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Students' Proof Scheme for Mathematical Proving and Disproving of Divisibility Proposition

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Abstract*—This study aimed to investigate and describe students' proof schemes for disproving mathematical proposition. Previous studies examined students' proof scheme of Calculus, Elementary Numbers Theory, Quadratic, and Geometry's Propositions. This study examined student proof schemes of Divisibility's Proposition which has never been studied before. The study participants were 20eleventh grade students from a private senior high school in Sidoarjo. All participants had to answer two tests, namely the mathematics ability test and the proof test. Three volunteer students (1 male and 2 female), with high mathematics ability and high score in proof test, were selected as research subjects. Semi-structured interview was conducted to three subjects for investigating the students' proof schemes. The qualitative data of proof schemes was categorized in Lee's proof scheme descriptors. The results show that all of the participants divisibility propositions in the 4th level because they were able to state that the falsity of the mathematics proposition by a specific counterexample. They couldn't change the specific counterexample to become a general counterexample with mathematics symbols. Meanwhile, for the true mathematical proposition, one subjects concluded that the proposition is true with informal deductive proof, the other subjects proved the proposition inductively by using specific examples. Therefore, the first subject is categorized into 5th level and 2 subjects are categorized into 2nd level.

Keywords—Proving, Disproving, Proof Schemes, Mathematical Proposition

I. INTRODUCTION

Mathematical proof is an important part of mathematics. Proof is a process carried out to explain the truth of a statement. The most important feature that distinguishes mathematics from other branches of science is the effort to identify, explain, and prove each instrument it uses [1]. Mathematics is developed through proven theorems so that knowledge of how to prove is needed in studying mathematics.

Learning about mathematical proof has become the main goal of the School Mathematics Curriculum in several countries for several generations [2]. The Ministry of Education of Australia in the School Mathematics Curriculum in Australia states that one of the four competencies students must achieve is reasoning ability including the ability to think and act logically, namely analyzing, proving, evaluating, explaining, concluding, justifying, and generalizing. The School Mathematics Curriculum in Indonesia also includes learning about proof as a competency that must be achieved. One of the basic competencies in question is to explain the method of proving mathematical statements in the form of lines, inequalities, and divisibility with mathematical induction.

The development and mental processes of students when they prove they have become the subject of research in mathematics education. Harel and Sowder have conducted research on the proof scheme in proving mathematical propositions. Lee's research on student evidentiary schemes was also conducted in 2015. If Harel and Sowder only focus on deductive evidence for true-value propositions, Lee complements his research with schemes to prove students in proving mathematical propositions that are of false value. Lee developed a proof scheme into schematic levels arranged hierarchically from level 0 to level 6 for proof schemes in the correct propositions and level 0 to level 5 for proof schemes in wrong proposition. This level contains all the examples and counter examples that students might make when proving proposition.

The study of student proof schemes for proving and disproving divisibility's propositions is limit. Even though, the study of student proof schemes for proving and disproving divisibility's propositions provides a great opportunity in understanding the construction of student proofs. The classification of the student proof schemes is one potential way to inform teachers / educators about the students' ability to refuse a mathematical proposition [2-4]. The classification of the students' proof schemes on mathematical propositions is needed to more understanding how students use examples, conclusions, and counterexamples while they develop more deductive reasoning.

Therefore, this study aimed to investigate and describe students' proof schemes for proving and disproving divisibility's proposition. The students' proof schemes are analyzed based on quality of proof that they constructed and the knowledge they have in proving. Then they are analyzed based on levels of proof schemes [2]. This qualitative research used the test of proof-by-counterexample and interview

II. METHOD

This qualitative research concerned to assess students 'proof construction scheme in proving divisibility's propositions. The sample was the 11th grade students of SMA Hang Tuah 5 Sidoarjo. 36 (14 males & 22 females) students had to answer two tests, both mathematics ability test and proof test. Three student-volunteers (1 male and 2 females), with high mathematics ability and high score in proof tests, were selected as research subjects. A semi-structured interview was individually conducted to each research subjects. Each student was interviewed during 20 minutes.

The data is categorized in seven levels of proof criteria and six levels of disproof criteria. This student proof scheme is analyzed based on their knowledge of proof construction and proof validation. To identify the levels of students' proof schemes are used descriptor of the proof scheme. These proof scheme descriptors are modified from the Lee's proof scheme descriptors [2]. The Descriptors of students' proof schemes for proving mathematical propositions that is used in this study explain in Table 1.

TABLE I. DESCRIPTORS OF STUDENTS' PROOF SCHEMES FOR PROVING MATHEMATICAL PROPOSITION

Level	Characteristics	Description of Students' Proofs
0	Irrelevant or minimal engagement in inferences	• Do not know how to prove or appeal to conviction in external knowledge sources • Unable to relate the antecedent and the consequent with examples
1	Novice use of examples or logical reasoning	• Conclude the implication is true (or false incorrectly) using one or more examples or an incorrect example or counterexample • Falsify the implication "if P then Q" through erroneous logical reasoning, e.g., "if not P then not Q" • Derive a mathematical property unrelated to the implication
2	Strategic use of examples for reasoning	• Generate examples based on random or case-based sampling, or extreme cases • Uses mathematical properties inferred from generated examples to make conclusions
3	Use of deductive inferences with major flaws in logical coherence and validity	• Deduce relevant mathematical properties but missing one or two deductive inferences for proving the implication • Deduce the implication to be true for some cases of the antecedent but omit other cases
4	Use of deductive inferences with minor flaws in logical coherence and validity	• Generate a chain of deductive inferences to justify conclusions but one or two inferences may be interpreted as inductive due to insufficient substantiation or as writing errors • Displays disorganization in the chain of deductive inferences
5	Construction of informal deductive proofs	• Generate a chain of deductive inferences to justify the implication using informal justifications
6	Construction of deductive proofs using formal representations	• Generate a chain of deductive inferences to justify the implication using formal representations

Descriptors of students' proof schemes for disproving mathematical propositions that is used in this study explain in Table 2.

TABLE II. DESCRIPTORS OF STUDENTS' PROOF SCHEMES FOR DISPROVING MATHEMATICAL PROPOSITION

Level	Characteristics	Description of Students' Proofs
0	Irrelevant or minimal engagement in inferences	• Do not know how to prove or appeal to conviction in external knowledge sources • Unable to relate the antecedent and the consequent with examples
1	Novice use of examples for reasoning or logical reasoning	• Conclude the implication is true (or false incorrectly) using one or more examples or an incorrect example or counterexample • Falsify the implication "if P then Q" through erroneous logical reasoning, e.g., "if not P then not Q" • Derive a mathematical property unrelated to the implication
2	Strategic use of examples for reasoning	• Generate examples based on random or case-based sampling, or extreme cases • Uses mathematical properties inferred from generated examples to make conclusions
3	Deductive inferences with major flaws in logical coherence and validity	• Deduce the implication to be true incorrectly for some cases of the antecedent but omit other falsifying cases due to misconceptions
4	Construction of Proof-by-counterexample	• Deduce the implication to be false by constructing one or few specific counterexamples
5	Construction of Proof-by-general-counterexample	• Deduce the implication to be false by constructing a general set of counterexamples and identifying the property of the set that falsifies the implication

III. RESULTS AND DISCUSSION

A total of 20 students have done on the mathematics ability test and 40 students' proof construction (38 students; each student doing on 2 types of proof test) have been analyzed. It presents the finding of students' proof schemes. 20 of 11th grade students answer the proof test and the results as Table 3.

TABLE III. THE RESULT OF STUDENTS' MATHEMATICS ABILITY AND PROOF TEST

Gender	Mathematics Ability			Proof Score		
	High	***Medium***	***Low***	$0 \le ss \le 60$	$60 \le ss \le 80$	$ss \ge 80$
Male	2	5	3	1	7	2
Female	4	4	2	3	5	2

Based on the results, students who have the highest score of each type of test are selected, so there were 3 students who are subsequently interviewed to know the students thinking in constructing the proof.

The three students (Komang, Sevialita, and Ayu Nanda) do two questions of proof. The results shown all of the participants were able to disprove divisibility's propositions in the 4th level, while for proving divisibility's proposition, the first subject is categorized into 5th level and 2 subjects are categorized into 2nd level. Here is the test, answers, and interview results of selected students.

> 1. Let x be an integer. If $3x$ is added to $3x^2$, the sum must be divisible by 2. Is the statement true or false? Prove your answer.
> 2. If two consecutive integers are added together, the result will be divisible by two. Is the statement true or false? Prove your answer.

Fig. 1. The Instruments of Proof

A. *Komang Proof Scheme*

Komang's solution in problem 1 and 2 showed in Fig. 2 and Fig. 3.

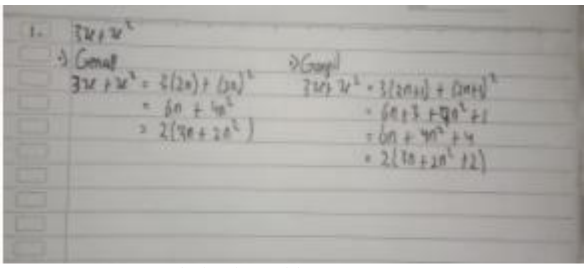

Fig. 2. Komang's solution in problem 1

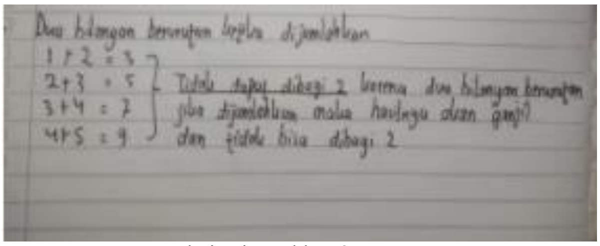

Fig. 3. Komang's solution in problem 2

Komang answered problem number one using two proof cases, that is if x is an even number and if x is an odd number. He makes an example if $x = 2n$ for even numbers and $x = 2n + 1$ for odd numbers. The permissions are substituted into the equation $3x + x^2$, then it is proven that the results can be divided into 2. The following is an excerpt from the interview regarding the answers given by Komang.

Researcher : Why is there even and odd writing here?
Komang : Because in this case integers consist of even and odd numbers.
Researcher : Why for even numbers x is assumed to be $2n$?
Komang : Because all numbers multiplied by 2 will be even numbers.
Researcher : Why is the odd number x suppose to be $2n + 1$?
Komang : Because all numbers multiplied by 2 then plus 1 will be odd numbers.
Researcher : Does it have to be $2n$ and $2n + 1$?
Komang : No, it can also be $4n$ or $6n$ for even numbers and $2n + 3$ for odd numbers. I chose the example to easily calculate it.

Based on the way Komang proves the proposition on problem 1, he can conclude that the proposition is true deductively by using informal representation. It shown that Komang's proof scheme in problem 1 was at fifth level.

In solving Problem 2, Komang constructed the proof by trying several numbers. Komang started it by adding 2 consecutive whole numbers. Then she divided it by 2 so the result was not divisible by 2. He done the same thing to different numbers. Furthermore, Komang done the same thing with several other numbers, then she divided the result by 2 so the result was not divisible by 2. Komang concluded that the proposition was false. This conclusion was based on specific examples that have been constructed.

Komang's proof construction on problems 2 indicated that the proposition was false based on a specific counterexample. Komang couldn't construct proof by using general mathematics symbols. The collection of random number samples was still classified in an inductive thinking [3-5]. Komang's proof scheme in problem 2 was at fourth level [2].

B. *Sevialita Proof Scheme*

Sevialita's solution in problem 1 and 2 showed in Fig. 4 and Fig. 5.

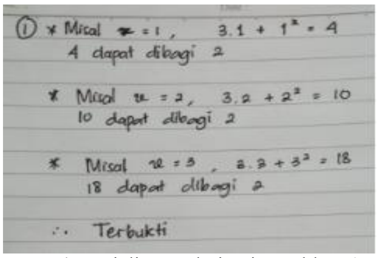

Fig. 4. Sevialita's solution in Problem 1

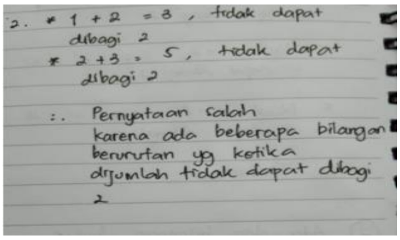

Fig. 5. Sevialita's solution in Problem 2

Sevialita begins to prove by taking several examples of the value of x then substituting into the equation. Based on these examples, the second subject concludes that the proposition is true. Because the second subject uses examples to prove the propositions given, the second subject is categorized into level 2 in the scheme of proving students to prove mathematical propositions. She argued that some of these numbers satisfied the proposition, so that in some cases used, it concludes that the proposition is true, as the following interview quotes.

Researcher : Why do you conclude that the proposition is true?
Sevialita : Because some numbers that I used is right. But not all-natural numbers.

In solving Problem 2, Sevialita constructed the proof by trying several numbers. Sevialita started it by adding 2 consecutive whole numbers. Then she divided it by 2 so the result was not divisible by 2. She done the same thing to different numbers. Furthermore, she did the same thing with several other numbers, then she divided the result by 2 so the result was not divisible by 2. She concluded that the proposition was false. This conclusion was based on specific examples that have been constructed.

Sevialita's proof construction on problems 2 indicated that the proposition was false based on a specific counterexample. She couldn't construct proof by using general mathematics symbols. The collection of random number samples was still classified in an inductive thinking [3-5]. It shown that Sevialita's proof scheme in problem 2 was at fourth level [2].

C. *Ayu Nanda Proof's Scheme*

Ayu Nanda's solution in problem 1 and 2 showed in Fig. 6 and Fig. 7.

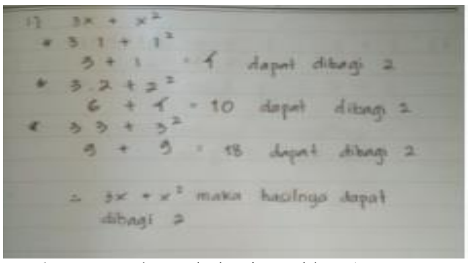

Fig. 6. Ayu Nanda's solution in Problem 1

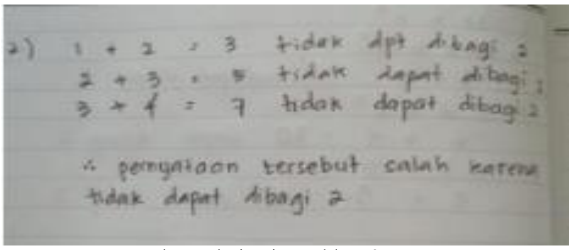

Fig. 7. Ayu Nanda's solution in Problem 2

Same with Sevialita, Ayu begins to prove by taking several examples of the value of x then substituting into the equation. Based on these examples, the third subject concludes that the proposition is true. Because the third subject uses examples to prove the propositions given, the third subject is categorized into level 2 in the scheme of proving students to prove mathematical propositions that are of true value. She argued that some of these numbers satisfied the proposition, so that in some cases used, it concludes that the proposition is true.

The construction of proof on question 2, Ayu also begin by making specific examples of whole numbers and some special cases. She started up with four consecutive numbers. Ayu summed each of two consecutive whole numbers, then she divided the result by so the result was divisible by 2. Ayu concluded that the proposition was false when two consecutive integers are added together, the result will be divisible by two.

Ayu showed that 1 was a counterexample that could refuse mathematical proposition [1,2,6,7]. She had an argument about counterexample as follows.

Researcher : Do you know what counterexample is?
Ayu : In my opinion, counterexample is one or more example that show the propotition is false

The construction of proof done on Problem 2 didn't use general examples. Ayu only used specific examples and based on specific cases. The collection of random number samples was still classified in an inductive thinking [3,4,8]. This shows that Ayu's proof scheme was at 4th level [2].

IV. CONCLUSION

Based on the result of the study, we concluded that the students' proof scheme for proving divisibility proposition is still low because propositions is still low because most students were able to show specific example and only one was able to demonstrate general proof by using mathematical symbol. We also concluded that the students' proof scheme for disproving divisibility propositions is still low because all of the students were able to show counterexample specific. Thus, there needs to increase proof-based learning so that students are able to improve logical reasoning and logical thinking as a provision to improve a higher thinking.

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