

Integrating the Hypothetical Learning Trajectory with Realistic Mathematics to In-Service Teachers' Professional Development

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Abstract— This study aimed at integrating the notion of Hypothetical Learning Trajectory (HLT) of teaching mathematics with Realistic Mathematics approach to in-service teachers' Professional Development. We constructed a teachers' training program to develop teachers' understanding on designing mathematics lesson with Realistic Approach. 60 secondary teachers were participated in the workshop and all have contribution in designing lesson in 20 groups. Most teachers tried to follow the direction to develop lesson with regards students' capacity that direct to one-way direction from teachers in which students' action is based on teachers' action. There were two groups of teachers consider to highlight hands on activity that lead to developing concept by using characteristics of Realistic Mathematics Education. We exemplify the process considering learning goals in lesson on similarities figures and exponential numbers.

Keywords—component, formatting, style, styling, insert (key words)

I. INTRODUCTION

Considerable research in Mathematics Education in the area of developing hypothetical learning trajectory (HLT) to conceptualize how students process through a learning sequence. The terminology Hypothetical Learning Trajectory was introduced primarily by [1] in which it included 'the learning goal, the learning activities, and the thinking and learning that students might engage. The creation of HLT is based on previous understanding of current knowledge of students. Within a Teacher Professional Development (TPD), we tried to develop secondary teachers' knowledge in designing mathematics lesson. The Realistic Mathematics Education (RME) approach that consider mostly on Guided reinvention was used within TPD. Guided reinvention on the mathematical concept has become important perspective in the mathematics education in recent curriculum [2]. This is in line with the particular view of [1] which consider teacher planning process in a constructivist framework. The teachers' competence in designing and doing the teaching process that

consider learners' perspective are regarded as pedagogical competence.

Several resources describe that many Indonesian teachers teach mathematics with mechanistically with drilling and practicing with procedural using algorithm. Therefore, students tend to avoid learning mathematics. Realistic Mathematics Education (RME) is considered as an alternative approach that give opportunity to students to learn mathematics meaningfully which consider students to have active discussion. The possible implementation of RME approach in Indonesia is a new innovation and curriculum for teachers. Freudenthal's view that mathematics as human activity and mathematics must be connected to reality [3]. Reference [4] explained six principles of mathematics teaching each of which reflects specific characteristic of the identity of RME such as Activity principle, Reality principle, Level principle, Intertwinement principle, Interaction principle and Guidance principles. Mathematics must be close to children and relevant to every life situation. With this regards, teachers need to reflect on their initial mathematics teaching.

Teacher Professional Development with Realistic Mathematics Education can be chosen as promising model of innovative teacher professional development to improve the quality of Indonesian mathematics teacher [5]. Many research identified key characteristics of effective teacher training. Several studies focused on the interactions between Teacher Professional Development facilitators and teachers and its impact of teaching such as [6], [7]. Furthermore, reference [8] suggested that teachers need the framework of references, together with the set of exemplary activities that serve as source of inspiration for his own designs. However, the integration of lesson design of Hypothetical Learning Trajectory (HLT) within Teacher Professional Development has not been done and elaborated. In this paper, first, we focus on the initial phase of teacher Professional Development that stimulate the HLT for teachers. Furthermore, we aim to answer the following research question: How is secondary

teachers' competence in designing Hypothetical Learning Trajectory with Realistic Mathematics Education approach?

Hypothetical Learning Trajectory

Based on [9], the Hypothetical Learning Trajectory henceforth HLT is a vehicle for planning the learning of a particular mathematical concept. The creation of the Hypothetical Learning Trajectory is based on the previous understanding of current knowledge of students.

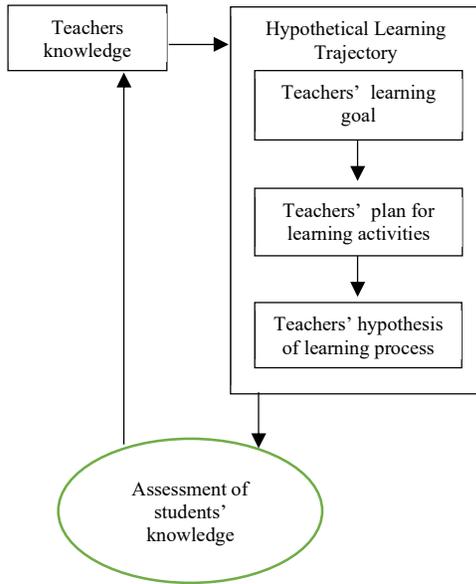


Fig 1. Hypothetical Learning Trajectory

In this study, we emphasized what teachers designed in terms of Hypothetical Learning Trajectory as an external products as [10].

II. METHODS

To obtain data of secondary teachers' competence in designing Hypothetical Learning Trajectory with RME approach, we initially developed validated instrument of teachers' work for lesson design. There were 60 secondary Mathematics teachers participated in the study. They were grouped to 20 groups for Group Discussion. Two groups of teachers were selected as research subject of this study. The Teacher Professional Development (TPD) program that was based on a conceptualization that the students' learning activities should promote students' learning. The cycle of TPD start with the overview of Realistic Mathematics Education approach which considered new for all teacher participants. Reference [11] suggested the opportunity for teacher to explore cultures of school and teacher work. Therefore, time for initial teachers' teaching reflection needs to considered. In addition, we provided the exemplary of mathematics activity with RME approach with the developed learning goal. Within this session, teachers and facilitators discussed how students develop mathematics concept within the activity. With regards developing Hypothetical Learning Trajectory, we created tables that lead teachers to design

students' activity with several hypotheses. The table was used and discussed in a workshop session with groups of teachers.

In General, there were three activities within Teacher Professionalism Development program in this study. First activity is stimulating about Realistic Mathematics Education and its examples. Afterwards, we gave time for teachers to do reflection on their initial teaching. By those activities, the initial mathematics teaching were recorded. The main activity is developing Hypothetical Learning Trajectory (HLT) design with Realistics Mathematics Approach for secondary students. The teachers' design were discussed afterwards. These activities were elaborated in Fig. 2.

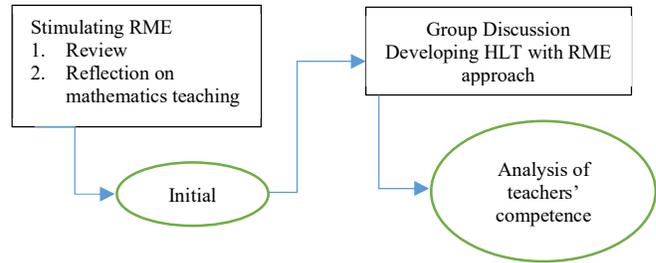


Fig 2. Teacher Professional Development Activities

III. RESULTS

Teachers' worksheet were shared to 20 groups of secondary Mathematics Teachers for their Hypothetical Learning Trajectory design. All participants were actively join and design mathematics lesson with Realistic Mathematics Approach with facilitators' guidance. Most teachers tried to follow the direction to develop lesson with regards students' capacity that direct to one-way direction from teachers in which students' action is based on teachers' action. Two Lesson design for two groups with hypothesis of students' activity were described in this paper. These two design were chosen due to its uniqueness. Initially, these two groups of teachers considered to have strong opinion and beliefs in doing direct instructions in mathematics teaching. After the stimulating activity with theory of RME, they tried to design the mathematics Hypothetical Learning Trajectory by accommodating the theory they received. The design consists of the choice of Mathematics topics and learning goals, Teachers' activity and hypothesis of students' activity. Table 1 shows design of Hypothetical Learning Trajectory (HLT) of one group of Mathematics Teachers with learning goal "Understand the exponential concept".

TABLE I. TEACHER GROUP 1 DESIGN OF HYPOTHETICAL LEARNING TRAJECTORY

Teachers' activity	Hypothesis of students' activity
Teacher prepares a piece of paper and scissors	Several shape of papers were prepared by students such as rectangle, square, triangle
Teacher asks students to fold the paper fairly	- Students do fair paper folding - Students might search symmetry fold first and fold the paper afterwards
Teacher asks students to cut the	Students cut the paper within then axis of symmetry fold and some might not

paper in the axis of symmetry fold	
Teacher ask students to pile up the paper	Student pile the cutting paper and cut within the symmetry fold
Teacher asks students to summarize the result of fi the first, second and so on	Students count all the paper fold Students write all the result of the used of paper for example $2 \times 2 \times 2 \dots \times 2 = 2^n$

By the hypothetical learning trajectory showed in Table 1, teacher 1 considered to lead students to show exponential concept by using hands on activity. The paper folding activity as well as its cutting might give opportunity to students to understand the meaning of 2^n . Different strategy of lesson design showed by group 2 teacher. Table 2 shows design of Hypothetical Learning Trajectory (HLT) of group 2 of Mathematics Teachers with learning goal “Understanding the meaning of two similar planes”.

TABLE II. TEACHER GROUP 2 LESSON DESIGN ON SIMILARITY TOPIC

Teachers' activity	Hypothesis of students' activity
Teacher asks students to observe whether the eternity of roof and the floor tile are similar	Student identify the form of the roof and floor tile are square. The size of the eternity of roof is 100 cm x 100 cm. The size of the floor tile is 30 cm x 30 cm
Teacher asks students identify the angle on the roof and floor tile	Students identify the angle are 90 degrees for every angle
Teacher asks students to compare the side on eternity and floor tile	Student compare the length of eternity and floor tile ($100 : 30 = 10 : 3$) and considered as similar
Teacher asks students to explain why eternity and floor tile are similar	Student might consider to read the students' book and state the two shapes are similar due to the angle and side has same ratio (similar)
Teacher asks students to observe others things such as book and table	Students observe the shape and measure the side of book and table

The concept of similarity was introduced by group of teacher in Table 2 by considering observation of nearby environment that close to students. Teacher would not just give the meaning of similarity directly. Though the steps of design follow the requirement of similarity, students were still be given opportunity to do observation.

With regards the teachers' Hypothetical Learning Trajectory design, teachers tried to oversimplify professional development materials. The two lesson design exemplify the leading of Mathematics' concept development. Furthermore,

based on observation, they have strong opinion and beliefs in self-teaching.

IV. CONCLUSION

Instead of having teacher to follow the direction to develop lesson with regards students' capacity that direct to one-way direction from teachers in which students' action is based on teachers' action, there were two groups of teachers consider to highlight hands on activity and environment observation that lead to develop mathematics concept by using characteristics of Realistic Mathematics Education. The steps of Teacher Professional Development by considering reflection of teachers' daily teaching and introduce Realistic Mathematics Education approach lead teacher. Future study on Teacher Professional Development might lead teacher to deepen, identify and analyze the principles and characteristics of Realistics Mathematics Education (RME) within the lesson design that they developed. Besides, the 'teachers' action' of the classroom can also be the focus of investigation.

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