

Efficient Algorithms for the Numerical Solution of the Coupled Sediment and Suspended Matter Transport Problems in Coastal Systems

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Abstract—This work is devoted to the development and numerical study of coupled models of sediment transport and suspended matter, taking into account coastal currents and stress near the bottom caused by wind waves, turbulent spatially 3D movement of the aquatic environment, the complex shape of the coastline, bottom topography, and other factors. Conservative stable difference schemes are constructed and investigated. A comparative analysis of the efficiencies of using explicit difference schemes with implicit schemes, as well as with schemes of a special form, explicitly implicit schemes, is given. The main idea is to use an additive difference scheme, consisting of a chain of spatially one-dimensional implicit and spatially two-dimensional explicit difference problems, approximating the original problem in the total sense. The explicit two-dimensional diffusion-convection-reaction problems, as well as the spatially two-dimensional sediment transport problems are approximated by regularized explicit schemes that involve the introduction of second-order difference derivatives with respect to the time variable with a relatively small time step - the regularizer as an additional term in the left-hand sides of the equations. This allows us to repeatedly increase the admissible time step in comparison with the unregularized explicit difference equations of the parabolic type. It is this method that has shown its advantage over explicit and implicit schemes.

Keywords—*coastal zone; sediment transport; suspended matter transport; mathematical model; linearization of the initial-boundary value problem; difference scheme.*

I. INTRODUCTION

The study of the dynamics of sediment transport and suspended matter in modern conditions in the coastal zones of water bodies is an important and urgent task. It has both fundamental and applied aspects. Among the important fundamental problems is, in particular, the problem of studying the patterns and characteristics of the global circulation of lithosphere matter in the coastal zones of water bodies. Applied research problems are directly related to the environmental and economic problems of the coast. The coastal zone, as the boundary part interacting with land, is a system of mechanical, hydrochemical, geochemical and biological barriers. The study of various aspects of substance transport during the passage of these barriers is another important area of the research area. In particular, it

seems relevant to assess the flows of pollutants and determine the amount of sediment deposited associated with the development of industrial and recreational activities in coastal areas. As a rule, research in this area requires the construction of mathematical models that are as close as possible to real processes and allow predicting the distribution of suspended matter in an aqueous medium [1] - [3].

For the first time, dynamic models of sediment attraction and weighing theoretically began to settle at the beginning of the 20th century. In particular, V.G. Golushkov was shown that the maintenance of heavy particles in a moving fluid occurs due to the vertical component of the pulsating velocity. In Russia, the work continued in the research of G.I. Marchuk, V.P. Dymnikova, A.S. Sargsyan, O.M. Belotserkovsky, R.A. Ibraeva, V.B. Zalesnogo, V.P. Shutyaeva, V.M. Belolipetskogo, V.K. Debolsky, R.V. Ozmidova, V.V. Zhmura, O.K. Leontiev, M.V. Flint, P.O. Zavyalova, and others. Among the international scientific research centers conducting research and development in this field, we note: International Association for the Physical Sciences of the Ocean, Scientific Committee on Oceanic Research (ICSU), Pacific Science Association, Engineering Committee on Oceanic Resources, Centre for Coastal and Marine Research, Centre for Applied Marine Sciences, Southampton Oceanography Centre, Proudman Oceanographic Laboratory (UK), GEOMAR Helmholtz Centre for Ocean Research, Leibniz Institute of Marine Sciences (Kiel, Germany), German Marine Research Consortium and etc.

This paper presents the coupled non-stationary models of sediment transport and suspended matter, taking into account the turbulent spatially 3D nature of the movement of the aquatic environment, the complex shape of the coastline, etc. Modeling of sediment transport and suspended particles has been considered in many dozens of publications, among which should be noted [4] - [7]. Earlier, the authors of [8] - [9] proved the correctness of the formulation of these models. To this end, a quadratic functional was constructed and an energy method was used to prove the uniqueness of the solution to the corresponding initial-boundary-value problem. Based on the transformation of the quadratic functional, a priori estimates of the norm of

the solution in the functional space L_2 were obtained depending on the integral estimates of the right-hand side, boundary conditions, and the initial condition.

We study parallel algorithms for the numerical implementation of the joint numerical solution of sediment and suspension transport problems based on explicit schemes with regularizing terms, implicit schemes and special types of schemes - explicit-implicit schemes. The proposed software package that implements the proposed models is of practical importance: it will significantly improve the accuracy of the operational forecast and the validity of engineering decisions made when creating coastal infrastructure facilities.

II. DESCRIPTION OF THE COMBINED MATHEMATICAL MODELS

A. Mathematical model of suspended matter transport

The main factors of weighing, redistribution and transport of bottom material is the combined effect of waves and currents. Under the influence of gravity, the particles in the water stream fall down. Vertical mixing occurs respectively in ascending and descending directions. The interaction between the two processes provides a vertical concentration profile.

Bottom sediments may consist of organic and inorganic substances. Inorganic minerals consist mainly of clay minerals (silica, alumina, montmorillonite, illite, etc.) and non-clay minerals (quartz, mica, etc.). Organic materials can exist in the form of plants and bacteria. In this paper, the model is adapted to describe the behavior of organic suspensions in a shallow reservoir under the assumption of a small vertical turbulent exchange.

We will use a rectangular Cartesian coordinate system $Oxyz$, where the axis Ox passes over the surface of the unperturbed water surface and is directed towards the sea, axis Oz directed vertically down. Let $h=H+\eta$ – total depth of the water area, [m]; H – depth at the unperturbed surface of the reservoir, [m]; η – elevation of the free surface relative to the geoid, [m].

Suppose that there are R types of particles in the water volume $V(x,y,z)=\{0 \leq x \leq L_x, 0 \leq y \leq L_y, 0 \leq z \leq L_z\}$, which at the point (x,y,z) and at the time t [sec] have a concentration $c_r(x,y,z,t)$, [mg/l], $r=\overline{1,R}$ (Fig 1).

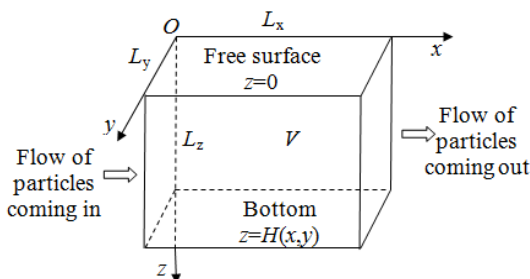


Fig. 1. Image of the water volume V where the process takes place

In the future I will omit the arguments of the function $c_r(x,y,z,t)$ and write just c_r .

The system of equations describing the behavior of particles will look like this:

$$\begin{cases} \frac{\partial c_r}{\partial t} + \frac{\partial(uc_r)}{\partial x} + \frac{\partial(vc_r)}{\partial y} + \frac{\partial((w+w_{g,r})c_r)}{\partial z} = \\ = \mu_r \left(\frac{\partial^2 c_r}{\partial x^2} + \frac{\partial^2 c_r}{\partial y^2} \right) + \frac{\partial}{\partial z} \left(\nu_r \frac{\partial c_r}{\partial z} \right) + F_r, r=\overline{1,R}; \\ F_1 = (\alpha_2 c_2 - \beta_1 c_1) + \Phi_1(x,y,z,t), \\ F_r = (\beta_{r-1} c_{r-1} - \alpha_r c_r) + \\ + (\alpha_{r+1} c_{r+1} - \beta_r c_r) + \Phi_r(x,y,z,t), r=\overline{2,R-1}, \\ F_R = (\beta_R c_R - \alpha_R c_R) + \Phi_R(x,y,z,t). \end{cases} \quad (1)$$

The following notation is used here: u,v,w – the components of the velocity vector \vec{U} of the fluid, [m/sec]; $w_{g,r}$ – the hydraulic size or the rate of deposition of particles of the r -th type, [m/sec]; μ_r, ν_r – the coefficients of horizontal and vertical diffusion of particles of the r -th type, [m²/sec]; α_r, β_r – particle conversion rates of the r -th type into $(r-1)$ -th and $(r+1)$ -th type, $\alpha_r \geq 0, \beta_r \geq 0$, [m/sec]; Φ_r – power of sources of particles of the r -th type, [mg/l sec].

The terms on the left side (apart from the time derivative) of the equation of system number r (3) describe the convection of particles: their transport under the influence of fluid flow and gravity. The terms on the right side describe the diffusion of suspensions and their conversion from one type to another, as well as decomposition. The coefficient of vertical microturbulent diffusion is substantially dependent on the vertical coordinate. Often it is enough to consider a model of sequential conversion of particles from one type to another. Inconsistent transformation would require the introduction of a transformation matrix, which would complicate the model without significantly expanding its scope.

To find the density of the aquatic environment, the formula is used

$$\rho = \left(1 - \sum_{r=1}^R V_r \right) \rho_0 + \sum_{r=1}^R V_r \rho_{v,r} c_r = V_r \rho_{v,r}, \quad (2)$$

where V_r – volume fraction of particles r -th type; ρ_0 – water density; $\rho_{v,r}$ – particle density r -th type.

Solutions of the system of equations (1) are searched for in a given region of continuous variation of the arguments $C_T(x,y,z,t)=V(x,y,z) \times (0,T)$, representing a four-dimensional cylinder with generators parallel to the time axis Ot heights T , whose upper base is the unperturbed surface of the water, the lower base is the bottom. The border S areas of C_T supposed smooth enough.

Add to the system (1) the initial and boundary conditions.

Initial conditions:

$$c_r(x,y,z,0) \equiv c_{r0}(x,y,z), r=\overline{1,R}. \quad (3)$$

Boundary conditions:

- on the lateral boundary of the region S cylinder C_T

$$\text{for } (\vec{U}_\Gamma, \vec{n}) \leq 0: \frac{\partial c_r}{\partial n} = 0, r = \overline{1, R}; \quad (4)$$

$$\text{for } (\vec{U}_\Gamma, \vec{n}) \geq 0: \frac{\partial c_r}{\partial n} = \frac{u_\Gamma}{\mu} c_r, r = \overline{1, R}, \quad (5)$$

where \vec{n} – the outer normal to the boundary of the domain S , \vec{U}_Γ – the velocity vector of the fluid at the boundary S , u_Γ – the velocity vector projection \vec{U}_Γ on the direction of the normal \vec{n} on the border of the region S ;

- on the water surface

$$\frac{\partial c_r}{\partial z} = 0, r = \overline{1, R}; \quad (6)$$

- at the bottom

$$\frac{\partial c_r}{\partial n} = \frac{w_{g,r}}{\nu} c_r, r = \overline{1, R}. \quad (7)$$

B. Mathematical model of sediment transport

During sediment movement, the following stages can be distinguished: sediment mobilization from bottom sediments into movement, sediment transport and moving sediment transfer to bottom sediments. In connection with the foregoing, the construction of a general model of sediment movement involves combining models of the individual components of the process into one. This paper presents a model of sediment transport taking into account their fractional composition, which is one of the main factors determining their water-physical and mechanical properties. Separation of sediments into fractions is carried out either by the geometric dimensions of the particles, or by hydraulic size (for particles smaller than 1 mm). The averaged expression of particle size and particle size for the study of sediment transport seems insufficient.

For simplicity, we assume that in the equation of sediment transport, the axes Ox , Oy are consistent with the directions of the coordinate axes of the hydrodynamic block models, in which the components of the velocity vector of the water medium and the coefficients of turbulent exchange along the vertical direction are calculated. Furthermore, for simplicity, the case is considered when the normal to the shore-line is directed to the north, coinciding with the axis Ox ; the axis Oy is directed to the east.

The reformation of the coastal zone of the water areas due to the movement of water and solid particles will be described for the case when the sediment particles move in one direction (the side of the shore). In this work it is assumed that particles move in the direction of the water flow along the axis Ox . The motion of the particles in the direction opposite to the direction of the resulting transfer will be neglected.

Let the sediments that participate in sediment transport consist Q of fractions, each of which has a relative fraction V_q in the total volume and density $\rho_q, q = \overline{1, 2, \dots, Q}$.

The sediment transport equation is written as:

$$(1 - \bar{\varepsilon}) \frac{\partial H}{\partial t} + \text{div} \left(\sum_{q=1}^Q V_q k_q \vec{\tau}_b \right) = \text{div} \left(\sum_{q=1}^Q V_q k_q \frac{\tau_{bc,q}}{\sin \varphi_0} \text{grad} H \right) + \sum_{r=1}^R \frac{w + w_{g,r}}{\rho_r^*} c_r, \quad (8)$$

where $\bar{\varepsilon}$ – the averaged over fractions porosity of bottom materials; $\vec{\tau}_b$ – the vector of tangential stress at the water bottom; $\tau_{bc,q}$ – the critical value of the tangential stress for the q -th fraction, $\tau_{bc,q} = a_q \sin \varphi_0$, φ_0 – an angle of repose of soil in the water; ρ_r^* – density of particles of suspended matter of the r -th type, which move in accordance with equations (1); $k_q = k_q(H, x, y, t)$ – the nonlinear coefficient, determined by the relation:

$$k_q = \frac{A \bar{\omega} d_q}{((\rho_q - \rho_0) g d_q)^\beta} \left| \frac{\tau_{bc,q}}{\sin \varphi_0} \text{grad} H \right|^{\beta-1}, \quad (9)$$

(ρ_q, d_q – density and characteristic particle size of the q -th fraction, respectively; ρ_0 – density of the aquatic environment; g – the gravity acceleration; $\bar{\omega}$ – the averaged wave frequency; A and β – dimensionless constants).

As with the equation of transport of suspensions, the region of specifying equation (8) is the cylinder C_T .

We supplement equation (8) by the initial condition assuming that the function of the initial conditions belongs to the corresponding class of smoothness:

$$H(x, y, 0) = H_0(x, y), \quad (10)$$

Let us formulate the conditions on the boundary of the region, starting from physical considerations:

$$|\vec{\tau}_b|_{y=0} = 0, \quad (11)$$

$$H(L_x, y, t) = H_2(y, t), 0 \leq y \leq L_y, \quad (12)$$

$$H(0, y, t) = H_1(y, t), 0 \leq y \leq L_y, \quad (13)$$

$$H(x, 0, t) = H_3(x), 0 \leq x \leq L_x, \quad (14)$$

$$H(x, L_y, t) = H_4(x, t), 0 \leq x \leq L_x, L_y' < L_y, \quad (15)$$

We assume that there is always a layer of liquid of finite thickness in the considered region and for the indicated time interval there is no dehydration of the region, that is

$$H(x, y, t) \geq C \equiv \text{const} > 0, 0 \leq x \leq L_x, 0 \leq y \leq L_y', 0 \leq t \leq T. \quad (16)$$

C. Linearization of sediment transport model

In order to create a linearized model on a time span $0 \leq t \leq T$ build a uniform grid ω_t с шагом τ , i.e. the set of points $\omega_t = \{t_n = n\tau, n = \overline{0, N}, N\tau = T\}$ and we linearize the initial-

boundary-value problem (8) - (16) by the methods described in [10-11].

We linearize the term $\text{div}\left(\sum_{q=1}^Q V_q k_q \frac{\tau_{bc,q}}{\sin\theta_0} \text{grad} H\right)$ and the coefficient k_q by choosing their values at the time $t=t_n, n=0,1,\dots,N$ and considering equation (8) in the time interval $t_{n-1} < t \leq t_n, n=1,2,\dots,N$. It is assumed that we know the function $H^{(n)}(x,y,t_{n-1})=H^{(n-1)}(x,y,t_{n-1})$ and its partial derivatives at spatial variables.

In the case, if $n=1$, it is enough to take the function with the initial conditions $H^1(x,y,t_0)$, i.e. $H^1(x,y,t_0)=H_0(x,y)$. If $n=2,\dots,N$, the function $H^n(x,y,t_{n-1})=H^{n-1}(x,y,t_{n-1})$ is assumed to be known, since it is assumed that the problem (8) - (16) for the previous time interval $t_{n-2} < t \leq t_{n-1}$.

We introduced the notation:

$$k_q^{n-1} = \frac{A \bar{\omega} d_q}{\left((\rho_q - \rho_0) g d_q\right)^\beta} \left| \tau_{bc,q} \frac{\text{grad} H^{n-1}}{\sin\theta_0} \right|^{\beta-1}, n=1, \bar{N}. \quad (17)$$

After linearization, we write equation (8) in the form:

$$(1-\bar{\varepsilon}) \frac{\partial H^n}{\partial t} + \text{div} \left(\sum_{q=1}^Q V_q (k_q)^{n-1} \bar{\tau}_b \right) = \text{div} \left(\sum_{q=1}^Q V_q (k_q)^{n-1} \frac{\tau_{bc,q}}{\sin\theta_0} \text{grad} H^n \right) + \sum_{r=1}^R \frac{w_r + w_{g,r}}{\rho_r^*} c_r. \quad (18)$$

We add to equations (18) the initial conditions:

$$H^1(x,y,t_0)=H_0(x,y), \dots, H^n(x,y,t_{n-1})=H^{n-1}(x,y,t_{n-1}), \quad n=2, \bar{N}. \quad (19)$$

The boundary conditions (11)-(15) are expected to be completed for all time intervals $t_{n-1} < t \leq t_n, n=1, \bar{N}$.

III. SOFTWARE IMPLEMENTATION OF THE TASK

The algorithms of numerical solution for the problems of sediment transport and transport of suspended particles have been presented in the framework of previous articles [12-15]. A comparison is made of the computational efficiency for explicit difference scheme and implicit scheme, as well as with a special type of scheme - an explicit-implicit scheme. To increase the stability margin of an explicit scheme, regularized schemes are used. Modification of the explicit scheme - the introduction of a second-order differential derivative with regularization can significantly reduce the restrictions on the admissible value of the time step [16-18]. In addition, explicit regularized schemes have shown their advantage in real-time costs compared to previously used traditional schemes.

A. Comparison of the efficiencies of explicit and implicit difference schemes

Labor intensity Q_{neiavn} of the solution of the problem of transport of suspensions by an implicit scheme is estimated:

$Q_{neiavn} = n_{tn} n(\varepsilon) N_x N_y q_{PTM}$, where q_{PTM} in the number of arithmetic operations for one iteration MPTM (modified alternating-triangular method) ($q_{PTM} \approx 50$); $n(\varepsilon) = O(N_{\max})$ in the number of iterations; N_{\max} in the number of nodes in space; $N_{\max} = \max\{N_x, N_y\}$, N_x, N_y in the number of steps along the coordinate axes Ox, Oy respectively; $n_{tn} = T/\tau_n$ in the number of time layers for the implicit scheme, T in calculated time interval, τ_n in the time step for an implicit scheme.

For an explicit scheme such an estimate has the form: $Q_{iavn} = n_{tr} N_x N_y q_{iavn}$, where q_{iavn} in the number of operations for the transition to the next time layer by an explicit regularized scheme ($q_{iavn} \approx 14$); $n_{tr} = T/\tau_r$ in the number of time layers for an explicit scheme, τ_r in the time step for an explicit scheme.

Calculation grids were used $k N_x \times k N_y \times N_z$, $N_x=122, N_y=102, N_z=13$, where $k=1,2,4$. Table 1 shows the values of the steps for the time variable, in which the accuracy of the calculation is about one percent of the solution for explicit regularized and implicit schemes for the different number of nodes in the calculation grids.

TABLE I. THE VALUES OF STEPS FOR A TEMPORARY VARIABLE FOR EXPLICIT AND IMPLICIT SCHEMES

Grid size	Explicit scheme		
	101 × 101 × 11	201 × 201 × 11	401 × 401 × 11
Step by time	0.072	0.036	0.025
Grid size	Implicit scheme		
	101 × 101 × 11	201 × 201 × 11	401 × 401 × 11
Step by time	0.2	0.1	0.075

Based on the results of numerical experiments, the following estimate is obtained, showing the time gain for the explicit scheme with respect to the implicit scheme, in the case of grid sizes with 101x101x11 (the average number of iterations 8): $Q_{neiavn}/Q_{iavn} \approx 10.286$, and in the case of grid sizes 201x201x11 (the average number of iterations 10): $Q_{neiavn}/Q_{iavn} \approx 12.857$, in the case of mesh sizes 401x401x11 (the number of iterations 12): $Q_{neiavn}/Q_{iavn} \approx 14.286$.

Table 2 shows the times of execution for one time step for explicit scheme and one iteration for implicit schemes as well as the values of the acceleration and efficiency of parallel algorithms. The calculated grid consisted of 101x101x11 knots. From the numerical estimates of the ratio of the times for solving the model problem to the explicit and implicit schemes, it can be concluded that when the size of the computational grid is increased, the gain in the calculation time of the explicit scheme only increases.

TABLE II. COMPARISON OF PARALLEL ALGORITHMS BASED ON EXPLICIT AND IMPLICIT SCHEMES

Number of calculators		1	2	4	8	16	32	64
Explicit scheme	Time, sec	0.00271	0.00074	0.00052	0.00029	0.00025	0.00060	0.00125
	Acceleration	1	3.662	5.212	9.345	10.84	4.517	2.168
	Efficiency	1	1.831	1.303	1.168	0.677	0.141	0.034
Implicit schema	Time, sec	0.01183	0.00446	0.00232	0.00179	0.00231	0.00365	0.00642
	Acceleration	1	2.652	5.099	6.609	5.121	3.241	1.843
	Efficiency	1	1.326	1.275	0.826	0.32	0.101	0.029
The time gain for an explicit scheme		10.286	14.201	10.513	14.544	21.772	14.334	12.102

When solving a problem on a grid containing 101x101x11 nots, the maximum acceleration for an explicit scheme was achieved on 16 cores and equal to 10.84. For the implicit scheme, the maximum acceleration, equal to 6.609 was achieved on 8 cores. Thus, the gain in time for the explicit scheme in relation to the implicit scheme was 16.871 times.

Table 3 shows the times for executing the transitions between layers for the explicit scheme and the execution of one iteration by an implicit scheme on the computational grid 5001x5001x101 nodes. In this case, the gain of the explicit scheme in time on 512 cores of the supercomputer system was 71.547 times.

TABLE III. TIME OF EXECUTION OF TIME STEPS BY AN EXPLICIT SCHEME AND ONE ITERATION BY AN IMPLICIT SCHEME AND VALUES ACCELERATION AND EFFICIENCY OF PARALLEL ALGORITHMS

Number of calculators		1	2	4	8	16	32	64	128	256	512
Explicit scheme	Time, sec	163	82	53	18	14	7.8	4.4	1.7	0.987	0.715
	Acceleration	1	1.98	3.06	8.68	11.5	20.93	37.1	96.05	165.434	228.36
Implicit schema	Efficiency	1	0.99	0.76	1.08	0.72	0.654	0.58	0.75	0.646	0.446
	Time, sec	370	188	127	48.9	47.2	31.8	18.2	7.6	6.318	5.8805
	Acceleration	1	1.967	2.924	7.555	7.837	11.61	20.29	48.33	58.563	62.921
	Efficiency	1	0.984	0.731	0.944	0.49	0.363	0.317	0.378	0.229	0.123
The time gain for an explicit scheme		19.74	19.94	20.76	23.63	29.33	35.47	36.38	38.89	55.691	71.547

B. Comparison of the efficiencies of explicit and implicit difference schemes

In order to obtain optimal values of time steps, the solution of the problem of transporting suspended particles was carried out on the basis of explicit and implicit schemes. The main idea of using explicitly implicit schemes is to reduce the transition from layer to layer to the sequential solution of one-dimensional and two-dimensional difference problems in spatial directions approximating the original problem. An explicit regularized scheme is used to approximate the two-dimensional diffusion-convection problem in horizontal directions, and an implicit scheme is used to approximate the one-dimensional diffusion-convection problem in the vertical direction.

Figure 2 shows the error values of the difference schemes (1 - the error function for the explicit circuit is indicated, 2 - the error for the explicit-implicit circuit is indicated). Notation used here: ψ - relative error value; τ_0 - time step. Magnitude τ_0 convenient to use to describe the error ψ , because when the grid size changes by spatial coordinates, there is practically no change in the function $\psi = \psi(\tau_0)$.

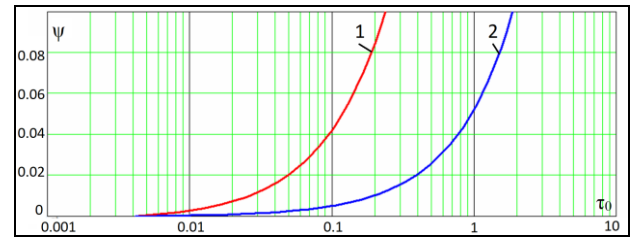


Fig. 2. The numerical study of the relative error as a function of the change in the time step according to explicit and explicit-implicit schemes

Fig. 2 shows that the achievement error for an explicit scheme is that the restriction on the time step is significantly greater than for the proposed explicit implicit scheme. The relative error of the explicit scheme is 1% if the value is 0.10087, in the case of using the proposed explicitly implicit scheme, the parameter is 0.01348. Thus, to achieve the accuracy of 1% of the explicitly-implicit scheme, it is necessary to make 7.483 times smaller steps than the explicit scheme, which will significantly improve the performance of the programs due to the better difference scheme. The proposed scheme is effective if the step in one spatial direction is significantly less than the steps in the remaining spatial directions.

IV. CONCLUSION

The main results of the work are the following:

- Advanced mathematical models of transport of multicomponent suspended particles and sediment transport in coastal systems are presented.
- Conservative stable difference schemes are constructed and investigated.
- A comparative analysis of the effectiveness of using an explicit difference scheme with an implicit scheme and with an explicit-implicit scheme is carried out. It is shown that the use of explicit regularized schemes leads to significant time savings (more than 10 times) compared to traditional implicit schemes. In turn, explicit-implicit schemes have shown their advantage over explicit schemes, since they allow the construction of parallel algorithms that are more economical with respect to the total time spent on arithmetic operations and information exchange operations between processors.

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