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The Layer Profile of the Students' Understanding of Image Making and Image Having in Completing Mathematical Problems

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***Abstract*—Mathematical understanding is an important aspect in learning mathematics. Pirie and Kieren's theory classified that eight layers of mathematical understanding, including image making and image having. In solving the problems, these layers can show the students such a way of solving problems. The purpose of this study is to describe the layers of understanding of the image making and image having for junior high school students with three dimensional material. The participants of this study were two students with equivalent mathematical abilities at Public Junior High School 3 Tulungagung. The results of the study show that the layers of understanding of image making and image having of those 2 students are almost the same with equal abilities. The findings show that both students can show an initial picture of solving the problem and the reason for thinking about the idea. There are no significant difficulties from the two subjects. However, for the second subject, there was a little difficulty in describing the completion step when she was given two different shapes.**

Keywords— Mathematical Understanding, Pirie and Kieren's Theory, Image making and image having layers.

I. INTRODUCTION

Education has a very important role in improving the existing human resources because it can create a superior and competitive generation as an effort to face the challenges in the future. The quality of mathematics education in Indonesia is low, it can be seen from the results of the Trends in International Mathematics and Science Study (TIMSS) survey which shows that since the first participation in 1999, Indonesia was ranked 34th out of 38 countries. Therefore, it is necessary to develop mathematical education in Indonesia in order that the mathematical ability of students in Indonesia increased.

The National Council of Teacher Mathematics [1] itself establishes that there are five standard mathematical abilities which should be achieved by the students. Those are problem solving abilities, communication skills, connection skills, reasoning abilities, and representation abilities. One of them is problem solving ability. The problem solving skills is important to be achieved by the students. In solving problems, the step of problem understanding has an important role for students. Problem solving has proven to be significant in facilitating the growth of students' mathematical understanding [2]. Because if students have an understanding that is still low will be an obstacle for students in solving problems. Van de Walle explains that "understanding can be defined as a measure of the quality and quantity of relations between existing knowledge" [3]. Mathematical understanding is also one of the goals of every material. According to Hewson and Thorleyn, "Understanding is a conception which can be analyzed by the students so that the students understand what is intended, are able to find ways to express that conception, and can explore the possibilities that are related" [4].

There are three models of mathematical understanding, namely understanding as a structured progress, understanding as forms of knowing something and understanding as process [5]. Understanding as structured progress describes that the development of understanding which follows the trend of constructivism, namely the process of constructing knowledge from basic to a higher level. Most understanding theories assume that understanding is a linear process. Pirie-Kieren assumes an understanding of the whole process of growth, dynamic, layered but not linear and never-ending. This understanding of the process is described as onion which has layers. The Pirie-Kieren understanding layer has eight levels, starting from the innermost layer, namely primitive knowing, image making, image having, noticing, formalising, observing, structuring to the outermost layer is inventising.

Of the eight layers of understanding that Pirie-Kieren created, two of them are the layer of image making and image having. These two layers illustrate how the initial steps taken by students in determining the solution of the problem to be carried out. So that both layers need to be emphasized more to help students to be able to determine the next steps in solving problems. Image making is the stage for students to describe the problem in accordance with their thoughts when they solve the problem given [6]. Image having is interpreted as the stage for the students when they have a mental image, it means that a picture which the students have when adapting information to solve problems are related, free and unlimited [7; 8]. It can be said that the image having layer is the first level of abstraction of students [9]. When someone tries to make a picture, physical or mental, to understand the problem, they are in the second layer, namely image making [10]. After creating and comparing images, one explains the picture which is most suitable for understanding the problem. In the next layer, image having, one can formulate a mental which can be used to solve problems [11]. Someone is stated on the image making layer when someone takes action to get

an idea of the concept [12]. Try to understand a topic, both mentally and physically, to get an idea about the topic. Someone develops certain ideas and draws a concept through images and examples [13]. It can be said that at this image making layer, someone takes various actions to get ideas from a concept which he/she wants.

The next layer of understanding is image having. Image having and image making is a person's activity to make a new picture or improve an existing picture which then manipulates it in the mind [14]. This level is the first level of abstraction of individuals [9]. It can be said that this level of image having is a level where mental abstractions substitute for ideas which have been obtained become mental images without any physical activity. On the other hand, mathematical understanding is a concept which is difficult to be defined and expressed [8]. It is not easy to formulate a definite definition of understanding. Therefore, there are various frameworks for what understanding is. Understanding is equated with building connections in the context of algorithm operations and problem solving. It can be said that understanding has a very close relationship with problem solving. In solving problems, the layer of image making and image having are very important. The layer is in line with the stage of students in solving problems according to Polya, namely at the planning stage. If there are no obstacles to student planning in solving problems, then there will be no obstacles as well in the steps to solve the problem.

Based on the explanations above, the researchers want to analyze the students' understanding activity in solving mathematical problems in detail. Especially in the layers of understanding image making and image having since these two layers can be categorized as students' initial steps in solving problems whether they use the correct steps or not.

II. METHOD

This research is a qualitative descriptive study. Because the data obtained from this research are written and verbal data. The purpose of this study is to describe the layers of mathematical understanding especially in the layers of image making and image having students in solving mathematical problems. This research was conducted at Junior High School 3 Tulungagung involving two students from the school. The researcher chooses participants with a purposive sampling method, namely selecting students by considering several aspects in order to expedite the research process, such as students with mathematical abilities and good communication so that data retrieval is not experiencing obstacles.

The Think Aloud method is applied to collect data. Think Aloud method is a method in which data is obtained by asking students to solve mathematical problems accompanied by verbal expressions of wise ideas [15]. Vygotsky described language as a tool for expressing ideas through words [16]. In this study, there are two types of instruments used to collect data. The first is the researcher itself as the main instrument and the second is supporting instruments in the form of worksheets, and unstructured interview guidelines.

The researcher took the data twice in order to obtain valid data. The valid data is compared to the indicators of the layer understanding of the image making and image having between two subjects. The collected data were analyzed based on Miles and Huberman's theory with the following steps: 1) Data reduction consisted of a selection process, a simplification process, and then extracting and changing raw data; 2) Display data by classifying and identifying data until data is organized; 3) Conclusions based on results. After that, the researcher applied a triangulation method to obtain valid data.

III. RESULTS AND DISCUSSION

Tests given to the two chosen subjects for 30 minutes. The test contains two items as follows:

1. A large cube can contain 8 small cubes in which the volume of 1 small cube is 8 cm^3. If the length of the edge of the large cube is increased by 2 cm, how many small cubes can be loaded into the large cube?
2. A cuboid is given to has a length of 7 cm and a width of 5 cm. If the sum of the lengths of the edges of the cuboid is the same as the sum of the lengths of the cube edges which has a volume of 125 cm^3, then determine the height of the cuboid and the volume.

A. The Understanding Profile of Pirie-Kieren theory: The Layer of Image Making:

In the first question, it can be seen that S1 can understand what is meant by the problem in question. S1 says that based on the first problem, he can imagine that the problem is about two cubes with different sizes. Then, when one of the side cubes is enlarged, the comparison of the size of the two sides will change.

From the way to complete the first problem, S1 concluded that because according to the problem there are two different cubes so that they must be completed one by one first then match the results.

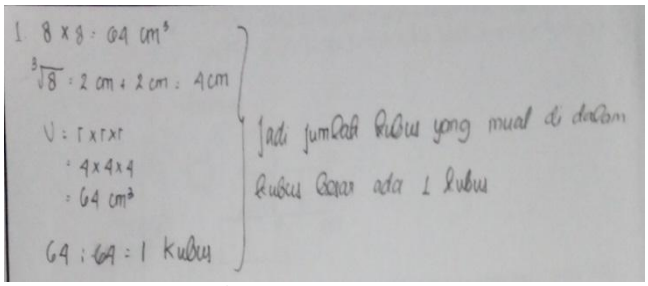

Fig. 1. S1 Answer on the first question

Then in the second problem, S1 can show the important points of the problem that will be used to determine the solution. According to S1, to solve the first problem, it must look for the length of the side of the cube to find the height of the cuboid. The Figure 2 show results of the second work of S1.

S1 concludes that the cube and the cuboid are the same as building a space composed of rectangles. However, the difference is just the type of quadrilateral.

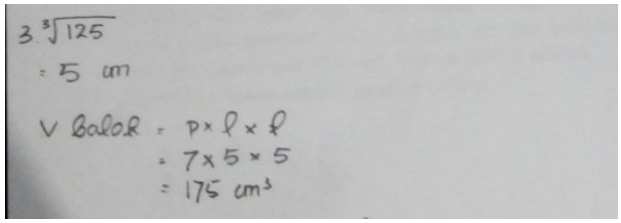

Fig. 2. S1 Answer on the second question

Then, in the first questioning process by S2, it can be seen that S2 can understand what is meant by the problem. According to S2 the first problem is about small cubes and large cubes. Then when the small cube is enlarged, the volume of the small cube will also increase. Figure 3 show s the work of the first question by S2.

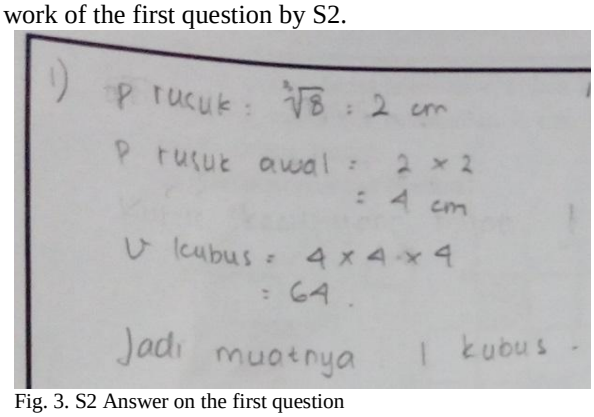

Fig. 3. S2 Answer on the first question

According to S2, he thought so because the first large cube can contain eight small cubes. Therefore, when the small cube is enlarged, the volume will increase so that the large cube will contain fewer small cubes than before.

In the second question, S2 said that the problem was aimed to find out the volume of the cuboid. However, to find the height of the cuboid, we must know the length of the side of the cube first. The work of S2 in the second question is presented in Figure 4.

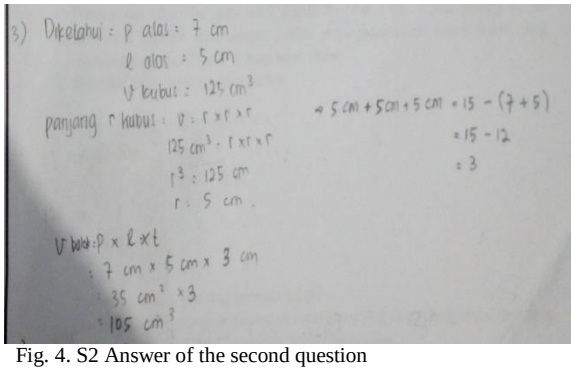

Fig. 4. S2 Answer of the second question

According to the S2, from those two building spaces, only the cube which can be searched for the whole element. While the cuboid only has the same number of ribs as the cube. So that, the height of the cuboid depends on the length of the side of the cube.

It shows that S1 and S2 do folding back when working on problems. However, folding back which occurs is ineffective folding back because S1 returns to the previous understanding, which is about the definition of cuboids and cubes, which are both the shape of a flat side space constructed of rectangles. However, folding back which occurs in S2 is to return to the initial understanding of the elements contained in building the space to solve the problem. S2 uses that understanding to solve problems.

Based on the description above, it can be concluded that both S1 and S2 understand what is meant by the first problem. The difference which occurs is in the folding back of the two subjects.

B. The Understanding of Profile Pirie-Kieren theory: The Layer of Image Having:

On the second problem, S1 reveals that he determines the picture of the second problem solving by looking for the side of the cube which is known by rooting the cube of the volume of the cube. Then, after finding the lengths of the sides of the cube, S1 concludes that the height of the cuboid is the same as the length of the side of the cube as shown in Figure 5.

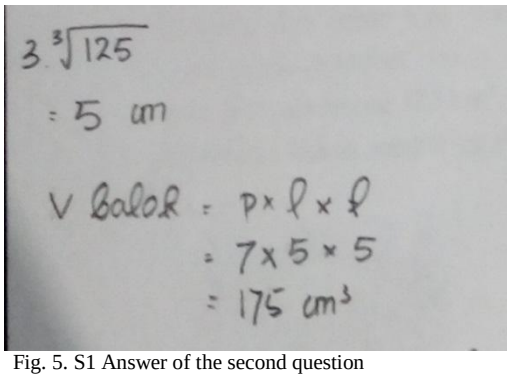

Fig. 5. S1 Answer of the second question

It can be seen that S1 made a mistake while working on the second question. Based on the results of the study, it turns out that S1 only reads the second question once and then immediately completes it.

Then, it can be seen that there were no difficulties for S2 in solving the second problem. S2 revealed that in solving the second problem, he separated the elements from each known building first. Then, from the two builds, the first thing to do is to find the side of the cube. In addition, according to the S2, to determine the height of the cuboid, he added 3 sides of the cube. Next, from the length of the sum of the three sides of the cube is subtracted by the length and width of the known cuboid, as presented in Figure 6.

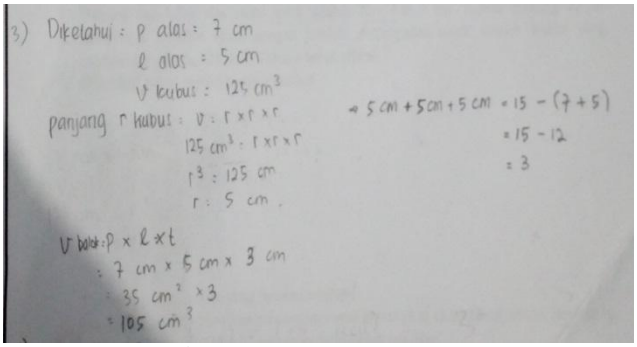

Fig. 6. S2 Answers of the second question

From the S2 explanation and the results of his work, it was seen that the S2 also had difficulty in understanding the second question. According to the results of the study, S2 read the second question three times. But the method used by S2 is appropriate and faster than it should be.

Based on the results of the explanation above, it can be concluded that S1 does not understand the purpose of the second question and is too hurry to complete the problem even though it does not yet know the purpose of the second question. It can be seen that after getting the side length from the S1 cube, it is immediately concluded that the side length is the height of the cuboid. The data above can be described in the Table I.

TABLE I. IMAGE MAKING AND IMAGE HAVING OF S1 AND S2

Subject 1 (S1)	Subject 2 (S2)
Students can understand what is meant in the problem. Students can explain and mention what is understood from the problem. Students can explain the description of the solution to be done. Students experience ineffective folding back, which is when returning to the initial definition to complete completion. Students experience errors when working on problems that contain different builds. Students are too hasty in determining questions without reading many times.	Students are able to understand correctly what is meant in the problem. Students can explain and describe what is understood from the problem. Students can explain the reason for choosing the description of the steps to solve the problem. Students read questions more than once to understand the problem. Students experience ineffective folding back which is when returning to the elements contained in the wake contained in the problem to determine the problem.

Based on the Table I, it can be seen that there are differences between S1 and S2, especially in the layer of image having. It is where the S2 is more able to explain the picture in solving problems correctly and in more detail. While S1 is too hasty in determining the settlement.

IV. CONCLUSION

Based on the results of the analysis and explanation it can be concluded that the layers of image making and image having illustrate the initial steps that can determine the next step of students in solving problems. This shows that if the initial steps taken by students in solving an inappropriate problem will have an impact on the next step. It appears when S1 experiences errors in the layer of image having so that in solving problems, he also experience difficulties. Therefore, the results of solving problems are not appropriate. Otherwise, S2 does not experience errors in each layer so that the resolution of the problem is correct and appropriate.

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