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Almost Automorphic Solutions to Cellular Neural Networks With Neutral Type Delays and Leakage Delays on Time Scales

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ABSTRACT

In this paper, cellular neural networks (CNNs) with neutral type delays and time-varying leakage delays are investigated. By applying the existence of the exponential dichotomy of linear dynamic equations on time scales, a fixed point theorem and the theory of calculus on time scales, a set of sufficient conditions which ensure the existence and exponential stability of almost automorphic solutions of the model are obtained. An example with its numerical simulations is given to support the theoretical findings.

1. INTRODUCTION

Since the cellular neural networks with delay were first introduced and investigated by Roska and Chua [1], they have been extensively applied in various different fields such as classification of pattern and processing of moving images. In recent years, extensive results on the existence and stability of equilibrium points, periodic solutions, almost periodic solutions and anti-periodic solutions for cellular neural networks have been reported. For example, Fan and Shao [2] investigated the positive almost periodic solutions for shunting inhibitory cellular neural networks with time-varying and continuously distributed delays, Li and Wang [3] analyzed the existence and exponential stability of the almost periodic solutions of shunting inhibitory cellular neural networks on time scales, Xia *et al.* [4] established the sufficient conditions for the existence and exponential stability of almost periodic solution for shunting inhibitory cellular neural networks with impulses, Peng and Wang [5] addressed the existence and exponential stability of anti-periodic solutions to shunting inhibitory cellular neural networks with time-varying delays in leakage terms. For more related work on shunting inhibitory cellular neural networks, one can see [4,6–17].

Many scholars [18–21] argue that neural networks usually contain some information about the derivative of the past state to further describe and model the dynamics for the complex neural reactions. Then some authors focused on the dynamical behaviors of neutral type neural networks. For example, Rakkiyappan *et al.* [22] considered the global exponential stability for neutral-type impulsive neural networks, Li *et al.* [23] discussed the existence of periodic solutions for neutral type cellular neural networks with delays, Bai [24] investigated the global stability of almost periodic solutions of Hopfield neural networks with neutral time-varying delays. In details, we refer the reader to [25–30].

Very recently, a typical time delay called Leakage (or "forgetting") delay may exist in the negative feedback terms of the neural network system, and these terms are variously known as forgetting or leakage terms [31,32]. Since time delays in the leakage term are difficult to handle but have great impact on the dynamical behavior of neural networks. Therefore, it is meaningful to consider neural networks with time delays in leakage terms [34].

It is well known that both continuous time and discrete time neural networks play an equal roles in various applications [34]. But it is troublesome to study the dynamical properties for continuous and discrete time systems, respectively. In 1990, Hilger [35] proposed the theory of time scales which can deal with both difference and differential calculus in a consistent way. Thus it is significant to investigate the dynamical behaviors of neural networks on time scales. For instance, some authors [3,36–40] investigated periodic solutions, almost periodic solutions and anti-periodic solutions of some neural networks on time scales.

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In addition, we shall point out that in real word, almost periodicity is universal than periodicity. Moreover, almost automorphic functions, which were introduced by Bochner, are much more general than almost periodic functions. In addition, the almost automorphic solutions of neural networks can be applied in many areas such as automatic control, image processing, psychophysics, robotics and so on [41–45]. Almost automorphic solutions in the context of differential equations were studied by several authors. We refer the reader to [46–53]. However, to the best of our knowledge, there is no paper published on the almost automorphic solutions of cellular neural networks with neutral type delays and time-varying leakage delays on time scales.

Inspired by the discussion above, in this paper, we consider the following cellular neural networks with neutral type delays and time-varying leakage delays on time scales

$$\begin{aligned} x_i^{\Delta}(t) = & -b_i(t)\,x_i\left(t-\eta_i(t)\right)+\sum_{j=1}^{n} a_{ij}(t)\,f_j\left(x_j(t)\right) \\ & +\sum_{j=1}^{n} b_{ij}(t)\,f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right) \\ & +\sum_{j=1}^{n} c_{ij}(t)\,f_j\left(x_i^{\Delta}\left(t-\sigma_{ij}(t)\right)\right)+I_i(t), \end{aligned} \tag{1}$$

where $\mathbb{T}$ is an almost periodic time scale, $i = 1, 2, \cdots, n$, n corresponds to the number of units in a neural network, x_i corresponds to the state vector of the ith unit at time t, $f_j\left(x_j(t)\right)$ denotes the output of the jth unit on ith unit at time t, b_{ij} denotes the strength of the jth unit on the ith unit at time $t-\tau_{ij}$, I_i denotes the external bias on the ith unit at time t, τ_{ij} corresponds to the transmission delay along the axon of the jth unit, b_i represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs, $\eta_i > 0$ with $t-\eta_i(t) \in \mathbb{T}$ for all $t \in \mathbb{T}$ denote s time delay in leakage term, $\tau_{ij}(t) \geq 0, \sigma_{ij}(t) \geq 0$ correspond the transmission delays and satisfy that for $t \in \mathbb{T}, t-\tau_{ij} \in \mathbb{T}, t-\sigma_{ij} \in \mathbb{T}$.

The main aim of this article is to establish some sufficient conditions for the existence and global exponential stability of almost automorphic periodic solutions of (1). By applying the existence of the exponential dichotomy of linear dynamic equations on time scales, a fixed point theorem and the theory of calculus on time scales, we obtain a set of sufficient conditions for the existence and exponential stability of almost automorphic solutions for model (1).

For convenience, we denote by $[a,b]_{\mathbb{T}} = \{t | t \in [a,b] \cap \mathbb{T}\}$. For a almost automorphic function $f\colon \mathbb{T} \to \mathbb{R}$, $f^+ = \sup_{t\in\mathbb{R}} |f(t)|$, $f^- = \inf_{t\in\mathbb{R}} |f(t)|$. We denote by $\mathbb{R}$ the set of real numbers, by $\mathbb{R}^+$ the set of positive real numbers, by $\mathbb{X}$ a real Banach space with the norm $||.||$. The initial conditions associated with system (1) are of the form:

$$x_i(s) = \varphi_i(s),\, x_i^{\Delta}(s) = \varphi_i^{\Delta}(s),\, s \in (-\tau, 0]_{\mathbb{T}}, \tag{2}$$

where $\tau = max\left\{max_{1\leq i\leq n}\eta_i^+, max_{(i,j)}\left\{\tau_{ij}^+, \sigma_{ij}^+\right\}\right\}, \varphi_i \in C^1([-\tau,0]_{\mathbb{T}}, \mathbb{R})$ and $i, j = 1, 2, \cdots, n$.

The remainder of the paper is organized as follows. In Section 2, we introduce some lemmas and definitions, which can be used to check the existence of almost automorphic solutions of system (1). In Section 3, we present some sufficient conditions for the existence of almost automorphic solutions of (1). Some sufficient conditions on the global exponential stability of almost automorphic solutions of (1) are established in Section 4. An example is given to illustrate the effectiveness of the obtained results in Section 5. A brief conclusion is drawn in Section 6.

2. PRELIMINARY RESULTS

In this section, we would like to recall some basic definitions and lemmas which are used in what follows.

Definition 2.1. [54] *Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}$ are defined, respectively, by*

$$\begin{aligned} \sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \rho(t) \\ = \sup\{s \in \mathbb{T} : s < t\} \text{ and } \mu(t) = \sigma(t) - t. \end{aligned}$$

Lemma 2.1. [54] *Assume that $p, q : \mathbb{T} \to \mathbb{R}$ are two regressive functions, then*

i. $e_0(t,s) \equiv 1$ *and* $e_p(t,t) \equiv 1$;

ii. $e_p(t,s) = \frac{1}{e_p(s,t)} = e_{\ominus p}(s,t)$;

iii. $e_p(t,s)e_p(s,r) = e_p(t,r)$;

iv. $(e_p(t,s))^{\Delta} = p(t)e_p(t,s)$.

Lemma 2.2. [54] *Let f, g be Δ-differentiable functions on $\mathbb{T}$, then*

i. $(\nu_1 f + \nu_2 g)^{\Delta} = \nu_1 f^{\Delta} + \nu_2 g^{\Delta}$, *for any constants* ν_1, ν_2;

ii. $(fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f(\sigma(t))g^{\Delta}(t) = f(t)g^{\Delta}(t) + f^{\Delta}(t)g(\sigma(t))$.

Lemma 2.3. [54] *Assume that $p(t) \geq 0$ for $t \geq s$, then $e_p(t,s) \geq 1$.*

Definition 2.2. [54] *A function $p : \mathbb{T} \to \mathbb{R}$ is called regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}^k$; $p : \mathbb{T} \to \mathbb{R}$ is called positively regressive provided $1 + \mu(t)p(t) > 0$ for all $t \in \mathbb{T}^k$. The set of all regressive and rd-continuous functions $p : \mathbb{T} \to \mathbb{R}$ will be denoted by $\boldsymbol{R} = \boldsymbol{R}(\mathbb{T}, \mathbb{R})$ and the set of all positively regressive functions and rd-continuous functions will be denoted by $\boldsymbol{R}^+ = \boldsymbol{R}^+(\mathbb{T}, \mathbb{R})$.*

Lemma 2.4. [54] *Suppose that $p \in \boldsymbol{R}^+$, then*

i. $e_p(t,s) > 0$, *for all* $t, s \in \mathbb{T}$;

ii. *if* $p(t) \leq q(t)$ *for all* $t \geq s$, *then* $e_p(t,s) \leq e_q(t,s)$ *for all* $t \geq s$.

Lemma 2.5. [54] *If $p \in \boldsymbol{R}$ and $a, b, c \in \mathbb{T}$, then*

$$\left[e_p(c,.)\right]^{\Delta} = -p\left[e_p(c,.)\right]^{\sigma}$$

and

$$\int_a^b p(t)\,e_p(c,\sigma(t))\,\Delta t = e_p(c,a) - e_p(c,b).$$

Lemma 2.6. [54] *Let $a \in \mathbb{T}^k, b \in \mathbb{T}$ and assume that $f : \mathbb{T}\times\mathbb{T}^k \to \mathbb{R}$ is continuous at (t,t) where $t \in \mathbb{T}^k$ with $t > a$. Also assume that $f^{\Delta}(t)$*

is rd-continuous on $[a, \sigma(t)]$. *Suppose that for each* $\varepsilon > 0$, *there exists a neighborhood* U *of* $\varepsilon \in [a, \sigma(t)]$ *such that*

$$\begin{aligned}&|f(\sigma(t), \epsilon) - f(s, \epsilon) - f^{\Delta}(t, \epsilon)(\sigma(t) - s)| \\ &\leq \varepsilon|\sigma(t) - s|, \quad \text{for all} \quad s \in U,\end{aligned}$$

where f^{Δ} *denotes the derivative of* f *with respect to the first variable. Then*

i. $g(t) := \int_a^t f(t, \epsilon)\Delta\epsilon$ *implies* $g^{\Delta}(t) := \int_a^t f^{\Delta}(t, \epsilon)\Delta\epsilon + f(\sigma(t), t)$;

ii. $h(t) := \int_t^b f(t, \epsilon)\Delta\epsilon$ *implies* $h^{\Delta}(t) := \int_t^b f^{\Delta}(t, \epsilon)\Delta\epsilon - f(\sigma(t), t)$.

Next, we recall some definitions of almost automorphic functions on time scales.

Definition 2.3. [55] *A time scale* $\mathbb{T}$ *is called an almost periodic time scale if*

$$\Pi := \{\epsilon \in \mathbb{R} : t \pm \epsilon \in \mathbb{T}, \forall t \in \mathbb{T} \neq \{0\}.$$

Definition 2.4. [54] *Let* $\mathbb{T}$ *be an almost periodic time scale.*

i. *A function* $f(t) : \mathbb{T} \to \mathbb{X}$ *is said to be almost automorphic, if for any sequence* $\{s_n\}_{n=1}^{\infty} \subset \Pi$, *there is a subsequence* $\{\epsilon_n\}_{n=1}^{\infty} \subset \{s_n\}_{n=1}^{\infty}$ *such that* $g(t) = \lim_{n\to\infty} f(t + \epsilon_n)$ *is well defined for each* $t \in \mathbb{T}$ *and* $\lim_{n\to\infty} g(t - \epsilon_n) = f(t)$ *for each* $t \in \mathbb{T}$. *Denote by* $AA(\mathbb{T}, \mathbb{X})$ *the set of all such functions*;

ii. *A continuous function* $f : \mathbb{T} \times \mathbb{X} \to \mathbb{X}$ *is said to be almost automorphic, if* $f(t, x)$ *is almost automorphic in* $t \in \mathbb{T}$ *uniformly in* $x \in \mathbb{B}$, *where* $\mathbb{B}$ *is any bounded subset of* $\mathbb{X}$. *Denote by* $AA(\mathbb{T} \times \mathbb{X}, \mathbb{X})$ *the set of all such functions.*

Lemma 2.7. [53] *Let* $f, g \in AA(\mathbb{T}, \mathbb{X})$. *Then we have the following*

i. $f + g \in AA(\mathbb{T}, \mathbb{X})$;

ii. $\alpha \in AA(\mathbb{T}, \mathbb{X})$ *for any constant* $\alpha \in \mathbb{R}$;

iii. *if* $\varphi : \mathbb{X} \to \mathbb{Y}$ *is a continuous function, then the composite function* $\varphi \circ f : \mathbb{T} \to \mathbb{Y}$ *is almost automorphic.*

Lemma 2.8. [34] *Let* $f \in AA(\mathbb{T} \times \mathbb{X}, \mathbb{X})$ *and* f *satisfies the Lipschitz condition in* $x \in \mathbb{X}$ *uniformly in* $t \in \mathbb{T}$. *If* $\varphi \in AA(\mathbb{T}, \mathbb{X})$, *then* $f(t, \varphi(t))$ *is almost automorphic.*

Definition 2.5. [55] *Let* $x \in \mathbb{R}^n$ *and* $A(t)$ *be a* $n \times n$ *matrix-valued function on* $\mathbb{T}$, *the linear system*

$$x^{\Delta}(t) = A(t)x(t), t \in \mathbb{T} \tag{3}$$

is said to admit an exponential dichotomy on $\mathbb{T}$ *if there exist positive constants* $k_i, \alpha_i, i = 1, 2$, *projection* P *and the fundamental solution matrix* $X(t)$ *of (3) satisfying*

$$|X(t)PX^{-1}(s)| \leq k_1 e_{\ominus\alpha_1}(t, s), s, t \in \mathbb{T}, t \geq s$$

and

$$|X(t)(I - P)X^{-1}(s)| \leq k_2 e_{\ominus\alpha_2}(t, s), s, t \in \mathbb{T}, t \leq s,$$

where $|.|$ *is a matrix norm on* $\mathbb{T}$, *that is, if* $A = (a_{ij})_{n\times n}$, *then we can take* $|A| = \left(\sum_{i=1}^{n}\sum_{j=1}^{n}|a_{ij}|^2\right)^{\frac{1}{2}}$.

Lemma 2.9. [53] *Suppose that* $A(t) \in AA(\mathbb{T}, \mathbb{R}^{n\times n})$ *such that* $\{A^{-1}(t)\}_{t\in\mathbb{T}}$ *and* $\{((I + \mu(t))A(t))^{-1}\}_{t\in\mathbb{T}}$ *are bounded. Moreover, suppose that* $g \in AA(\mathbb{T}, \mathbb{R}^n)$ *and (3) admits an exponential dichotomy, then the following system*

$$x^{\Delta}(t) = A(t)x(t) + g(t) \tag{4}$$

has a solution $x(t) \in AA(\mathbb{T}, \mathbb{R}^n)$ *and* $x(t)$ *is expressed as follows*

$$\begin{aligned}x(t) = &\int_{-\infty}^{t} X(t)PX^{-1}(\sigma(s))g(s)\Delta s \\ &- \int_{t}^{+\infty} X(t)(I - P)X^{-1}(\sigma(s))g(s)\Delta s,\end{aligned}$$

where $X(t)$ *is the fundamental solution matrix of (3),* I *denotes the* $n \times n$*-identity matrix.*

Lemma 2.10. [55] *Let* $c_i > 0$ *and* $-c_i(t) \in \boldsymbol{R}^+, \forall t \in \mathbb{T}$. *If* $\min_{1\leq i\leq n}\{\inf_{t\in\mathbb{T}} c_i(t)\} = m > 0$, *then the linear system*

$$x^{\Delta}(t) = diag(-c_1(t), -c_2(t), \cdots, -c_n(t))x(t) \tag{5}$$

admits an exponential dichotomy on $\mathbb{T}$.

Definition 2.6. [34] *Let* $x^*(t) = (x_1^*(t), x_2^*(t), \cdots, x_n^*(t))^T$ *be an almost automorphic solution of (1) with initial value* $\varphi^*(t) = (\varphi_1^*(t), \varphi_2^*(t), \cdots, \varphi_n^*(t)^T$. *If there exist positive constants* λ *with* $\ominus\lambda \in \boldsymbol{R}^+$ *and* $M > 1$ *such that for an arbitrary solution* $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ *of (1) with initial value* $\varphi(t) = (\varphi_1(t), \varphi_2(t), \cdots, \varphi_n(t))^T$ *satisfies*

$$||x - x^*|| \leq M||\varphi - \varphi^*||e_{\ominus\lambda}(t, t_0), t_0 \in [-\tau, \infty)_{\mathbb{T}}, t \geq t_0.$$

Then the solution $x^*(t)$ *is said to be globally exponentially stable.*

3. EXISTENCE OF ALMOST AUTOMORPHIC SOLUTIONS

In this section, we will establish sufficient conditions on the existence of pseudo almost periodic solutions of (1). Let $X^* = \{f \in C^1(\mathbb{T}, \mathbb{R}|f, f^{\Delta} \in AA(\mathbb{T}, \mathbb{R}^n)\}$ with the norm $||f||_{X^*} = \max\{|f|_1, |f^{\Delta}|_1\}$, where $|f|_1 = \max_{1\leq i\leq n} f_i^+, |f^{\Delta}|_1 = \max_{1\leq i\leq n}(f_i^{\Delta})^+$. Then X^* is a Banach space. Let $\varphi^0(t) = (\varphi_1^0(t), \varphi_2^0(t), \cdots, \varphi_n^0(t))^T$, where $\varphi_i^0(t) = \int_{-\infty}^{t} e_{-b_i}(t, \sigma(s)I_i(s)\Delta s)$, $i = 1, 2, \cdots, n$ and L be a constant satisfying $L \geq \max\{||\varphi^0||_{X^*}, \max_{1\leq i\leq n}\{f_j(0)\}\}$. Throughout this article, we assume that

(H1) $b_i \in C(\mathbb{T}, \mathbb{R}^+)$ with $-b_i \in \boldsymbol{R}^+$ and $\inf_{t\in\mathbb{T}}\{1 - \mu(t)b_i(t)\} = \overline{b} > 0, a_{ij}, b_{ij}, c_{ij}, I_i \in C(\mathbb{T}, \mathbb{R}), \tau_{ij}, \sigma_{ij} \in C(\mathbb{T}, \mathbb{R}^+)$ are almost automorphic, where $i, j = 1, 2, \cdots, n$.

(H2) $f_j \in C(\mathbb{R}, \mathbb{R})$ and there exist constants $L_j > 0$ and $M_j > 0$ such that for any $u, v \in \mathbb{R}$,

$$|f_j(u) - f_j(v)| \leq L_j|u - v|, |f_j(u)| \leq M_j,$$

where $j = 1, 2, \cdots, n$.

(H3)

$$\max_{1\le i\le n}\left\{\frac{\varrho_i}{b_i^-}, \left(1+\frac{b_i^+}{b_i^-}\right)\varrho_i\right\} \le \frac{1}{2}, \max_{1\le i\le n}\left\{\frac{\varsigma_i}{b_i^-}, \left(1+\frac{b_i^+}{b_i^-}\right)\varsigma_i\right\} \le 1$$

where

$$\varrho_i = b_i^+\eta_i^+ + \sum_{j=1}^n \left[\left(a_{ij}^+ + b_{ij}^+ + c_{ij}^+\right)(L_j+1)\right],$$
$$\varsigma_i = b_i^+\eta_i^+ + \sum_{j=1}^n \left[\left(a_{ij}^+ + b_{ij}^+ + c_{ij}^+\right)L_j\right].$$

Theorem 3.1. *If (H1)–(H3) are satisfied. Then there exists a unique almost automorphic solution of system (1) in* $\mathbb{X}_0 = \{\varphi \in \mathbb{X}^* | \|\varphi - \varphi^0\|_{\mathbb{X}^*} \le L\}$.

Proof For any given $\varphi \in \mathbb{X}^*$, we consider the following system

$$x_i^\Delta(t) = -b_i(t)x_i(t) + \Theta_i(t,\varphi) + I_i(t), i = 1, 2, \cdots, n, \tag{6}$$

where

$$\Theta_i(t,\varphi) = b_i(t)\int_{t-\eta_i(t)}^t \varphi_i^\Delta(s)\Delta s + \sum_{j=1}^n a_{ij}(t)f_j(\varphi_j(t)) + \sum_{j=1}^n b_{ij}(t)f_j(\varphi_j(t-\tau_{ij}(t))) + \sum_{j=1}^n c_{ij}(t)f_j(\varphi_i^\Delta(t-\sigma_{ij}(t))), i = 1, 2, \cdots, n. \tag{7}$$

It follows from Lemma 2.10 that the linear system

$$x_i^\Delta(t) = -b_i(t)x_i(t), i = 1, 2, \cdots, n, \tag{8}$$

admits an exponential dichotomy on $\mathbb{T}$. Thus, in view of Lemma 2.9, we derive that system (6) has exactly one almost automorphic solution as follows

$$x_i^\varphi(t) = \int_{-\infty}^t e_{-b_i}(t,\sigma(s))[\Theta_i(s,\varphi) + I_i(s)]\Delta s, i = 1, 2, \cdots, n, \tag{9}$$

For $\varphi \in \mathbb{X}^*$, then

$$\|\varphi\|_{\mathbb{X}^*} \le \|\varphi - \varphi_0\|_{\mathbb{X}^*} + \|\varphi_0\|_{\mathbb{X}^*} \le 2L. \tag{10}$$

Define an operator as follows

$$\Phi : \mathbb{X}^* \to \mathbb{X}^*, (\varphi_1, \varphi_2, \cdots, \varphi_n)^T \to \left(x_1^\varphi, x_2^\varphi, \cdots, x_n^\varphi\right)^T. \tag{11}$$

First we show that for any $\varphi \in \mathbb{X}^*$, we have $\Phi\varphi \in \mathbb{X}^*$. Note that, for $i = 1, 2, \cdot, n$, we have

$$\begin{aligned}|\Theta_i(s,\varphi)| &= \Big| b_i(s)\textstyle\int_{s-\eta_i(s)}^s \varphi_i^\Delta(\theta)\Delta\theta + \sum_{j=1}^n a_{ij}(s)f_j(\varphi_j(s)) \\ &\quad + \sum_{j=1}^n b_{ij}(s)f_j(\varphi_j(s-\tau_{ij}(s))) \\ &\quad + \sum_{j=1}^n c_{ij}(s)f_j(\varphi_i^\Delta(s-\sigma_{ij}(s)))\Big| \\ &\le b_i^+\eta_i^+\|\varphi\|_{\mathbb{X}^*} + \sum_{j=1}^n a_{ij}^+ \left(|f_j(\varphi_j(s)) - f_j(0)| + |f_j(0)|\right) \\ &\quad + \sum_{j=1}^n b_{ij}^+\left(|f_j(\varphi_j(s-\tau_{ij}(s))) - f_j(0)| + |f_j(0)|\right) \\ &\quad + \sum_{j=1}^n c_{ij}^+\left(|f_j(\varphi_i^\Delta(s-\sigma_{ij}(s))) - f_j(0)| + |f_j(0)|\right) \\ &\le b_i^+\eta_i^+\|\varphi\|_{\mathbb{X}^*} + \sum_{j=1}^n a_{ij}^+\left(L_j\|\varphi\|_{\mathbb{X}^*} + |f_j(0)|\right) \\ &\quad + \sum_{j=1}^n b_{ij}^+\left(L_j\|\varphi\|_{\mathbb{X}^*} + |f_j(0)|\right) \\ &\quad + \sum_{j=1}^n c_{ij}^+\left(L_j\|\varphi\|_{\mathbb{X}^*} + |f_j(0)|\right) \\ &= \left[b_i^+\eta_i^+ + \sum_{j=1}^n \left(a_{ij}^+ + b_{ij}^+ + c_{ij}^+\right)L_j\right]\|\varphi\|_{\mathbb{X}^*} \\ &\quad + \sum_{j=1}^n \left[a_{ij}^+ + b_{ij}^+ + c_{ij}^+\right)|f_j(0)| \\ &\le 2L\left\{b_i^+\eta_i^+ + \sum_{j=1}^n \left[\left(a_{ij}^+ + b_{ij}^+ + c_{ij}^+\right)(L_j+1)\right]\right\}.\end{aligned} \tag{12}$$

Thus we get

$$\begin{aligned}&|(\Phi(\varphi - \varphi^0))_i(t)| \\ &= \left|\int_{-\infty}^t e_{-b_i}(t,\sigma(s))\Theta_i(s,\varphi)\Delta s\right| \\ &\le \int_{-\infty}^t e_{-b_i}(t,\sigma(s))|\Theta_i(s,\varphi)|\Delta s \\ &\le 2L\int_{-\infty}^t e_{-b_i^-}(t,\sigma(s))\left\{b_i^+\eta_i^+ + \sum_{j=1}^n\left[\left(a_{ij}^+ + b_{ij}^+ + c_{ij}^+\right)(L_j+1)\right]\right\}\Delta s \\ &\le \frac{2L\varrho_i}{b_i^-}, i = 1, 2, \cdots, n.\end{aligned} \tag{13}$$

On the other hand, for $i = 1, 2, \cdots, n$, we have

$$\begin{aligned}&|(\Phi(\varphi - \varphi^0))_i^\Delta(t)| \\ &= \left|\left(\int_{-\infty}^t e_{-b_i}(t,\sigma(s))\Theta_i(s,\varphi)\Delta s\right)_t^\Delta\right| \\ &= \left|\Theta_i(t,\varphi) - b_i(t)\int_{-\infty}^t e_{-b_i}(t,\sigma(s))\Theta_i(s,\varphi)\Delta s\right| \\ &\le |\Theta_i(t,\varphi)| + |b_i(t)|\int_{-\infty}^t e_{-b_i}(t,\sigma(s))|\Theta_i(s,\varphi)|\Delta s \\ &\le 2L\varrho_i\left(1+\frac{b_i^+}{b_i^-}\right), i = 1, 2, ..., n.\end{aligned} \tag{14}$$

It follows from (H3) that

$$\|\Phi\varphi - \varphi^0\|_{\mathbb{X}^*} \le \max_{1\le i\le n}\left\{\frac{\varrho_i}{b_i^-}, \left(1+\frac{b_i^+}{b_i^-}\right)\varrho_i\right\} \le L \tag{15}$$

which implies that $\Phi\varphi \in \mathbb{X}^*$. Next, we show that Φ is a contraction. For any $\varphi = (\varphi_1, \varphi_2, \cdots, \varphi_n)^T, \psi = (\psi_1, \psi_2, \cdots, \psi_n)^T \in \mathbb{X}^*$, for $i = 1, 2, \cdots, n$, we denote

$$
\begin{aligned}
&\Upsilon_i(s,\varphi,\psi) \qquad (16)\\
&= b_i(t)\int_{s-\eta_i(s)}^{s}\left(\varphi_i^{\Delta}(s)-\psi_i^{\Delta}(s)\right)\Delta s\\
&+\sum_{j=1}^{n} a_{ij}(t)\left[f_j\left(\varphi_j(t)\right)-f_j\left(\psi_j(t)\right)\right]\\
&+\sum_{j=1}^{n} b_{ij}(t)\left[f_j\left(\varphi_j\left(t-\tau_{ij}(t)\right)\right)-f_j\left(\psi_j\left(t-\tau_{ij}(t)\right)\right)\right]\\
&+\sum_{j=1}^{n} c_{ij}(t)\left[f_j\left(\varphi_i^{\Delta}\left(t-\sigma_{ij}(t)\right)\right)-f_j\left(\psi_i^{\Delta}\left(t-\sigma_{ij}(t)\right)\right)\right].
\end{aligned}
$$

Then

$$
\begin{aligned}
&\left|(\Phi\varphi-\Phi\psi)_i(t)\right| \qquad (17)\\
&=\left|\int_{-\infty}^{t} e_{-b_i}(t,\sigma(s))\Upsilon_i(s,\varphi,\psi)\Delta s\right|\\
&\le \int_{-\infty}^{t} e_{-b_i}(t,\sigma(s))\left|\Upsilon_i(s,\varphi,\psi)\right|\Delta s\\
&\le \int_{-\infty}^{t} e_{-b_i}(t,\sigma(s))\left\{b_i^{+}\eta_i^{+}+\sum_{j=1}^{n}\left[\left(a_{ij}^{+}+b_{ij}^{+}+c_{ij}^{+}\right)L_j\right]\right\}\\
&\quad \Delta s\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\le \frac{\varsigma_i}{b_i^{-}}\|\varphi-\psi\|_{\mathbb{X}^*}, i=1,2,\cdots,n
\end{aligned}
$$

and

$$
\begin{aligned}
&\left|(\Phi\varphi-\Phi\psi)_i^{\Delta}(t)\right| \qquad (18)\\
&=\left|\int_{-\infty}^{t} e_{-b_i}(t,\sigma(s))\left(\Upsilon_i(s,\varphi,\psi)\right)_t^{\Delta}\Delta s\right|\\
&=\left|\Upsilon_i(s,\varphi,\psi)-b_i(t)\int_{-\infty}^{t} e_{-b_i}(t,\sigma(s))\Upsilon_i(s,\varphi,\psi)\Delta s\right|\\
&\le \left|\Upsilon_i(s,\varphi,\psi)\right|+\left|b_i(t)\right|\int_{-\infty}^{t} e_{-b_i}(t,\sigma(s))\left|\Upsilon_i(s,\varphi,\psi)\right|\Delta s\\
&\le \left\{b_i^{+}\eta_i^{+}+\sum_{j=1}^{n}\left[\left(a_{ij}^{+}+b_{ij}^{+}+c_{ij}^{+}\right)L_j\right]\right\}\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\quad + b_i^{+}\int_{-\infty}^{t} e_{-b_i}(t,\sigma(s))\\
&\quad \times\left\{b_i^{+}\eta_i^{+}+\sum_{j=1}^{n}\left[\left(a_{ij}^{+}+b_{ij}^{+}+c_{ij}^{+}\right)L_j\right]\right\}\Delta s\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\le \left(1+\frac{b_i^{+}}{b_i^{-}}\right)\|\varphi-\psi\|_{\mathbb{X}^*}, i=1,2,\cdots,n
\end{aligned}
$$

In view of (H3), we get that $\|\Phi\varphi-\Phi\varphi\| < \|\varphi-\psi\|$. Then Φ is a contraction. Thus Φ has a fixed point in $\mathbb{X}_0$, that is, (1) has a unique almost automorphic solution in $\mathbb{X}_0$. The proof of Theorem 3.1 is completed.

4. EXPONENTIAL STABILITY OF ALMOST AUTOMORPHIC SOLUTIONS

In this section, we will obtain the exponential stability of the almost automorphic solutions of system (1).

Theorem 4.1. *Suppose that (H1)–(H3) are fulfilled. Then the almost automorphic solution of system (1) is globally exponentially stable.*

Proof By Theorem 3.1, we know that (1) has an almost automorphic solution $x(t) = \left(x_1(t), x_2(t), \cdots, x_n(t)\right)^T$ with initial condition $\varphi(t) = \left(\varphi_1(t), \varphi_2(t), \cdots, \varphi_n(t)\right)^T$. Suppose that $y(t) = \left(y_1(t), y_2(t), \cdots, y_n(t)\right)^T$ is an arbitrary solution of (1) with initial condition $\psi(t) = \left(\psi_1(t), \psi_2(t), \cdots, \psi_n(t)\right)^T$. Denote $u(t) = \left(u_1(t), u_2(t), \cdots, u_n(t)\right)^T$, where $u_i(t) = y_i(t) - x_i(t), i = 1, 2, \cdots, n$. Then it follows from (1) that

$$
\begin{aligned}
u_i^{\Delta}(t) &= -b_i(t)u_i\left(t-\eta_i(t)\right)+\sum_{j=1}^{n} a_{ij}(t)\left[f_j\left(y_j(t)\right)-f_j\left(x_j(t)\right)\right]\\
&+\sum_{j=1}^{n} b_{ij}(t)\left[f_j\left(y_j\left(t-\tau_{ij}(t)\right)\right)-f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right)\right]\\
&+\sum_{j=1}^{n} c_{ij}(t)\left[f_j\left(y_i^{\Delta}\left(t-\sigma_{ij}(t)\right)\right)-f_j\left(x_i^{\Delta}\left(t-\sigma_{ij}(t)\right)\right)\right],\\
&i=1,2,\cdot,n.
\end{aligned} \qquad (19)
$$

The initial condition of (19) is

$$
\begin{aligned}
\phi_i(s) &= \varphi_i(s)-\psi_i(s), \phi_i^{\Delta}(s)\\
&= \varphi_i^{\Delta}(s)-\psi_i^{\Delta}(s), s\in[-\tau,0]_{\mathbb{T}}, i=1,2,\cdots,n.
\end{aligned} \qquad (20)
$$

Rewrite (19) as the form

$$
\begin{aligned}
u_i^{\Delta}(t) &= -b_i(t)u_i(t)+b_i(t)\int_{t-\eta_i(t)}^{t} u_i^{\Delta}(s)\Delta s\\
&+\sum_{j=1}^{n} a_{ij}(t)\left[f_j\left(y_j(t)\right)-f_j\left(x_j(t)\right)\right]\\
&+\sum_{j=1}^{n} b_{ij}(t)\left[f_j\left(y_j\left(t-\tau_{ij}(t)\right)\right)-f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right)\right]\\
&+\sum_{j=1}^{n} c_{ij}(t)\left[f_j\left(y_i^{\Delta}\left(t-\sigma_{ij}(t)\right)\right)-f_j\left(x_i^{\Delta}\left(t-\sigma_{ij}(t)\right)\right)\right],\\
&i=1,2,\ldots,n.
\end{aligned} \qquad (21)
$$

It follows from (21) that for $i = 1, 2, \cdots, n$ and $t \ge t_0, t_0 \in [-\tau, 0]_{\mathbb{T}}$,

$$
\begin{aligned}
u_i(t) = {}& u_i(t_0)e_{-b_i}(t,t_0)+\int_{t_0}^{t} e_{-b_i}(t,\sigma(s))\left\{b_i(s)\int_{s-\eta_i(s)}^{s} u_i^{\Delta}(\vartheta)\Delta\vartheta\right.\\
&+\sum_{j=1}^{n} a_{ij}(s)\left[f_j\left(y_j(s)\right)-f_j\left(x_j(s)\right)\right]\\
&+\sum_{j=1}^{n} b_{ij}(s)\left[f_j\left(y_j\left(s-\tau_{ij}(s)\right)\right)-f_j\left(x_j\left(s-\tau_{ij}(s)\right)\right)\right]\\
&\left.+\sum_{j=1}^{n} c_{ij}(s)\left[f_j\left(y_i^{\Delta}\left(s-\sigma_{ij}(s)\right)\right)-f_j\left(x_i^{\Delta}\left(s-\sigma_{ij}(s)\right)\right)\right]\right\}\Delta s,
\end{aligned} \qquad (22)
$$

where $i = 1, 2, \cdots, n$. For $\mu \in \mathbb{R}$, define $\Pi_i(\omega)$ and $\Gamma_i(\omega)$ as follows

$$
\begin{aligned}
\Pi_i(\omega) = {}& b_i^{-}-\omega-e^{\omega sup_{s\in\mathbb{T}}\mu(s)}\left[b_i^{+}\eta_i^{+}e^{\omega\eta_i^{+}}+\sum_{j=1}^{n} a_{ij}^{+}L_j\right.\\
&\left.+\sum_{j=1}^{n} b_{ij}^{+}L_j e^{\omega\tau_{ij}^{+}}+\sum_{j=1}^{n} c_{ij}^{+}L_j e^{\omega\sigma_{ij}^{+}}\right],
\end{aligned} \qquad (23)
$$

$$\Gamma_i(\omega) = b_i^- - \omega - \left(b_i^+ e^{\omega \sup_{s\in\mathbb{T}}\mu(s)} + b_i^- - \omega\right) \left[b_i^+\eta_i^+ e^{\omega\eta_i^+} + \sum_{j=1}^n a_{ij}^+ L_j + \sum_{j=1}^n b_{ij}^+ L_j e^{\omega\tau_{ij}^+} + \sum_{j=1}^n c_{ij}^+ L_j e^{\omega\sigma_{ij}^+}\right], \tag{24}$$

where $i = 1, 2, \cdots, n$. By (H3), we get

$$\Pi_i(0) = b_i^- - \left[b_i^+\eta_i^+ + \sum_{j=1}^n \left(a_{ij}^+ + b_{ij}^+ + c_{ij}^+\right) L_j\right] > 0, \tag{25}$$

$$\Gamma_i(0) = b_i^- - \left(b_i^+ + b_i^-\right) \left[b_i^+\eta_i^+ + \sum_{j=1}^n \left(a_{ij}^+ + b_{ij}^+ + c_{ij}^+\right) L_j\right] > 0. \tag{26}$$

Since $\Pi_i(\omega)$ and $\Gamma_i(\omega)$ are continuous on $[0, +\infty)$ and $\lim_{\omega\to+\infty}\Pi_i(\omega) = -\infty, \lim_{\omega\to+\infty}\Gamma_i(\omega) = -\infty$, then there exist $\omega_i, \omega_i^* > 0$ such that $\Pi_i(\omega_i) = 0, \Gamma_i\left(\omega_i^*\right) = 0$ and $\Pi_i(\omega) > 0$ for $\omega \in (0, \omega_i), \Gamma_i(\omega) > 0$ for $\omega \in \left(0, \omega_i^*\right), i = 1, 2, \cdots, n$. By choosing a positive constant $\omega_0 = \min\{\omega_1, \omega_2, \cdots, \omega_n, \omega_1^*, \omega_2^*, \cdots, \omega_n^*\}$, we get $\Pi_i(\omega_0) \geq 0$ and $\Gamma_i(\omega_0) \geq 0, i = 1, 2, \cdots, n$. Thus we can choose a positive constant $0 < \xi < \min\{\omega_0, \min_{1\leq i\leq n}\{b_i^-\}\}$ such that

$$\Pi_i\left(\xi\right) > 0, \Gamma_i\left(\xi\right) > 0, i = 1, 2, \cdots, n,$$

which implies that

$$\frac{e^{\xi \sup_{s\in\mathbb{T}}\mu(s)}}{b_i^- - \xi} \left[b_i^+\eta_i^+ e^{\xi\eta_i^+} + \sum_{j=1}^n a_{ij}^+ L_j + \sum_{j=1}^n b_{ij}^+ L_j e^{\xi\tau_{ij}^+} + \sum_{j=1}^n c_{ij}^+ L_j e^{\xi\sigma_{ij}^+}\right] < 1 \tag{27}$$

and

$$\left[1 + \frac{b_i^+ e^{\xi \sup_{s\in\mathbb{T}}\mu(s)}}{b_i^- - \xi}\right] \left[b_i^+\eta_i^+ e^{\xi\eta_i^+} + \sum_{j=1}^n a_{ij}^+ L_j + \sum_{j=1}^n b_{ij}^+ L_j e^{\xi\tau_{ij}^+} + \sum_{j=1}^n c_{ij}^+ L_j e^{\xi\sigma_{ij}^+}\right] < 1, \tag{28}$$

where $i = 1, 2, \cdots, n$. Let

$$M = \max_{1\leq i\leq n}\left\{\frac{b_i^-}{b_i^+\eta_i^+ + \sum_{j=1}^n \left(a_{ij}^+ + b_{ij}^+ + c_{ij}^+\right) L_j}\right\} \tag{29}$$

By (H3), we know that $M > 1$. Then we get

$$\frac{1}{M} < \frac{e^{\xi sup_{s\in\mathbb{T}}\mu(s)}}{b_i^- - \xi} \left[b_i^+\eta_i^+ e^{\xi\eta_i^+} + \sum_{j=1}^n a_{ij}^+ L_j + \sum_{j=1}^n b_{ij}^+ L_j e^{\xi\tau_{ij}^+} + \sum_{j=1}^n c_{ij}^+ L_j e^{\xi\sigma_{ij}^+}\right]. \tag{30}$$

Moreover, we have that $e_{\ominus\xi}(t, t_0) > 1$, where $t \in [-\tau, t_0]_{\mathbb{T}}$. Then

$$||u||_{\mathbb{X}^*} \leq Me_{\ominus\xi}(t, t_0)\,||\varphi - \psi||_{\mathbb{X}^*}, \text{ for all } t \in [-\tau, t_0]_{\mathbb{T}}. \tag{31}$$

We claim that

$$||u||_{\mathbb{X}^*} \leq Me_{\ominus\xi}(t, t_0)\,||\varphi - \psi||_{\mathbb{X}^*}, \text{ for all } t \in [-t_0, +\infty]_{\mathbb{T}}. \tag{32}$$

To prove this (32), we show that for any $p > 1$, the following inequality holds

$$||u||_{\mathbb{X}^*} \leq pMe_{\ominus\xi}(t, t_0)\,||\varphi - \psi||_{\mathbb{X}^*}, \text{ for all } t \in [-t_0, +\infty]_{\mathbb{T}}. \tag{33}$$

which implies that

$$|u_i(t)| \leq pMe_{\ominus\xi}(t, t_0)\,||\varphi - \psi||_{\mathbb{X}^*}, \text{ for all } t \in [-t_0, +\infty]_{\mathbb{T}}. \tag{34}$$

and

$$|u_i^\Delta(t)| \leq pMe_{\ominus\xi}(t, t_0)\,||\varphi - \psi||_{\mathbb{X}^*}, \text{ for all } t \in [-t_0, +\infty]_{\mathbb{T}}. \tag{35}$$

By way of contradiction, assume that (33) does not hold. Now we consider the two cases.

Case 1. (34) is not true and (35) is true. Then there exists $t^* \in (t_0, +\infty)_{\mathbb{T}}$ and $i^* \in \{1, 2, \cdots, n\}$ such that

$$|u_{i^*}(t^*)| \geq pMe_{\ominus\xi}(t^*, t_0)\,||\varphi - \psi||_{\mathbb{X}^*},$$
$$|u_{i^*}(t)| < pMe_{\ominus\xi}(t, t_0)\,||\varphi - \psi||_{\mathbb{X}^*}, \text{for all } \quad t \in [-t_0, t^*]_{\mathbb{T}},$$

$$|u_k(t)| < pMe_{\ominus\xi}(t, t_0)\,||\varphi - \psi||_{\mathbb{X}^*}, \text{for } \quad k \neq i^*, t \in [-t_0, t^*]_{\mathbb{T}}, \; k = 1, 2, \cdots, n.$$

Therefore, there exists a constant $\gamma_1 \geq 1$ such that

$$|u_{i^*}(t^*)| = \gamma_1 pMe_{\ominus\xi}(t^*, t_0)\,||\varphi - \psi||_{\mathbb{X}^*},$$
$$|u_{i^*}(t)| < \gamma_1 pMe_{\ominus\xi}(t, t_0)\,||\varphi - \psi||_{\mathbb{X}^*}, \quad \text{for all} \quad t \in [-t_0, t^*]_{\mathbb{T}}.$$

$$|u_k(t)| < \gamma_1 pMe_{\ominus\xi}(t, t_0)\,||\varphi - \psi||_{\mathbb{X}^*}, \text{for } \quad k \neq i^*, t \in [-t_0, t^*]_{\mathbb{T}}, \; k = 1, 2, \cdots, n.$$

By (22), for $i = 1, 2, \cdots, n$, we get

$$\begin{aligned}|u_{i^*}(t^*)| = \Bigg| & u_{i^*}(t_0)\, e_{-b_{i^*}}(t^*, t_0) + \int_{t_0}^{t^*} e_{-b_{i^*}}(t^*, \sigma(s)) \\ & \Bigg\{ b_{i^*}(s) \int_{s-\eta_{i^*}(s)}^{s} u_{i^*}^\Delta(\vartheta)\,\Delta\vartheta \\ & + \sum_{j=1}^n a_{i^*j}(s)\left[f_j\left(y_j(s)\right) - f_j\left(x_j(s)\right)\right] \\ & + \sum_{j=1}^n b_{i^*j}(s)\left[f_j\left(y_j\left(s - \tau_{i^*j}(s)\right)\right) - f_j\left(x_j\left(s - \tau_{i^*j}(s)\right)\right)\right] \\ & + \sum_{j=1}^n c_{i^*j}(s)\left[f_j\left(y_{i^*}^\Delta\left(s - \sigma_{i^*j}(s)\right)\right)\right. \\ & \left.\left. - f_j\left(x_{i^*}^\Delta\left(s - \sigma_{i^*j}(s)\right)\right)\right]\right\}\Delta s \Bigg|\end{aligned}$$

$$
\begin{aligned}
&\leq e_{-b_{i^*}}\left(t^*,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}+\gamma_1 pMe_{\ominus\xi}\left(t^*,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\quad\times\left|\int_{t_0}^{t^*} e_{-b_{i^*}}\left(t^*,\sigma(s)\right)e_{\xi}\left(t^*,\sigma(s)\right)\left\{b^+_{i^*}\int_{s-\eta_{i^*}(s)}^{s} e_{\xi}\left(\sigma(s),\vartheta\right)\Delta\vartheta\right.\right.\\
&\quad+\sum_{j=1}^{n} a^+_{i^*j}L_j e_{\xi}\left(\sigma(s),s\right)+\sum_{j=1}^{n} b^+_{i^*j}L_j e_{\xi}\left(\sigma(s),s-\tau_{i^*j}(s)\right)\\
&\quad\left.\left.+\sum_{j=1}^{n} c^+_{i^*j}L_j e_{\xi}\left(\sigma(s),s-\sigma_{i^*j}(s)\right)\right\}\Delta s\right|\\
&\leq e_{-b_{i^*}}\left(t^*,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}+\gamma_1 pMe_{\ominus\xi}\left(t^*,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\quad\times\left|\int_{t_0}^{t^*} e_{-b_{i^*}\oplus\xi}\left(t^*,\sigma(s)\right)\left\{b^+_{i^*}\eta^+_{i^*}e_{\xi}\left(\sigma(s),s-\eta_{i^*}(s)\right)\right.\right.\\
&\quad+\sum_{j=1}^{n} a^+_{i^*j}L_j e_{\xi}\left(\sigma(s),s\right)+\sum_{j=1}^{n} b^+_{i^*j}L_j e_{\xi}\left(\sigma(s),s-\tau_{i^*j}(s)\right)\\
&\quad\left.\left.+\sum_{j=1}^{n} c^+_{i^*j}L_j e_{\xi}\left(\sigma(s),s-\sigma_{i^*j}(s)\right)\right\}\Delta s\right|\\
&\leq e_{-b_{i^*}}\left(t^*,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}+\gamma_1 pMe_{\ominus\xi}\left(t^*,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\quad\times\left|\int_{t_0}^{t^*} e_{-b_{i^*}\oplus\xi}\left(t^*,\sigma(s)\right)\left\{b^+_{i^*}\eta^+_{i^*}e^{\xi\left(\eta^+_{i^*}+\sup_{t\in\mathbb{T}}\mu(s)\right)}\right.\right.\\
&\quad+\sum_{j=1}^{n} a^+_{i^*j}L_j e^{\xi\sup_{t\in\mathbb{T}}\mu(s)}+\sum_{j=1}^{n} b^+_{i^*j}L_j e^{\xi\left(\tau^+_{i^*j}+\sup_{t\in\mathbb{T}}\mu(s)\right)}\\
&\quad\left.\left.+\sum_{j=1}^{n} c^+_{i^*j}L_j e^{\xi\left(\sigma^+_{i^*j}+\sup_{t\in\mathbb{T}}\mu(s)\right)}\right\}\Delta s\right|\\
&=\gamma_1 pMe_{\ominus\xi}\left(t^*,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\left\{\frac{1}{\gamma_1 pM}e_{-b_{i^*}\oplus\xi}\left(t^*,t_0\right)+e^{\xi\sup_{t\in\mathbb{T}}\mu(s)}\right.\\
&\quad\times\left[b^+_{i^*}\eta^+_{i^*}e^{\xi\eta^+_{i^*}}+\sum_{j=1}^{n} a^+_{i^*j}L_j+\sum_{j=1}^{n} b^+_{i^*j}L_j e^{\xi\tau^+_{i^*j}}+\sum_{j=1}^{n} c^+_{i^*j}L_j e^{\xi\sigma^+_{i^*j}}\right]\\
&\quad\left.\times\int_{t_0}^{t^*} e_{-b_{i^*}\oplus\xi}\left(t^*,\sigma(s)\right)\Delta s\right\}
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
&\leq\gamma_1 pMe_{\ominus\xi}\left(t^*,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\left\{\frac{1}{M}e_{-\left(b_{i^*}-\xi\right)}\left(t^*,t_0\right)+e^{\xi sup_{t\in\mathbb{T}}\mu(s)}\right.\\
&\quad\times\left[\left(b^+_{i^*}\eta^+_{i^*}e^{\xi\eta^+_{i^*}}+\sum_{j=1}^{n} a^+_{i^*j}L_j+\sum_{j=1}^{n} b^+_{i^*j}L_j e^{\xi\tau^+_{i^*j}}+\sum_{j=1}^{n} c^+_{i^*j}L_j e^{\xi\sigma^+_{i^*j}}\right)\right.\\
&\quad\left.\left.\times\frac{1}{-\left(b_{i^*}-\xi\right)}\int_{t_0}^{t^*}\left(-\left(b_{i^*}-\xi\right)\right)e_{-\left(b_{i^*}-\xi\right)}\left(t^*,\sigma(s)\right)\Delta s\right]\right\}\\
&=\gamma_1 pMe_{\ominus\xi}\left(t^*,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\left\{\left[\frac{1}{M}-\frac{e^{\xi sup_{t\in\mathbb{T}}\mu(s)}}{b^-_{i^*}-\xi}\left(b^+_{i^*}\eta^+_{i^*}e^{\xi\eta^+_{i^*}}\right.\right.\right.\\
&\quad\left.\left.+\sum_{j=1}^{n} a^+_{i^*j}L_j+\sum_{j=1}^{n} b^+_{i^*j}L_j e^{\xi\tau^+_{i^*j}}+\sum_{j=1}^{n} c^+_{i^*j}L_j e^{\xi\sigma^+_{i^*j}}\right)\right]e_{-\left(b_{i^*}-\xi\right)}\left(t^*,t_0\right)\\
&\quad+\frac{e^{\xi sup_{t\in\mathbb{T}}\mu(s)}}{b^-_{i^*}-\xi}\left(b^+_{i^*}\eta^+_{i^*}e^{\xi\eta^+_{i^*}}+\sum_{j=1}^{n} a^+_{i^*j}L_j+\sum_{j=1}^{n} b^+_{i^*j}L_j e^{\xi\tau^+_{i^*j}}\right.\\
&\quad\left.\left.+\sum_{j=1}^{n} c^+_{i^*j}L_j e^{\xi\sigma^+_{i^*j}}\right)\right\}\\
&<\gamma_1 pMe_{\ominus\xi}\left(t^*,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*},
\end{aligned}
\tag{37}
$$

which is a contradiction.

Case 2. (35) is not true and (34) is true. Then there exists $t^{**}\in(t_0,+\infty)_{\mathbb{T}}$ and $i^{**}\in\{1,2,\cdots,n\}$ such that

$$
\begin{aligned}
&\left|u^{\Delta}_{i^{**}}\left(t^{**}\right)\right|\geq pMe_{\ominus\xi}\left(t^{**},t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*},\\
&\left|u^{\Delta}_{i^{**}}(t)\right|<pMe_{\ominus\xi}\left(t,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*},\ \text{for all}\ t\in\left[-t_0,t^{**}\right]_{\mathbb{T}},
\end{aligned}
$$

$$
\begin{aligned}
&\left|u^{\Delta}_{k}(t)\right|<pMe_{\ominus\xi}\left(t,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*},\ \text{for}\ k\neq i^{**},t\in\left[-t_0,t^{**}\right]_{\mathbb{T}},\\
&k=1,2,\cdots,n.
\end{aligned}
$$

Therefore, there exists a constant $\gamma_2\geq 1$ such that

$$
\begin{aligned}
\left|u^{\Delta}_{i^{**}}\left(t^{**}\right)\right|&=\gamma_2 pMe_{\ominus\xi}\left(t^{**},t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*},\left|u_{i^{**}}(t)\right|\\
&<\gamma_2 pMe_{\ominus\xi}\left(t,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*},\ \text{for all}\ t\in\left[-t_0,t^{**}\right]_{\mathbb{T}}.
\end{aligned}
$$

$$
\begin{aligned}
&\left|u^{\Delta}_{k}(t)\right|<\gamma_2 pMe_{\ominus\xi}\left(t,t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*},\quad\text{for}\quad k\neq i^{**},\\
&t\in\left[-t_0,t^{**}\right]_{\mathbb{T}},k=1,2,\cdots,n.
\end{aligned}
$$

By (22), for $i=1,2,\cdots,n$, we have

$$
\begin{aligned}
\left|u^{\Delta}_{i^{**}}\left(t^{**}\right)\right|&=\left|-b_{i^{**}}\left(t^{**}\right)u_{i^{**}}\left(t_0\right)e_{-b_{i^{**}}}\left(t^{**},t_0\right)+b_{i^{**}}\left(t^{**}\right)\right.\\
&\quad\int_{t^{**}-\eta_{i^{**}}}^{t^{**}} u^{\Delta}_{i^{**}}(s)\Delta s\\
&\quad+\sum_{j=1}^{n} a_{i^{**}j}\left(t^{**}\right)\left[f_j\left(y_j\left(t^{**}\right)\right)-f_j\left(x_j\left(t^{**}\right)\right)\right]\\
&\quad+\sum_{j=1}^{n} b_{i^{**}j}\left(t^{**}\right)\left[f_j\left(y_j\left(t^{**}-\tau_{i^{**}j}\left(t^{**}\right)\right)\right)\right.\\
&\quad\left.-f_j\left(x_j\left(t^{**}-\tau_{i^{**}j}\left(t^{**}\right)\right)\right)\right]\\
&\quad+\sum_{j=1}^{n} c_{i^{**}j}\left(t^{**}\right)\left[f_j\left(y^{\Delta}_{i^{**}}\left(t^{**}-\sigma_{i^{**}j}\left(t^{**}\right)\right)\right)\right.\\
&\quad\left.-f_j\left(x^{\Delta}_{i^{**}}\left(t^{**}-\sigma_{i^{**}j}\left(t^{**}\right)\right)\right)\right]\\
&\quad-b_{i^{**}}\left(t^{**}\right)\int_{t_0}^{t^{**}} e_{-b_{i^{**}}}\left(t^{**},\sigma(s)\right)\\
&\quad\times\left\{b_{i^{**}}(s)\int_{s-\eta_{i^{**}}}^{s} u^{\Delta}_{i^{**}}(\vartheta)\Delta\vartheta\right.\\
&\quad+\sum_{j=1}^{n} a_{i^{**}j}(s)\left[f_j\left(y_j(s)\right)-f_j\left(x_j(s)\right)\right]\\
&\quad+\sum_{j=1}^{n} b_{i^{**}j}(s)\left[f_j\left(y_j\left(s-\tau_{i^{**}j}(s)\right)\right)\right.\\
&\quad\left.-f_j\left(x_j\left(s-\tau_{i^{**}j}(s)\right)\right)\right]\\
&\quad+\sum_{j=1}^{n} c_{i^{**}j}(s)\left[f_j\left(y^{\Delta}_{s}\left(s-\sigma_{i^{**}j}(s)\right)\right)\right.\\
&\quad\left.\left.\left.-f_j\left(x^{\Delta}_{i^{**}}\left(s-\sigma_{i^{**}j}(s)\right)\right)\right]\right\}\Delta s\right|\\
&\leq e_{-b_{i^{**}}}\left(t^{**},t_0\right)e_{-b^{**}_{i}}\left(t^{**},t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\quad+\gamma_2 pMe_{\ominus\xi}\left(t^{**},t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\quad\times\left(b^+_{i^{**}}\int_{s-\eta_{i^{**}}}^{s} e_{\xi}\left(\sigma(s),\vartheta\right)\Delta\vartheta\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=1}^{n} a^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma\left(t^{**}\right), t^{**}\right)\\
&+\sum_{j=1}^{n} b^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma\left(t^{**}\right), t^{**}-\tau_{i^{**}j}\left(t^{**}\right)\right)\\
&+\sum_{j=1}^{n} c^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma\left(t^{**}\right), t^{**}-\sigma_{i^{**}j}\left(t^{**}\right)\right)\Bigg)\\
&+b^{+}_{i^{**}}\gamma_2 pMe_{\ominus\xi}\left(t^{**}, t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\Bigg\{\int_{t_0}^{t^{**}} e_{-b_{i^{*}*}}\left(t^{**}, \sigma(s)\right) e_{\xi}\left(t^{**}, \sigma(s)\right)\\
&\quad\times\Bigg[b^{+}_{i^{**}}\int_{s-\eta_{i^{**}}}^{s} e_{\xi}\left(\sigma(s), \vartheta\right)\Delta\vartheta+\sum_{j=1}^{n} a^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma(s), s\right)\\
&+\sum_{j=1}^{n} b^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma(s), s-\tau_{i^{**}j}(s)\right)\\
&+\sum_{j=1}^{n} c^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma(s), s-\sigma_{i^{**}j}(s)\right)\Bigg]\Delta s\Bigg\}\\
\le\ & e_{-b_{i^{**}}}\left(t^{**}, t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}+\gamma_2 pMe_{\ominus\xi}\left(t^{**}, t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\times\Bigg(b^{+}_{i^{**}}\eta^{+}_{i^{**}}e_{\xi}\Bigg(t^{**}, t^{**}-\eta_{i^{**}}\left(t^{**}\right)+\sum_{j=1}^{n} a^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma\left(t^{**}\right), t^{**}\right)\\
&+\sum_{j=1}^{n} b^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma\left(t^{**}\right), t^{**}-\tau_{i^{**}j}\left(t^{**}\right)\right)\\
&+\sum_{j=1}^{n} c^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma\left(t^{**}\right), t^{**}-\sigma_{i^{**}j}\left(t^{**}\right)\right)\Bigg)\\
&+b^{+}_{i^{**}}\gamma_2 pMe_{\ominus\xi}\left(t^{**}, t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\Bigg\{\int_{t_0}^{t^{**}} e_{-b_{i^{*}*}\oplus\xi}\left(t^{**}, \sigma(s)\right)\\
&\times\Bigg[b^{+}_{i^{**}}\eta^{+}_{i^{**}}e_{\xi}\left(\sigma(s), s-\eta_{i^{**}}(s)\right)+\sum_{j=1}^{n} a^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma(s), s\right)\\
&+\sum_{j=1}^{n} b^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma(s), s-\tau_{i^{**}j}(s)\right)\\
&+\sum_{j=1}^{n} c^{+}_{i^{**}j}L_j e_{\xi}\left(\sigma(s), s-\sigma_{i^{**}j}(s)\right)\Bigg]\Delta s\Bigg\}\\
\le\ & e_{-b_{i^{**}}}\left(t^{**}, t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}+\gamma_2 pMe_{\ominus\xi}\left(t^{**}, t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\\
&\times\Bigg(b^{+}_{i^{**}}\eta^{+}_{i^{**}}e^{\xi\eta_{i^{**}}}+\sum_{j=1}^{n} a^{+}_{i^{**}j}L_j+\sum_{j=1}^{n} b^{+}_{i^{**}j}L_j e^{\xi\tau_{i^{**}j}}\\
&+\sum_{j=1}^{n} c^{+}_{i^{**}j}L_j e^{\xi\sigma_{i^{**}j}}\Bigg)\\
&\times\left(1+b^{+}_{i^{**}}e^{\xi sup_{t\in\mathbb{T}}\mu(s)}\int_{t_0}^{t^{**}} e_{-b_{i^{**}}\oplus\xi}\left(t^{**}, \sigma(s)\right)\Delta s\right)\\
\le\ & \gamma_2 pMe_{\ominus\xi}\left(t^{**}, t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\Bigg\{\frac{b^{+}_{i^{**}}}{M}e_{-b_{i^{**}}\oplus\xi}\left(t^{**}, t_0\right)\\
&+\Bigg(b^{+}_{i^{**}}\eta^{+}_{i^{**}}e^{\xi\eta^{+}_{i^{**}}}+\sum_{j=1}^{n} a^{+}_{i^{**}j}L_j\\
&+\sum_{j=1}^{n} b^{+}_{i^{**}j}L_j e^{\xi\tau^{+}_{i^{**}j}}+\sum_{j=1}^{n} c^{+}_{i^{**}j}L_j e^{\xi\sigma^{+}_{i^{**}j}}\Bigg)\\
&\times\left(1+b^{+}_{i^{**}j}e^{\xi sup_{t\in\mathbb{T}}\mu(s)}\int_{t_0}^{t^{**}} e_{-b_{i^{**}}\oplus\xi}\left(t^{**}, \sigma(s)\right)\Delta s\right)\Bigg\}\\
\le\ & \gamma_2 pMe_{\ominus\xi}\left(t^{**}, t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*}\Bigg\{\Bigg[\frac{1}{M}-\frac{e^{\xi sup_{t\in\mathbb{T}}\mu(s)}}{b^{-}_{i^{**}}-\xi}\\
&\Bigg(b^{+}_{i^{**}}\eta^{+}_{i^{**}}e^{\xi\eta^{+}_{i^{**}}}+\sum_{j=1}^{n} a^{+}_{i^{**}j}L_j+\sum_{j=1}^{n} b^{+}_{i^{**}j}L_j e^{\xi\tau^{+}_{i^{**}j}}\\
&+\sum_{j=1}^{n} c^{+}_{i^{**}j}L_j e^{\xi\sigma^{+}_{i^{**}j}}\Bigg)\Bigg]\\
&\times b^{+}_{i^{**}}e_{-(b_{i^{**}}-\xi)}\left(t^{**}, t_0\right)+\left(1+\frac{b^{+}_{i^{**}j}e^{\xi sup_{t\in\mathbb{T}}\mu(s)}}{b^{-}_{i^{**}j}-\xi}\right)\\
&\Bigg(b^{+}_{i^{**}}\eta^{+}_{i^{**}}e^{\xi\eta^{+}_{i^{**}}}+\sum_{j=1}^{n} a^{+}_{i^{**}j}L_j+\sum_{j=1}^{n} b^{+}_{i^{**}j}L_j e^{\xi\tau^{+}_{i^{**}j}}\\
&+\sum_{j=1}^{n} c^{+}_{i^{**}j}L_j e^{\xi\sigma^{+}_{i^{**}j}}\Bigg)\Bigg\}\\
<\ & \gamma_2 pMe_{\ominus\xi}\left(t^{**}, t_0\right)\|\varphi-\psi\|_{\mathbb{X}^*},
\end{aligned}
\tag{38}
$$

which is also a contradiction. Based on the two cases above, we can conclude that (33) holds. Let $p \to 1$, then (32) holds. We can take $\ominus\lambda = \ominus\xi$, then $\lambda > 0$ and $\ominus\lambda \in \boldsymbol{R}^{+}$. Then we derive

$$\|u\|_{\mathbb{X}^*} \le M\|\varphi-\psi\|_{\mathbb{X}^*}e_{\ominus\lambda}\left(t, t_0\right), t \in [-\tau, \infty)_{\mathbb{T}}, t \ge t_0, \tag{39}$$

which means that the almost automorphic solution of (1) is globally exponentially stable. The proof of Theorem 4.1 is completed.

Remark 4.1

In [2–4,7], the scholars considered the almost periodic solution for different type neural networks. In [5,8,10,11,39], the authors investigated anti-periodic solutions to various neural networks. The research topic of [2–5,7,8,10,11,39] did not involve almost automorphic solution. In this article, we have analyzed the almost automorphic solutions to cellular neural networks with neutral type delays and time-varying leakage delays on time scales. The obtained theoretical results in [2–5,7,8,10,11,39] cannot be applied to system (1) to derive the existence and the exponential stability of almost automorphic solutions for system (1). From this viewpoint, we can say that the main results on the existence and the exponential stability of almost automorphic solutions for system (1) are completely new and complement previous publications.

5. AN EXAMPLE

Considering the following cellular neural networks with neutral type delays and time-varying leakage delays on time scales

$$
\begin{cases}
x_1^{\Delta}(t) = -b_1(t)x_1\left(t-\eta_1(t)\right)+\sum_{j=1}^{2} a_{1j}(t)f_j\left(x_j(t)\right)\\
\qquad+\sum_{j=1}^{2} b_{1j}(t)f_j\left(x_j\left(t-\tau_{1j}(t)\right)\right)\\
\qquad+\sum_{j=1}^{2} c_{1j}(t)f_j\left(x_1^{\Delta}\left(t-\sigma_{1j}(t)\right)\right)+I_1(t),\\
x_2^{\Delta}(t) = -b_2(t)x_2\left(t-\eta_2(t)\right)+\sum_{j=1}^{2} a_{2j}(t)f_j\left(x_j(t)\right)\\
\qquad+\sum_{j=1}^{2} b_{2j}(t)f_j\left(x_j\left(t-\tau_{2j}(t)\right)\right)\\
\qquad+\sum_{j=1}^{2} c_{2j}(t)f_j\left(x_2^{\Delta}\left(t-\sigma_{2j}(t)\right)\right)+I_2(t),
\end{cases}
\tag{40}
$$

where $f_1(u_1) = sin0.3u_1, f_2(u_2) = sin0.2u_2$ and

$$\begin{bmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.01 + 0.04\cos\sqrt{2}t & 0.02 + 0.01\cos\sqrt{3}t \\ 0.01 + 0.03\cos\sqrt{5}t & 0.01 + 0.02\cos\sqrt{3}t \end{bmatrix},$$

$$\begin{bmatrix} b_{11}(t) & b_{12}(t) \\ b_{21}(t) & b_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.01 + 0.02\cos\sqrt{5}t & 0.02 + 0.03\cos\sqrt{2}t \\ 0.02 + 0.03\cos\sqrt{3}t & 0.02 + 0.03\cos\sqrt{3}t \end{bmatrix},$$

$$\begin{bmatrix} c_{11}(t) & c_{12}(t) \\ c_{21}(t) & c_{22}(t) \end{bmatrix} = \begin{bmatrix} 0.02 + 0.02\cos\sqrt{5}t & 0.02 + 0.03\cos\sqrt{2}t \\ 0.03 + 0.01\cos\sqrt{3}t & 0.03 + 0.02\cos\sqrt{2}t \end{bmatrix},$$

$$\begin{bmatrix} b_1(t) & b_2(t) \\ \eta_1(t) & \eta_2(t) \end{bmatrix} = \begin{bmatrix} 0.02 + 0.01\cos\sqrt{2}t & 0.03 + 0.01\cos\sqrt{3}t \\ 0.02 + 0.01\cos\sqrt{5}t & 0.01 + 0.02\cos\sqrt{2}t \end{bmatrix},$$

$$\begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} = \begin{bmatrix} 0.03 + 0.01\sin\sqrt{5}t \\ 0.04 + 0.01\sin\sqrt{3}t \end{bmatrix}.$$

Then we get $L_1 = 0.3, L_2 = 0.2, M_1 = 0.3, M_2 = 0.2$ and

$$\begin{bmatrix} a_{11}^+ & a_{12}^+ \\ a_{21}^+ & a_{22}^+ \end{bmatrix} = \begin{bmatrix} 0.05 & 0.03 \\ 0.04 & 0.03 \end{bmatrix}, \begin{bmatrix} b_{11}^+ & b_{12}^+ \\ b_{21}^+ & b_{22}^+ \end{bmatrix} = \begin{bmatrix} 0.03 & 0.05 \\ 0.05 & 0.05 \end{bmatrix},$$

$$\begin{bmatrix} c_{11}^+ & c_{12}^+ \\ c_{21}^+ & c_{22}^+ \end{bmatrix} = \begin{bmatrix} 0.04 & 0.05 \\ 0.04 & 0.05 \end{bmatrix}, \begin{bmatrix} b_1^- & b_2^- \\ \eta_1^+ & \eta_2^+ \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.3 \end{bmatrix}.$$

It is not difficult to verify that all assumptions in Theorems 4.1 are fulfilled. Thus we can conclude that (1) has an almost automorphic solution, which is globally exponentially stable. The results are verified by the numerical simulations in Figures 1 and 2.

6. CONCLUSIONS

In this paper, we investigate a class of cellular neural networks with neutral type delays and time-varying leakage delays. Applying the existence of the exponential dichotomy of linear dynamic equations on time scales, a fixed point theorem and the theory of calculus on time scales, we establish a series of sufficient conditions for the existence and exponential stability of almost automorphic solutions for the cellular neural networks with neutral type delays and time-varying leakage delays on time scales. We show that the existence and global exponential stability of almost automorphic solutions for system (1) only depends on time delays η_i $(i = 1, 2, \cdots, n)$ (the delays in the leakage term) and does not depend on time delays $\tau_{ij}\left(i, j = 1, 2, \cdots, n\right)$ and $\sigma_{ij}\left(i, j = 1, 2, \cdots, n\right)$, which implies that the delays in the leakage term do harm to the existence and global exponential stability of almost automorphic solutions. To the best of our knowledge, it is the first time to deal with the almost automorphic solution for cellular neural networks with neutral type delays and time-varying leakage delays on time scales. The idea of this manuscript can be applied directly to investigate a lot of numerous network systems. The theoretical predictions of this manuscript show that under a suitable parameter condition, the cellular neural networks with neutral type delays and leakage delays will display almost automorphic oscillatory phenomenon. In real life, the almost automorphic oscillatory behavior plays an important role in helping us process visual information successfully. It can be effectively applied in predicting the law of brain cell activity, which is useful to serve the diagnosis of diseases. In addition, we know that the quaternion-valued cellular neural networks can be regarded as a generalization of real-valued and complex-valued cellular neural networks. So far, there are very few publications that consider almost automorphic solutions of quaternion-valued cellular neural networks. In the near future, we will focus on this topic.

CONFLICT OF INTEREST

The authors declare that they have no competing interests.

AUTHORS' CONTRIBUTIONS

The study was conceived and designed by Changjin Xu and Maoxin Liao and experiments performed by Peiluan Li and Zixin Liu. All authors read and approved the manuscript.

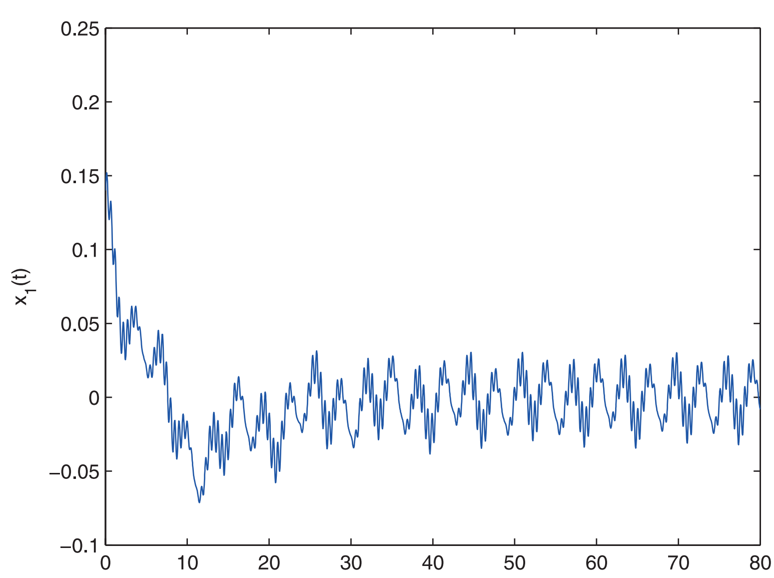

Figure 1 The relation of t and x_1.

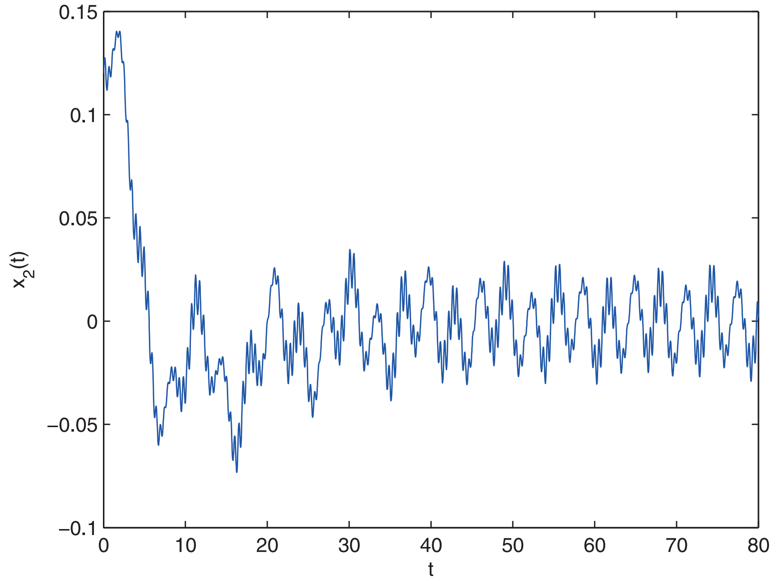

Figure 2 The relation of t and x_2.

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