Advances in Social Science, Education and Humanities Research, volume 397
3rd International Conference on Learning Innovation and Quality Education (ICLIQE 2019)

Identification of Mathematical Reasoning Ability in Solving Higher Order Thinking Skills Problems

Arini Alfa Mafada[1] , Tri Atmojo Kusmayadi[1], Laila Fitriana[1]

[1]Postgraduate of Mathematics Education, Universitas Sebelas Maret, Indonesia.

alfamafada@gmail.com[1], tri.atmojo.kusmayadi@staff.uns.ac.id[1], lailafitriana_fkip@staff.uns.ac.id[1]

Abstract: Mathematical reasoning is one of high-level thinking ability that can be developed through mathematics learning. The student with mathematical reasoning ability can develop their concept in mathematics to solve any problems. Problem with Higher Order Thinking Skills (HOTS) is one of kind problem that needs mathematical reasoning ability that contain of three highest cognitive aspects in revised Taxonomy Bloom. They are analyzing, evaluating, and creating. The research aim is to describe students' mathematical reasoning ability in solving HOTS problems of geometry. This study is descriptive qualitative. The research subjects were 6 students in 8th grade of Junior High School in Surakarta. The data was collected by a test and followed by an interview. The test instrument was based on the indicator of HOTS problem and then was analyzed by the indicator of mathematical reasoning to know the process of their reasoning in solving the problem. The results show that among 6 students, the average percentage of students who can make conjectures is 90%, 88% of students can use the correlation of quadrilateral characteristics, 65% can create valid arguments with systematic steps, and 59% are able to make conclusion. It indicates that mathematical reasoning ability of students in solving HOTS problem is on the medium level.

Keywords: *mathematical reasoning ability, higher order thinking skills problem*

INTRODUCTION

Permendikbud No.58 of 2014 mentions that one of the goals of mathematics learning is to use reasoning in doing mathematical manipulation both in simplification and analyze components exist in problem-solving in the context of mathematics and in real life, science, and technology that includes the ability to understand problems, complete models and interpret solutions obtained to solve problems in life. Therefore, one important capability must be owned in mathematics learning is the ability in reasoning. Reasoning is an ability that is used as a foundation in mathematical thinking, which is curriculum in almost all the world want the teachers can facilitate the development of reasoning. Furthermore, according to NCTM (2000) which is emphasizes the importance of developing mathematical reasoning in school. This is because through mathematical reasoning habits well, students will be able not only in understanding but also to using what they have learned at school. Meanwhile, according to Khalimi reasoning is the process of making conclusions or creating arguments based on the information that are available, or based on certain conclusions that have been verified (2011: 180). Besides, according to Linola (2017), reasoning is a dialectical process which means during the process of reasoning or thinking, thoughts in a way can automatically put the relationship between the knowledge possessed. Logicists put forward there are three processes that must be passed in reasoning, namely understanding, making opinions, and creating conclusions (Baharudin, 2007: 121). However, based on a national and international assessment of mathematical reasoning abilities in Indonesia is still low.

In Astuty's research (2019) conducted research on junior high school students based on their independence shows that students with high independence can fulfill 6 indicators of reasoning ability. While, students with low independence are only able to fulfill 1 indicator, that is put

forward a presumption. While when is seen by the Field Dependent cognitive style, as in the study of Mardiyah (2018) found that students with weak Field Dependent cognitive styles did not able to manipulate mathematics, compile evidence, and make conclusions. And also the research conducted by Rizqy (2017) on 8th-grade junior high school students showed that it was less than 30% of students who can fulfill the indicator of mathematical reasoning ability are in compiling prove and make conclusions. Even, Faradillah's research (2018) carried out on the mathematics pre-service teacher showed that no one was able to analyze problems given and all subjects have difficulty in generalizing, synthesize and solving non-routine problems provided by the researcher. Beside that, mathematical reasoning ability of 8th grade student is still low which is researched by Rizqi (2017) that use submitting discussion, arranging the proof and give the reason/proof to the truth solution, checking the validity of an argument, and taking a conclusion of a statement as the mathematical reasoning ability indicators. Furthermore, Poon and Leung (2016) said that there are strong correlations between the students' achievement in geometry and their fundamental logical reasoning ability.

The low of mathematical reasoning ability students is relate with the achievement of Indonesian students in international assessments like TIMSS. The last data of TIMSS in 2015, Indonesia was in 45th position of 50 countries. According to Johar (2018), the low achievement of Indonesian students in international assessments is influenced by several factors. One of them, Indonesian students are not familiar with the form of the questions demand to reason. In line with this, according to Linola (2017), teachers are expected to get used to their students to get used to deal with questions that contain reasoning. One of the questions now is a trend and is used in international assessments, namely Higher -based questions Order Thinking Skills (HOTS). Unlike Linola (2017), who analyzes reasoning abilities mathematically students in solving story problems, researchers in this study will see how students' mathematical reasoning ability in solving HOTS problems for the topic of two-dimensional shape. The mathematical reasoning indicators analyzed in this study are (1) able to make conjectures; (2) able to use the correlation of two-dimensional characteristics; (3) able to create valid arguments with steps systematic; and (4) able to make conclusions.

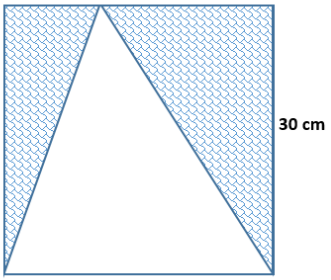
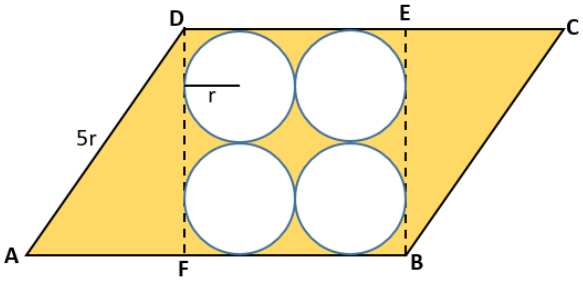
RESEARCH METHODS

This study use descriptive qualitative method. The analysis referred to in this research is on students' mathematical reasoning ability in solving HOTS problems. The subject consisted of 6 students of Junior High School in Surakarta. They were students of 8th grade. Subjects were selected by purposive random sampling technique. Furthermore, the data needed in this study was taken through giving a test. The questions are about two-dimensional shape problems. The problems contain of HOTS indicator, they are analyzing and evaluating which are C4 and C5 of revised Taxonomy Bloom. The obtained data were analyzed based on indicator of mathematical reasoning ability.

RESULTS AND DISCUSSION

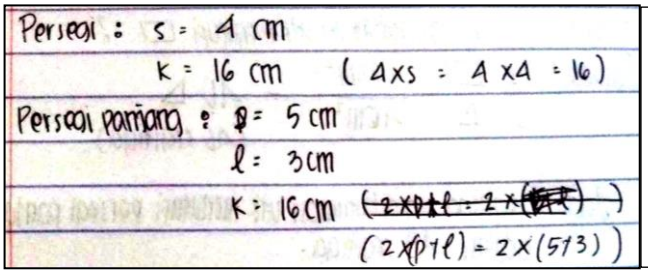
In this study, indicators of mathematical reasoning abilities used are results in the synthesis of Lithner, Astuty and Rizky namely: (1) able to make conjectures; (2) able to use the correlation of two-dimensional shape characteristics; (3) able to create valid arguments with steps systematic; and (4) able to make conclusions. The subjects in this study amounted to 6 subjects chosen by purposive sampling technique. The six subjects are then given 4 problems related to two-dimensional shape containing indicators of HOTS problems. The problems are shown on the Table 1.

Table 1. Items Developed Based on HOTS

Item	Problem
1	Is it possible for a square to have the same circumference with a rectangle? If possible, determine the size of the square and the rectangle!
2	Johan's room will be set with floortile. The area of Johan's room is 30 m^2 while the area of the floortile is 150 cm^2. If one box contains of 30 floortiles, prove that Johan only needs at least 70 boxes of floortiles?
3	Rinjani wants to make a toy with square patterned carton with the side length is 30 cm. If she cuts the carton as shown as below, how many the remaining of unused carton? 
4	If a parallelogram with the length of AD is 5r and there are 4 circles in the parallelogram that has the same size of the radius is r. Is it true that the yellow area is $4r^2$ (7 - π)? Explain your answer. 

From four problems above, item 1 is a problem about analyzing the circumference of square and rectangle. Then, item 2 is about evaluating a statement given, item 3 is about finding the area of triangle by analyzing the information given, and item 4 is about evaluating the problem of parallelogram and circles area that relate each other. Item 1 and 3 contain of HOTS indicator, it is C4. Beside, item 2 and 4 contain of HOTS indicator, it is C5.

For the first problem, all subjects cannot understand the question. Then after getting explanation from the researcher, subject can write down the answer. The following is one of the answer of the subject.

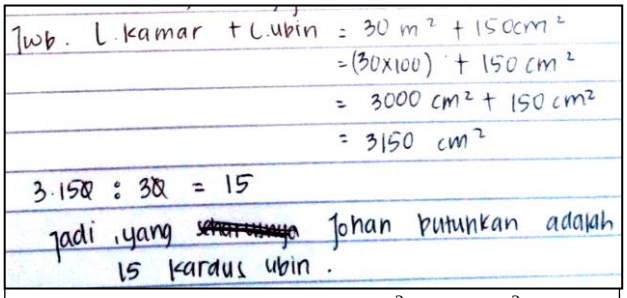


Square: s = 4 cm
Circumference = 16 cm
(4 x s = 4 x 4 = 16)
Rectangle: length = 5 cm
Width = 3 cm
Circumference = 16 cm
(2 x (l + w) = 2 x (5 + 3))

Figure 1. Subject Can Make Conjectures

The Figure 1 is an answer from subject who tried to give an example of the size of the square and the rectangle that has the same circumference. From the answers of the subject, it can be seen that the subject fulfills the first indicator, namely is able to make conjecture. The conjecture is in the form of square and rectangular sizes that might have same circumference. However, students have not been able to show the correlation from the information and do not conclude the answers to the problems given. Thus, it appears that students have difficulty in writing the answers referred to in question so that only the first indicator is fulfilled by the subject.

Furthermore, for the second problem, some subjects are able to answer correctly although not all indicators of mathematical reasoning can be fulfilled. Here's the answer from one of the subjects that can be seen on the Figure 2.

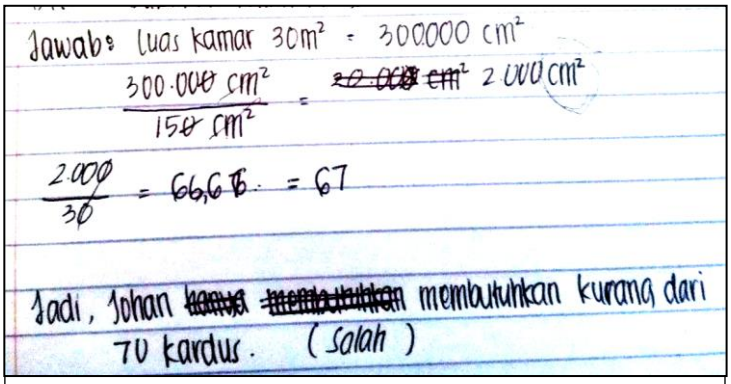


Room area + floortile area = 30 m^2 + 150 cm^2
= (30 x 100) + 150 cm^2
= 3000 cm^2 + 150 cm^2
= 3150 cm^2

3150 : 30 = 15

Figure 2. Subject Failed in Making Conjectures

The subject's answer of the second problem shows that the subject is able to write down the information of the question. However, the subject failed in making conjectures because of the weakness in understanding of the problem. Subject made summary of the room area and the floortile area. However, other subjects are able to solve the problem correctly even though it is still wrong in compiling the conclusion like the answer on the Figure 3.



So, Johan needs at least 70 boxes (wrong)

Figure 3. Subject Can Answer Correctly But Not Perfect in Making Conclusion

Then, for the third problem which contains the HOTS indicator, it is analyze the problem by connecting the characteristics of quadrilateral. On the problem, the average subject can answer correctly even though it is not perfect in answering as in the following figure.

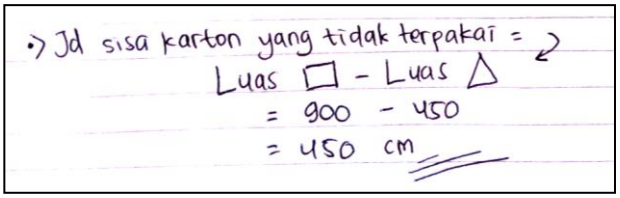
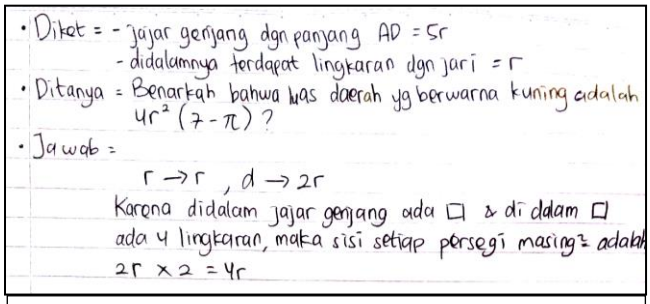


Figure 4. Subject Make a Mistake in Using The Unit

From the figure 5, the subject has arrived at the ability to make conclusions. However, there is an error in the final answer. That is an error in using the unit. Subject writes down 450 cm as her answer, even though it should be 450 cm^2. Finally, for the last problem, none of subject is able to answer correctly. Some of them get errors in the calculation process that contains algebra. It can be seen as in one of the answers to the following subject.



r → r , d → 2r

Because in the parallelogram there is a square and in the square there are 4 circles, so every side of the square is 2r x 2 = 4r

Figure 5. Subject is able to use the correlation of circle and parallelogram characteristics

According to Figure 5 above, actually subject has been able to make conjecture by identifying the size of the square which is part of the parallelogram through the radius of the circles. In addition, the subject also knows what to look for firstly. Subject is looking for AF which is part of the ABCD parallelogram base using Pythagoras Theorem as shown on the Figure 6.

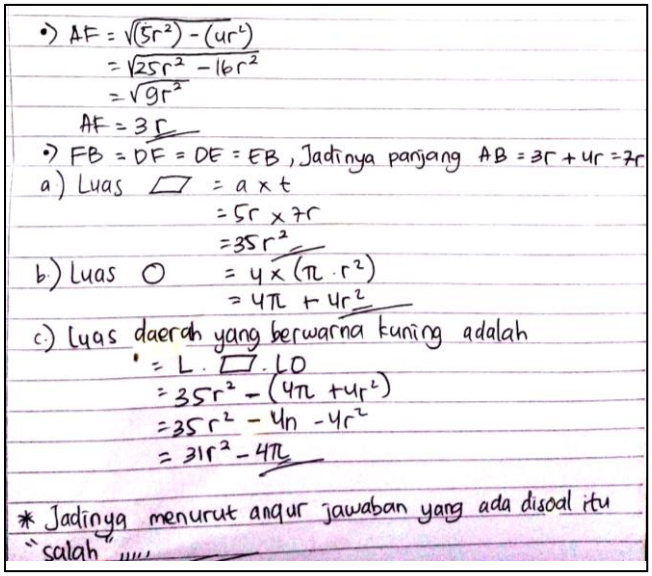


Figure 6. Subject Failed in Creating Systematic Steps

Subject failed to identify the base and height of parallelogram. Subject thinks that the base is 5r while it should be 7r, while the height according to the subject is 7r whereas the truth is 4r. In this case, the subject falls/ fails on the second and third indicators, namely in using patterns and relationships, in this case, is which link the parallelogram the base and height should be perpendicular to each other. Besides, the subject also failed on the third indicator that is in compiling valid arguments with systematic steps, namely the actual subject already in the correct process in finding the area of a circle, but there is an error in the process algebraic multiplication calculations.

After the student's work is corrected by the researcher, then the researcher calculates the total score obtained by the students, after that it is changed to percent form. Table 2 is the percentage of results from student answers based on indicators of mathematical reasoning ability.

Table 2. The Results of The Percentage Mathematical Reasoning Ability of Each Item

Indicators of Mathematical Reasoning Ability	Percentage of Correct Answer (%)				Average
	Item 1	Item 2	Item 3	Item 4	
Able to make conjecture	100%	92%	92%	75%	90%
Able to use the correlation of two-dimensional shape characteristics.	100%	83%	92%	75%	88%
Able to create valid arguments with systematic steps.	50%	75%	92%	42%	65%
Able to make conclusion.	50%	70%	83%	33%	59%

Based on Table 2, it can be seen that most of the students get difficulties in solving HOTS problems that are given when viewed from the mathematical reasoning ability indicator especially on third and fourth indicator that get the lowest average. Actually students are able to fulfill the first indicator, namely in making conjectures. It can be seen on the test result is 90% which indicates that most of them know what must have to be solved first and know what the formula that can be used to solve the problem. It is same with Gunhan (2014) that students can identify the existing pattern as well as the mathematical relationship within the pattern by using the self-similarity and repetition rules of fractals. However, students begin to get difficulties when face second and third indicators, namely in connecting the characteristics of two-dimensional shape and doing systematic steps in solving problems. It can be seen from the result is 88%.

Some students face few difficulties when use the correlation of two-dimensional shape. They were confused when they have to differentiate the concept of area and circumference. It can occurred when students did not understand the concept well. They just knew the formula of area and circumference but they did not know the meaning of area and circumference. Sarfaty and Patkin (2013) said that examples of solids in different positions and nonexamples of the same concepts are an important stage in building children's conceptual comprehension. Therefore, teacher should let the students know which is the area and which is the circumference from the same of two-dimensional shape and the concept in daily life.

In creating valid arguments with systematic steps, students tend to get difficulties when face algebra in their calculation. Only 65% that can fulfill this indicator. Most of them have a weakness in algebra, so when they do calculation process and they face algebra, they will make error in calculation. Relate with Samson (2012) stated, it was difficult for students to transition from a visual expression to an algebraic expression, where this balance can be expected to help students. Students usually face a problem of geometry with number as the size of the two-dimensional shape. They were not usual to solve the problem in algebra form. It can be shown that there are no students that can solve the problem of item 4 correctly.

On the fourth indicators, actually most of students can create conclusion correctly but because of the error in calculation, the conclusion be wrong. Only 59% that can fulfill this indicator. Meanwhile, even some students did not write down the conclusion. They just wrote the calculation without the conclusion. It means that some of students mathematical reasoning ability is still fair. For this reason, as mentioned by Sandy (2019) that student with fair mathematical reasoning ability face difficulty in making conclusion based on the problem. However, in this case, based on the result there are 59% of the student can create conclusion. It can be indicated that student mathematical reasoning ability in solving Higher Order Thinking Skills (HOTS) problems is on medium level.

CONCLUSION

Based on the results and discussions, it can be concluded that mathematical reasoning ability student in solving HOTS problem for the topic of two-dimensional shape is on the medium level. It is based on the test analysis. When viewed from the indicators of mathematical reasoning abilities, the number of students who can make conjectures is 90%, 88% students can use the correlation of two-dimensional shape is characteristics, 65% students can create valid arguments with systematic steps, and 59% students are able to make conclusion. According to this result, one of the efforts that can be used to improve mathematical reasoning ability is by

using HOTS problems or non-routine problems in the student's daily studying. Besides that, choosing the right strategies' process can develop a meaningful learning process that will make students having more flexibility decisions in solving anything mathematical problems.

REFERENCES

Ahmad, S., Prahmana, R.C.I., Kenedi, A.K., Helsa, Y., Arianil, Y., & Zaini, M. (2017). The Instruments of Higher Order Thinking Skills. *J. Phys.: Conf. Ser.* 943 012053

Astuty, E. S. W., Waluya, B., Sugianto. (2019). Mathematical Reasoning Ability Based on Self Regulated Learning by Using The Learning of *Reciprocal Teaching* With RME Approach. *UJMER : Unnes Journal of Mathematics Education Research,* 8 (1) (2019) 49 – 56. http://journal.unnes.ac.id/sju/index.php/ujmer.

Baharudin. (2007). *Psikologi Pendidikan*. Yogyakarta: Ar Ruzz Media.

Faradillah, A. (2018). Analysis of mathematical reasoning ability of pre-service mathematics teachers in solving algebra problem based on reflective and impulsive cognitive style. *Formatif: Jurnal Ilmiah Pendidikan MIPA*, *8(2), 119-128*. http://dx.doi.org/10.30998/formatif.v8i2.2333

Gunhan, B. C. (2014). A Case Study on The Investigation of Reasoning Skills in Geometry. *South African Journal of Education, 34(2), 910-929.* http://www.sajournalofeducation.co.za.

Johar, R., Yusniarti, S., & Saminan. (2018). The analysis of proportional reasoning problem in the Indonesian mathematics textbook for the junior high school. *Journal on Mathematics Education, 9(1),* 55-68.

Khalimi. (2011). *Logika; Teori dan Aplikasi.* Jakarta: Gaung Persada Press.

Kemdikbud. (2014). Permendikbud Nomor 58 Tahun 2014

Linola, M. D., Marsitin, R., & Wulandari, TC. (2017). Analisis Kemampuan Penalaran Matematis Peserta Didik Dalam Menyelesaikan Soal Cerita Di Sman 6 Malang. *Pi: Mathematics Education Journal, 1(1)*, *27-33.* http://ejournal.unikama.ac.id/index.php/pmej.

Mardiyah, I., Suhito, & Safaatullah, M. F. (2018). Analysis of mathematical reasoning ability of junior high school students of grade VII viewed from cognitive style on problem based learning with mind mapping. *Unnes Journal of Mathematics Education*, *7(2), 122-128*. doi: 10.15294/ujme.v7i2.24883

NCTM. (2000). *Principle and Standards for School Mathematics*. Reston VA: NCTM.

Poon, K. K. & Leung, C. K. (2016) A study of geometric understanding via logical reasoning in Hong Kong *International Journal for Mathematics Teaching and Learning (IJMT),* 17(3).

Rizqi, N.R., & Surya, E. (2017). An Analysis Of Students' Mathematical Reasoning Ability In VIII Grade Of Sabilina Tembung Junior High School. *JARIIE-ISSN(O)-2395-4396, 3(2), 3527-3533.* www.ijariie.com.

Samson D (2012). Pictorial pattern generalisation: Tension between local and global visualisation. *Pythagoras: Journal of The Association for Mathematics Education of South Africa, 33(3), 172-181.* http://dx.doi.org/10.4102/pythagoras.v33i3.172

Sandy, W. R., Inganah, S., & Jamil, A. F. (2019). The Analysis of Students' Mathematical Reasoning Ability in Completing Mathematicalproblems on Geometry. *Mathematics Education Journals, ISSN: 2579-5260 (Online), 3(1), 72-79.* http://ejournal.umm.ac.id/index.php/MEJ.

Sarfaty, Y., & Patkin, D. (2013). The ability of second graders to identify solids in different positions and to justify their answer. *Pythagoras: Journal of The Association for Mathematics Education of South Africa*, *34(1), 212-222*. http://dx.doi.org/10.4102/pythagoras.v34i1.212