$\times e^{-j(\varphi_{1,2}+\varphi_{2,3}+\ldots+\varphi_{k-1,k})} \cdot (-1)^{k-\rho-1} \times$$

$$\times \frac{\alpha \cdot \Delta_{z,k-\rho-1}}{q_{1,2} \cdot q_{2,3} \cdot \ldots \cdot q_{k-\rho-1,k-\rho}} \cdot e^{j(\varphi_{1,2}+\ldots+\varphi_{k-1-\rho,k-\rho})} =$$

$$= \frac{1}{m} \sum_{k=\rho+1}^{m} (-1)^{2k-\rho-2} \frac{\alpha^2 \cdot \Delta_{y,k-1} \cdot \Delta_{z,k-\rho-1}}{(q_{1,2} \cdot \ldots \cdot q_{k-1,k}) \cdot (q_{1,2} \cdot \ldots \cdot q_{k-\rho-1,\kappa-\rho})} \times$$

$$\times e^{-j(\varphi_{1,2}+\ldots+\varphi_{k-1,k}) + j(\varphi_{1,2}+\ldots+\varphi_{k-1-\rho,k-\rho})}$$

$$=\frac{1}{m}\sum_{k=\rho+1}^{m}(-1)^{2k-\rho-2}\frac{\alpha^2\cdot\Delta_{y,k-1}\cdot\Delta_{z,k-\rho-1}}{(q_{1,2}\cdot q_{2,3}\cdot\ldots\cdot q_{k-\rho-1,k-\rho})^2}\times$$

$$\times\frac{1}{q_{k-\rho,k-\rho+1}\cdot\ldots\cdot q_{k-1.k}}\cdot e^{-j(\varphi_{k-\rho,k-\rho+1}+\ldots+\varphi_{k-1,k})}, \quad (12)$$

where at $y = z$ expression (12) describes the envelope of the autocorrelation function of the signal, and for $y \neq z$ - intercorrelation envelope signal functions.

To assess the spectral characteristics EDOSSS [9, 10] use known from [2], the formula of the relative effective width of the spectrum of the signal, provided the energy equation and the reference analysed signals:

$$\frac{W_{yy}^2}{W_0^2}=\frac{T}{\tau}\cdot[1\pm R_1(\tau)], \quad (13)$$

wherein the duration T - signals in the ensemble, τ - the duration of a single chip, $R_1(\tau)$ - the first side peak of the autocorrelation function (ACF) signal.

Based on (12) represent the first side peak in the ensemble ACF signal [11, 12] in the following form:

$$R_{yy}(1)=\frac{1}{m}\cdot\sum_{k=2}^{m}\left[(-1)^{k-1}\cdot\alpha\cdot\frac{\Delta_{y,k-1}}{q_{12}\cdot q_{23}\cdot\ldots\cdot q_{k-1,k}}\times\right.$$

$$\times e^{-j(\varphi_{1,2}+\varphi_{2,3}+\ldots+\varphi_{k-1,k})}\cdot(-1)^{k-2}\cdot\alpha\cdot\frac{\Delta_{y,k-2}}{q_{12}\cdot q_{23}\cdot\ldots\cdot q_{k-2,k-1}}\times$$

$$\left.\times e^{j(\varphi_{1,2}+\varphi_{2,3}+\ldots+\varphi_{k-2,k-1})}\right]=\frac{1}{m}\cdot\sum_{k=2}^{m}\left[(-1)^{2k-3}\times\right.$$

$$\left.\times\frac{\alpha^2\cdot\Delta_{y,k-1}\cdot\Delta_{y,k-2}}{(q_{12}\cdot q_{23}\cdot\ldots\cdot q_{k-2,k-1})^2\,q_{k-1,k}}\cdot e^{-j\varphi_{k-1,k}}\right]. \quad (14)$$

Substituting (14) into (13) obtain an expression

$$\frac{W_{yy}^2}{W_0^2}=m\pm\sum_{k=2}^{m}\left[(-1)^{2k-3}\cdot\frac{\alpha^2\cdot\Delta_{y,k-1}\cdot\Delta_{y,k-2}}{(q_{12}\cdot q_{23}\cdot\ldots\cdot q_{k-2,k-1})^2\,q_{k-1,k}}\times\right.$$

$$\left.\times e^{-j\varphi_{k-1,k}}\right]. \quad (15)$$

V. CONCLUSION

From (7) and (8) implies that for constant values of the diagonal coefficients modules $q_{1,2}$, $q_{2,3}$, …, $q_{k-1,k}$, the arguments of these factors do not affect the values of the moduli a_k individual elements EDOSSS. Therefore, when any of these arguments, the value of the coefficients Δ_1, Δ_2, …, $\Delta_{k-1,k}$ It will be the same.

Analysis of the relations (7) and (8) demonstrates that the arguments $\psi_{1,2},\psi_{2,3},\ldots,\psi_{k-1,k}$ eigenvectors diagonal Hermitian matrix Q defined arguments $\varphi_{1,2},\varphi_{2,3},\ldots,\varphi_{k-1,k}$ its diagonal coefficients.CF of signals with equal values of the modules of the unit cells is determined only by the values of their arguments [13-15]. From relations analysis (12, 14, 15) it follows that under certain ratios modules $q_{1,2}$, $q_{2,3}$, …, $q_{k-1,k}$ and arguments $\varphi_{1,2},\varphi_{2,3},\ldots,\varphi_{k-1,k}$ second diagonal coefficients Hermitian matrix and the corresponding minors required levels CF lateral peaks and relative values of the effective width of the spectrum of the synthesized EDOSSS can be obtained.

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