

The Fraction Magnitude Knowledge through Representations at Students with Mathematics Difficulties

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Abstract—Elementary school students' knowledge of fraction magnitude may predict their mathematics achievement. Students' mathematics achievement is influenced by their mathematics difficulties. This study exposes students' representations in estimating fractions to indicate their fraction magnitude knowledge. This study is conducted on the 4th grade of elementary students in Surabaya. A class which consists of 10 girls and 18 boys, had chosen purposively from 3 available classes. All students had to answer two tests, namely the diagnostic test and the fraction magnitude test. Four volunteer students were selected as research subjects which have different and interesting mathematics difficulty in fraction. Semi-structured interview was conducted to all subjects to reveal students' thinking process when solving diagnostic and fractions magnitude tests. Results suggest that 46% of 26 students who have bad score (low ability) have mathematics difficulties in representing and estimating simple fraction because they considered nominator and denominator as independent numbers (i.e. $\frac{1}{3}$ is represented as a bar model which is divided by 4 equally parts and a part is in shaded). More than 20% of 26 students had mathematics difficulties which were not able to estimate non-simple fraction but they were able to represent it. Their ability is influenced by difficulties in fraction magnitude.

Keywords—fraction magnitude knowledge, representation, students' difficulties

I. INTRODUCTION

Fraction is important not only in mathematics but also in other subjects. Understanding fraction is the gateway to understand the higher concept in mathematics [1, 2, 3]. Fraction is defined as part of whole [4], and also defined as numbers that were represented by the ordered pair of a/b , where a and b are natural numbers with $b \neq 0$ [5].

The concept of fraction is still difficult for Indonesian students in particularly for elementary students. This difficulty emphasizes that students did not understand about fraction magnitude [1, 3, 6, 7]. Fraction magnitude knowledge is the ability to estimate, to represent, and to compare fractions [8, 9, 10]. A mistake which Indonesian students had are comparing fractions, both unit and non-unit fractions [6]. Another mistake is from 28 students in Surabaya, 11 students had low ability in estimating fractions because they do not know the concept of fraction and fraction magnitude [11].

Fraction magnitude knowledge is important to master higher mathematics concept for example algebra [1, 2, 3]. Moreover, fraction magnitude knowledge is linked to student mathematics achievement or support each other [12, 13]. Poor mathematics achievement can be affected by learning problems. Two types of learning problems can be distinguished into a learning disability is situated in the child's own cognitive, while learning difficulty is situated outside the child [14]. Because of these reasons, mathematics difficulty in fraction becomes special attention [15].

Although many students understand fractions easily, fraction magnitude is challenging for students with mathematics difficulty [16]. Research investigation to fraction magnitude knowledge with mathematics difficulty students and their typically achieving peers have shown significant and widening gaps in fraction achievement between two groups at the upper elementary level [11, 17, 18] and the middle school level [7, 19]. Mathematics difficulty students in the 3rd and 6th grades are slower in understanding the concepts and procedures of fraction than typically achieving peers [18]. It contributes to mathematics achievement gap [16].

One factor which affects poor fraction performance for mathematics difficulty students is weak fraction magnitude knowledge. Fraction magnitude knowledge includes learning about the way in which properties of whole number and fractional number are similar and the ways in which they diverge [13]. Fraction magnitude knowledge is the basic fraction concept [13] and weak fraction magnitude understanding is one of characteristics of student with mathematics difficulty [18].

II. METHOD

This study is conducted on the 4th grade of elementary students in Surabaya A class consists of 10 girls and 18 boys, had chosen purposively from 3 available classes. All students had to answer two tests, namely the diagnostic test and the fraction magnitude test.

The diagnostic test investigates student mathematics difficulty gradually. It examines from the easiest to the hardest concept of fraction magnitude. There are 5 topics of fractions in this test [10]. Each topic consists of 4 questions about fraction comparison through representation. The

representations used are bar and circle models of representations. The fraction magnitude test is focused on the ability to estimate fraction through representation. There are 2 questions about fraction estimation into number line [8, 10]. There are 5 fractions of each question and range of number line is 0 to 1. For example “put and represent these fractions, $1/8, 1/2, 2/4, 5/8, 3/4$ into number line 0 to 1. All these tests are validated by an expert.

Two tests are analysed and categorized into high, medium and low abilities in which take into consideration of students’ document scores test. Based on these results, will be further analysed students’ mathematics difficulties in fraction. There are many difficulties in fraction which are conceptually and procedurally in estimating and representing simple or non-simple fractions. Four volunteer students were selected as research subject which have different and interesting mathematics difficulties in fraction. Semi-structured interview was conducted to all subjects to reveal students’ thinking process when solving diagnostic and fractions magnitude tests. Table I shows the topics to examine mathematics difficulty in fraction magnitude. There are 5 topics about simple fraction and non-simple fraction.

TABLE I. THE TOPICS AND EXAMPLE TO EXAMINE MATHEMATICS DIFFICULTY IN FRACTION MAGNITUDE

Topics	Examples
The simple fraction	$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$
Fractions with the same denominator (similar fraction)	$\frac{2}{6}, \frac{4}{6}, \frac{5}{6}$
Fractions with the same numerator	$\frac{3}{6}, \frac{3}{7}, \frac{3}{8}$
Equivalent fractions	$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$
Fractions that have common multiple	$\frac{3}{4}, \frac{5}{12}, \frac{1}{6}$

III. RESULTS AND DISCUSSION

The diagnostic test has 5 topics which investigate students’ difficulties in fraction. The result of the diagnostic test showed that from 28 students 1 student has high ability, 1 student has medium ability, and 26 students have low mathematics abilities in fraction. Students with low mathematics ability had mathematics difficulties in understanding fractions. Question number 1 is about simple fraction in which 12 students did not know how to compare simple fractions and represent them. Therefore, they were not able to compare 4 other topics (similar fractions, fractions with same numerator, equivalent fractions, and fractions that have common multiple). Question number 2 is about similar fraction in which 3 students had difficulty in compare and represent them. Question number 3 is about fractions with same numerator; 4 students had difficulty in represent them. Question number 4 is about equivalent fraction; 2 students had difficulty in represent them. Question number 5 is the hardest and 5 students had difficulty in decide the greatest fractions with different numerator and denominator. Meanwhile, there are a student categorized into high ability had not mathematics difficulty, and a student with medium ability had difficulty to understand equivalent fractions.

Results of fraction magnitude test, 95% of 26 students with mathematics difficulty were not able to represent and to estimate fractions, for example in estimating $3/4$, they did not know the relationship between numerator and denominator. They considered numerator and denominator as an independent number. Meanwhile, a student with medium ability could not take fractions appropriately into number line.

This study underlined that students with mathematics difficulties were not able to represent fractions with different denominators and estimate them. Most students had difficulties in understanding simple fractions and fractions with different numerator and denominator (non-unit fractions). A student who had difficulties in understanding simple fractions (SF), a student with mathematics difficulty in representing similar fractions and fractions with same numerator (SNF), a student with difficulty in represent equivalent fractions (EF), and a student who had difficulties in fractions that have common multiple (CM) were selected as a research subjects. Figure 1 shows the example of student SF who had difficulty in simple fractions and some dialogs between SF and interviewer (I).

	I :	How could you represent these fractions?
	SF:	I represented $1/2$ as a bar which is divided into 3 equal parts and 1 part is shaded
	I :	How it could be?
	SF:	the upper side is 1 so I shaded a part and the lower side is 2 so I did not shade 2 parts of a bar
	I :	What is the name of the upper and lower sides?
	SF:	I forget, one of them is denominator but I do not know which one is the denominator.
I :	So did you think that $1/5$ is the greatest fractions?	
SF:	Yes because $1/5$ has 6 parts and a part is shaded which is greatest than all fractions here	

Fig. 1. Student SF fractions representation and estimation.

Student SF with mathematics difficulties represent numerator and denominator as an independent number, for instance, $1/2$ is represented as a bar which is divided into 3 equal parts while one part is in shaded. He considered 1 and 2 (in the number $1/2$) as independent numbers. Therefore he concluded that $1/5$ is the greatest fraction.

Student SF had difficulty in understanding simple fraction, hence he also could not represent and estimate simple fractions in number line. This invention is supported by statement that fraction magnitude knowledge is linked to student mathematics achievement or support each other and poor mathematics achieving can be affected by learning problems, one of them is learning difficulties especially in this case is mathematics difficulty [11, 12, 13].

Student SNF was able to represent and compare similar fraction but she did not know conceptually because she was not able to apply this concept into fraction with same

denominator. While represented $2/6$ and $4/6$ into bar she got true representation and she said that $4/6$ is greater than $2/6$ because has same denominator so she just take consideration of numerator. At the otherwise she confused to represent fraction with same numerator. In compare $3/6$ and $3/7$, she said $3/7$ is greater than $3/6$ because these two fractions have same numerator. In this case she decided fraction with greater denominator is bigger than fraction with smaller denominator. Because of it, she could not estimate these fraction into number line. Figure 2 shows the example of student SNF work.

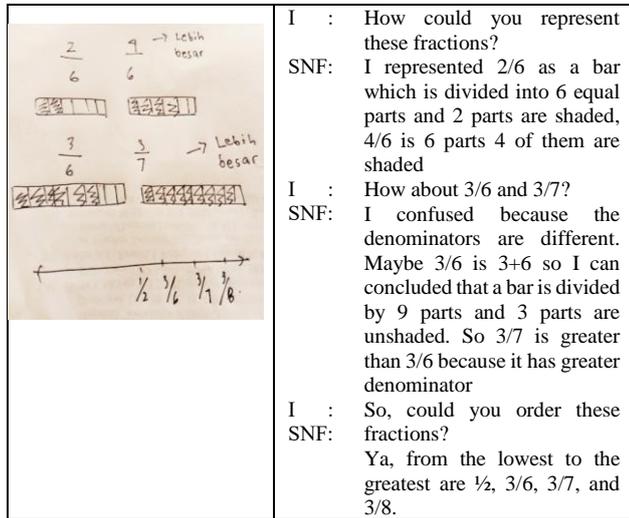


Fig. 2. Student SNF fractions representation and estimation.

Student EF had difficulty in estimating equivalent fractions. She was able to represent them in bar models and circle models, but from depth interview she was not able to estimate these fractions in number line. She could represent fractions into bar or circle models procedurally but not conceptually. Figure 3 shows the example of student EF fractions representation in estimating fractions.

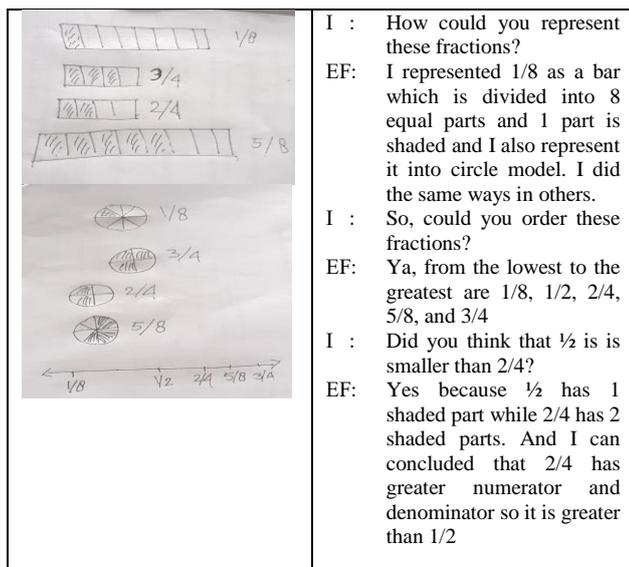


Fig. 3. Student EF fractions representation and estimation.

In figure 3, student EF understood from pictures were created by him that $1/8$ is the least fraction and $3/4$ is the greatest fraction. But she did not understand the appropriate location

of these fractions in number line. She put $2/4$ after $1/2$ because numerator and denominator of $2/4$ are greater than $1/2$, just $1/8$ is located before $1/2$. Because of these argument, there is a finding that student EF did not understand equivalent fraction. This error is influenced by he had difficulty in fraction magnitude knowledge.

Student CM had difficulty in concept of fraction comparison especially fractions that have common multiple. He consider that fractions which have greater numerator and denominator is the greatest. In case of fraction comparison between $3/4$ and $5/12$, he stated that $5/12$ is greater than $3/4$ because it has greater numerator and denominator than $3/4$. He also did wrong in a procedure of cross multiple. He argued that in fraction comparison we allowed to use cross multiple i.e. 3 is multiplied by 12 and 5 is multiplied by 4, but he did wrong in put these results. Figure 4 illustrates the example of student CM work.

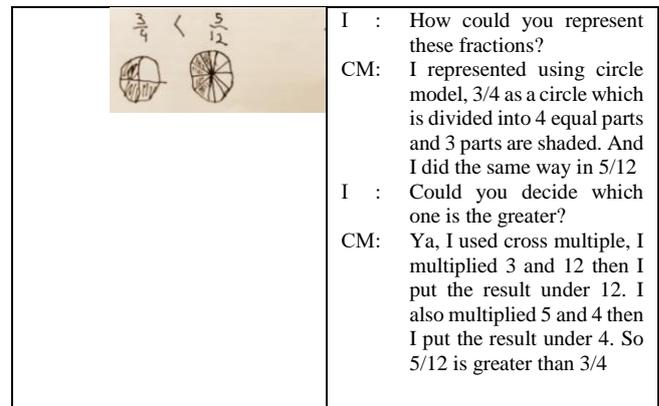


Fig. 4. Student CM fractions representation and estimation.

IV. CONCLUSION

Fraction magnitude knowledge is linked to student's mathematics achievement or support each other, while poor mathematics achievement in fraction can be affected by learning difficulties, one of them is mathematics difficulty in fraction. Student fraction difficulty can influence student fraction magnitude knowledge. Student with mathematics difficulty in understanding simple fraction for example $1/4$ is stated by a bar is divided into 5 and a bar is shaded, he could not estimate and represent simple fraction. Likewise student with mathematics difficulty in understanding similar fractions, he could represent fraction but could not estimate them. Student with mathematics difficulty in fractions with different numerator and denominator, or fractions that have common multiple they cannot estimate these fractions in number line. These problems is because they did not understand fraction magnitude therefore they applied procedures without knowing the concept. It had consequences they did wrong in these procedures.

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