

MADM Approach for Evaluating Learning Community-Based Programs for Students: IVDHF_UUBLS -VIKOR

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Abstract—Due to the high complexity in the process of learning community-based programs for students, decision makers' assessments for evaluating the programs unavoidably will involve decision hesitancy and uncertainty. Therefore, this paper develops multiple attributes of the decision making (MADM) method. Firstly, the interval-valued dual hesitant fuzzy uncertain unbalanced linguistic set (IVDHF_UUBLS) is utilized for eliciting complicate assessments from decision makers more effectively. Then an IVDHF_UUBLS-VIKOR approach is further proposed by extending the conventional VIKOR method to the IVDHF_UUBLS environments. Furthermore, an illustrative case study has also been conducted to demonstrate our proposed decision making approach.

Keywords—MADM; VIKOR; decision hesitancy; linguistic decision making; learning programs evaluation

I. INTRODUCTION

Learning community-based programs (LCBPs) have been recognized as effective approaches in improving learning effects no matter in class teaching for students or for teachers professional development education [1]. Inevitably situated in educational systems [2], LCBPs are often driven and oriented to topics of macro trends [3,4] and innovative pedagogical frameworks [5,6], which carries new knowledge and involves teachers and students [7]. Thus operational process of LCBPs still generally features teacher, student and content as its instructional core [6]. Apparently, to improve operational performance of teacher professional development, more efforts should also be made to construct appropriate approaches to quality evaluation of LCBPs. However, evaluating LCBPs generally require decision maker (experts) to assess from specific aspects, which intrinsically involves impreciseness and uncertainty. Therefore, appropriate approaches for evaluating LCBPs comprehensively and effectively, that are also capable of accommodating vagueness of assessments including decision hesitancy and uncertainty, need to be well investigated.

When tackling complicate comprehensive evaluation problems, MADM methodologies have shown great suitability

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and adaptability [8], such as, the straightforward MADM frameworks of TOPSIS method and VIKOR method. Especially, in comparison with TOPSIS, VIKOR can derive compromise solution(s) that exhibit closest to the ideal solution and also hold both minimum individual regret and maximum group utility [9]. For addressing vagueness in decision assessments because of increasingly high complexity in real socioeconomic scenarios, MADM methodologies have been enhanced by fuzzy set theories [10,11]. Most recently, the preference expression tool of interval-valued dual hesitant fuzzy uncertain unbalanced linguistic set (IVDHF_UUBLS) introduced by Ref. [12] exhibits more comprehensive and flexible in eliciting decision makers' complicate assessments. Therefore, on the strength of VIKOR and the IVDHF_UUBLS, we extend VIKOR to the IVDHF_UUBLS environments and develop an IVDHF_UUBLS-VIKOR approach for evaluating learning community-based programs for students.

II. BASIC NOTIONS OF IVDHF_UUBLS

Definition 2.1 [12] Let X be a fixed set and S be a continuous unbalanced linguistic label set. Then IVDHF_UUBLS SD on X is defined as

$$SD = \left\{ \left\langle x, \tilde{s}_{g(x)}, \tilde{h}(x), \tilde{g}(x) \right\rangle \mid x \in X \right\},$$

where $\tilde{s}_{g(x)} = [s_\alpha, s_\beta]$ represents judgment to object x , s_α and s_β are unbalanced linguistic variables from set S which denotes experts' assessments on the object X under evaluation. $\tilde{h}(x) = \cup \{ \tilde{\mu} \} = \bigcup_{\{\mu^L, \mu^U\} \in \tilde{h}(x)} \{ [\mu^L, \mu^U] \}$ and $\tilde{g}(x) = \cup \{ \tilde{\nu} \} = \bigcup_{\{\nu^L, \nu^U\} \in \tilde{g}(x)} \{ [\nu^L, \nu^U] \}$ are given sets of closed intervals from $[0,1]$. $\tilde{h}(x)$ collects membership degrees of x to $\tilde{s}_{g(x)}$, and $\tilde{g}(x)$ collects non-membership degrees of x to $\tilde{s}_{g(x)}$. $\tilde{h}(x)$ and $\tilde{g}(x)$ have: $\tilde{\mu}, \tilde{\nu} \in [0,1]$ and $0 \leq (\mu^U)^+ + (\nu^U)^+ \leq 1$, where, for all $x \in X$, $(\mu^U)^+ \in \tilde{h}^+(x) = \bigcup_{\{\mu^L, \mu^U\} \in \tilde{h}(x)} \max \{ \mu^U \}$, $(\nu^U)^+ \in \tilde{g}^+(x) = \bigcup_{\{\nu^L, \nu^U\} \in \tilde{g}(x)} \max \{ \nu^U \}$.

Certain element in the set SD is normally called an interval-valued dual hesitant fuzzy uncertain unbalanced linguistic number (IVDHF_UUBLN).

Definition 2.2 [12] Given an IVDHF_UUBLN $sd = (\tilde{s}_g, \tilde{h}, \tilde{g})$, $\tilde{s}_g = [s_{\alpha}, s_{\beta}]$, then score function $S(sd)$ and accuracy function $P(sd)$ can be defined as follows

$$S(sd) = \frac{\Delta_{t_0}^{-1}(TF_{t_0}^{t_{k_1}}(\psi(s_{\alpha}))) + \Delta_{t_0}^{-1}(TF_{t_0}^{t_{k_2}}(\psi(s_{\beta})))}{2} \times \frac{1}{2} \left(\frac{1}{l(\tilde{h})} \sum_{[\mu^L, \mu^U] \in \tilde{h}} \mu^L - \frac{1}{l(\tilde{g})} \sum_{[v^L, v^U] \in \tilde{g}} v^L + \frac{1}{l(\tilde{h})} \sum_{[\mu^L, \mu^U] \in \tilde{h}} \mu^U - \frac{1}{l(\tilde{g})} \sum_{[v^L, v^U] \in \tilde{g}} v^U \right), \quad (1)$$

$$P(sd) = \frac{\Delta_{t_0}^{-1}(TF_{t_0}^{t_{k_1}}(\psi(s_{\alpha}))) + \Delta_{t_0}^{-1}(TF_{t_0}^{t_{k_2}}(\psi(s_{\beta})))}{2} \times \frac{1}{4} \left(\frac{1}{l(\tilde{h})} \sum_{[\mu^L, \mu^U] \in \tilde{h}} \mu^L + \frac{1}{l(\tilde{g})} \sum_{[v^L, v^U] \in \tilde{g}} v^L + \frac{1}{l(\tilde{h})} \sum_{[\mu^L, \mu^U] \in \tilde{h}} \mu^U + \frac{1}{l(\tilde{g})} \sum_{[v^L, v^U] \in \tilde{g}} v^U \right), \quad (2)$$

where $l(\tilde{h})$ and $l(\tilde{g})$ denotes numbers of elements in \tilde{h} and \tilde{g} respectively; t_{k_1} and t_{k_2} are indicate levels of s_{α} and s_{β} in a linguistic hierarchy; t_0 indicates the maximum level of t_k .

Definition 2.3 [12] Given any two elements of $sd_1 = (\tilde{s}_{g_1}, h_1, g_1)$ and $sd_2 = (\tilde{s}_{g_2}, h_2, g_2)$, we have

- (1) If $S(sd_1) < S(sd_2)$, then $sd_1 \prec sd_2$.
- (2) If $S(sd_1) = S(sd_2)$, then
 - (a) If $P(sd_1) < P(sd_2)$, then $sd_1 \prec sd_2$;
 - (b) If $P(sd_1) = P(sd_2)$, then $sd_1 \sim sd_2$.

Definition 2.4 Let $sd_1 = (\tilde{s}_{g_1}, \tilde{h}_1, \tilde{g}_1)$ and $sd_2 = (\tilde{s}_{g_2}, \tilde{h}_2, \tilde{g}_2)$, where $\tilde{s}_{g_1} = [s_{\alpha_1}, s_{\beta_1}]$, $\tilde{s}_{g_2} = [s_{\alpha_2}, s_{\beta_2}]$. $l_{\tilde{h}_1}$, $l_{\tilde{h}_2}$, $l_{\tilde{g}_1}$ and $l_{\tilde{g}_2}$ denotes lengths of \tilde{h}_1 , \tilde{h}_2 , g_1 and g_2 . Suppose

$$I_1 = \frac{1}{n(t_{i_1}) - 1} \Delta_{t_0}^{-1}(TF_{t_0}^{t_{i_1}}(\psi(s_{\alpha_1}))),$$

$$I_2 = \frac{1}{n(t_{i_2}) - 1} \Delta_{t_0}^{-1}(TF_{t_0}^{t_{i_2}}(\psi(s_{\beta_1}))),$$

$$I_3 = \frac{1}{n(t_{j_1}) - 1} \Delta_{t_0}^{-1}(TF_{t_0}^{t_{j_1}}(\psi(s_{\alpha_2}))),$$

$$I_4 = \frac{1}{n(t_{j_2}) - 1} \Delta_{t_0}^{-1}(TF_{t_0}^{t_{j_2}}(\psi(s_{\beta_2}))),$$

where t_{i_1} , t_{i_2} , t_{j_1} and t_{j_2} respectively indicate levels of s_{α_1} , s_{α_2} , s_{β_1} and s_{β_2} in a certain linguistic hierarchy. Then based on the widely adopted

normalized Hamming distance, we define a distance measure d for IVDHF_UUBLS as follows,

Situation 1. When $l_{\tilde{h}_1} = l_{\tilde{h}_2} = l_1$ and $l_{\tilde{g}_1} = l_{\tilde{g}_2} = l_2$, then

$$D = \left(\frac{1}{2} \left(\frac{1}{l_1} \sum_{k=1}^{l_1} \left(\left| I_1 \mu_{\tilde{h}_1}^{L_j} - I_3 \mu_{\tilde{h}_2}^{L_k} \right|^2 + \left| I_2 \mu_{\tilde{h}_1}^{U_j} - I_4 \mu_{\tilde{h}_2}^{U_k} \right|^2 \right) + \frac{1}{l_2} \sum_{k=1}^{l_2} \left(\left| I_1 v_{\tilde{g}_1}^{L_j} - I_3 v_{\tilde{g}_2}^{L_k} \right|^2 + \left| I_2 v_{\tilde{g}_1}^{U_j} - I_4 v_{\tilde{g}_2}^{U_k} \right|^2 \right) \right) \right)^{\frac{1}{2}}; \quad (3)$$

Situation 2. When $l_{\tilde{h}_1} \neq l_{\tilde{h}_2}$ and $l_{\tilde{g}_1} \neq l_{\tilde{g}_2}$, then

$$D = \left(\frac{1}{2} \left(\frac{1}{l_{\tilde{h}_1} l_{\tilde{h}_2}} \sum_{j=1}^{l_{\tilde{h}_1}} \sum_{k=1}^{l_{\tilde{h}_2}} \left(\left| I_1 \mu_{\tilde{h}_1}^{L_j} - I_3 \mu_{\tilde{h}_2}^{L_k} \right|^2 + \left| I_2 \mu_{\tilde{h}_1}^{U_j} - I_4 \mu_{\tilde{h}_2}^{U_k} \right|^2 \right) + \frac{1}{l_{\tilde{g}_1} l_{\tilde{g}_2}} \sum_{j=1}^{l_{\tilde{g}_1}} \sum_{k=1}^{l_{\tilde{g}_2}} \left(\left| I_1 v_{\tilde{g}_1}^{L_j} - I_3 v_{\tilde{g}_2}^{L_k} \right|^2 + \left| I_2 v_{\tilde{g}_1}^{U_j} - I_4 v_{\tilde{g}_2}^{U_k} \right|^2 \right) \right) \right)^{\frac{1}{2}}. \quad (4)$$

Theorem 2.1. The distance measure D that defined in the above Definition 2.4 holds the properties below:

- (a) $0 \leq D(sd_1, sd_2) \leq 1$;
- (b) $D(sd_1, sd_2) = 0$ if and only if sd_1 and sd_2 are exactly the same;
- (c) $D(sd_1, sd_2) = D(sd_2, sd_1)$.

III. EXTENDED VIKOR APPROACH FOR MCDM WITH IVDHF_UUBLS INFORMATION

Let $A = \{A_1, A_2, \dots, A_n\}$ be a set of alternatives, $C = \{C_1, C_2, \dots, C_m\}$ be a set of criteria. $\omega = (\omega_1, \omega_2, \dots, \omega_m)^T$ is the weighting vector for C , where $\omega_j \geq 0$ and $\sum_{j=1}^m \omega_j = 1$. Suppose that $R = (r_{ij})_{n \times m}$ is the decision matrix based on information form of unbalanced linguistic term set S , where r_{ij} is an IVDHF_UUBLN. The procedure of IVDHF_UUBLS-VIKOR is shown as follows.

Step 1. Determine criteria weighting vector associated with decision matrix, where $\omega = \{\omega_1, \dots, \omega_j, \dots, \omega_m\}$ is obtained by deviation maximizing method:

$$\omega_j = \frac{\sum_{k=1}^n d(r_{ij}, r_{kj})}{\sum_{i=1}^n \sum_{k=1}^n d(r_{ij}, r_{kj})}, \quad (5)$$

Step 2. Obtain f_i^* and f_i^- for all attributes ratings in decision matrix, where

$$f_j^* = \max_i r_{ij}, \quad f_j^- = \min_i r_{ij}. \quad (6)$$

where $\max_i r_{ij}$ and $\min_i r_{ij}$ are derived by use of Eq. (1).

Step 3. Calculate normalized fuzzy distance d_{ij} :

$$d_{ij} = \frac{d(f_j^*, r_{ij})}{d(f_j^*, f_j^-)}, \tag{7}$$

where $d(f_j^*, r_{ij})$ and $d(f_j^*, f_j^-)$ are calculated by the distance measure D .

Step 4. Obtain t group utility S_i and maximal regret R_i according to

$$S_i = \sum_{j=1}^m \omega_j d_{ij}, j = 1, 2, \dots, m, \tag{8}$$

$$R_i = \max_j \omega_j d_{ij}, j = 1, 2, \dots, m, \tag{9}$$

where ω_j is ensured in Step 1;

Step 5. Obtain the index values Q_i by

$$Q_i = v \frac{S_i - S^*}{S^- - S^*} + (1-v) \frac{R_i - R^*}{R^- - R^*}, \tag{10}$$

where $S^* = \min_i S_i, S^- = \max_i S_i, R^* = \min_i R_i, R^- = \max_i R_i$.

Step 6. Sort S, R and Q in descending order to generate three ranked lists.

Step 7. Propose a compromise solution of $A^{(1)}$ which is the best ranked by the measure Q ; or to determine compromise solutions according to rules given in the conventional VIKOR method.

IV. ILLUSTRATIVE CASE STUDY

Currently in universities all over the world, learning community-based teaching programs have been widely adopted and developed to enhance traditional teaching models. To continuously improve the operational quality of those programs, the task of comprehensive evaluation has been an imperative part of teaching management practices in universities. As suggested by [6], teacher, student and content still are the instructional core of LCBPs. Therefore, according to processes of LCBPs, we here take three attributes to comprehensively evaluate LCBPs: content arrangement (C_1), instructional design (C_2) and learning community (C_3).

TABLE I. DECISION MATRIX WITH IVDHF_UUBLNS

	C_1	C_2	C_3
A_1	{([AL,M],[0.2,0.3]), {[0.2,0.4],[0.3,0.4])}	{([QM,VH],[0.4,0.5],[0.5,0.6]),{[0.2,0.3])}	{([M,QM],[0.1,0.3]) ,{[0.6,0.7])}
A_2	{([AN,QL],[0.5,0.6]),{ [0.2,0.3])}	{([VL,L],[0.6,0.7]), {[0.1,0.2])}	{([M,H],[0.2,0.3]), {[0.5,0.6],[0.6,0.7])}
A_3	{([QL,M],[0.3,0.4]), {[0.4,0.5],[0.5,0.6])}	{([AL,VH],[0.6,0.7],[0.7,0.8]),{[0.1,0.2])}	{([VL,QL],[0.4,0.5] ,{[0.2,0.3])}
A_4	{([VL,QL],[0.2,0.4]) ,{[0.5,0.6])}	{([H,VH],[0.3,0.5]), {[0.2,0.3])}	{([VH,T],[0.6,0.7]), {[0.1,0.2],[0.2,0.3])}
A_5	{([QL,M],[0.4,0.5]), {[0.1,0.2],[0.4,0.5])}	{([AL,QM],[0.6,0.7]), {[0.1,0.2])}	{([QL,QM], {[0.4,0.5],[0.6,0.7]), {[0.1,0.3])}

TABLE II. RANKED RESULTS AND THE COMPROMISE SOLUTIONS FOR ALL ALTERNATIVES

	Alternatives					Ranking Orders	Compromise Solution
	x_1	x_2	x_3	x_4	x_5		
S	0.81	0.83	0.48	0.97	0.27	$x_4 \succ x_2 \succ x_1$ $\succ x_3 \succ x_5$	x_4
R	0.55	0.54	0.31	0.75	0.14	$x_4 \succ x_1 \succ x_2$ $\succ x_3 \succ x_5$	x_4
Q ($v=0.5$)	0.716	0.724	0.287	1	0	$x_4 \succ x_2 \succ x_1$ $\succ x_3 \succ x_5$	x_4

Suppose that a panel of teaching affairs supervisors have been organized to evaluate five alternative LCBPs: A_1, A_2, A_3, A_4 and A_5 by considering the above three attributes. Due to the complexity during the task, the experts generally will have decision hesitancy and uncertainty in expressing their assessments. Thus the proposed IVDHF_UUBLNS-VIKOR approach is adopted and applied to solve the task. The corresponding linguistic variables in $R = (r_{ij})_{5 \times 3}$ are chosen from an unbalanced linguistic term set S , where $S = \{N, AN, VL, QL, L, M, QM, H, VH, T\}$. Table I collects assessments by the supervisor panel. The details of computation are listed in the following.

Step 1. Determine weighting vector by Eq. (5), we have

$$\omega = (0.17, 0.3, 0.53).$$

Step 2. Determine the values of best f_i^* and the worst f_i^- for all criteria ratings in decision matrix, where

$$f_1^* = r_{21}, f_1^- = r_{31}; f_2^* = r_{32}, f_2^- = r_{22}; f_3^* = r_{53}, f_3^- = r_{23}.$$

Step 3. Calculate normalized fuzzy distance d_{ij} by Eq. (7), where

$$d_{11}=0.76, d_{21}=0, d_{31}=1, d_{41}=0.37, d_{51}=0.83;$$

$$d_{12}=0.46, d_{22}=1, d_{32}=0, d_{42}=0.55, d_{52}=0.43;$$

$$d_{13}=1, d_{23}=1, d_{33}=0.57, d_{43}=1.34, d_{53}=0.$$

Step 4. Determine the values of group utility S_i and maximal regret R_i according to Eqs. (8)-(9):

$$S_1 = 0.81, S_2 = 0.83, S_3 = 0.48, S_4 = 0.97, S_5 = 0.27;$$

$$R_1 = 0.55, R_2 = 0.54, R_3 = 0.31, R_4 = 0.75, R_5 = 0.14.$$

Step 5. Suppose $v = 0.5$, then we can obtain Q_i ($i = 1, 2, 3, 4, 5$) by Eq. (10):

$$Q_1 = 0.7159, Q_2 = 0.724, Q_3 = 0.287, Q_4 = 1, Q_5 = 0.$$

Step 6. Rank the alternatives according to the, respectively. Get three ranked lists as collected in Table II by the descending order of S, R and Q .

Step 7. According to VIKOR method, x_4 is thus the unique compromise solution.

V. CONCLUSION

For tackling the complicate decision making problems of evaluating learning community-based programs for students, we have developed an IVDHF_UUBLS –VIKOR approach as an indispensable extension of classic VIKOR framework. The proposed IVDHF_UUBLS –VIKOR approach is capable of processing the evaluation task in a straightforward manner and concurrently accommodating practical complicate situations where decision makers are inclined to have decision hesitancy and decision uncertainty in denoting their preferences. For future study, research direction could be oriented to wide applications to various management fields.

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