

Comparison of Dissonance Distributions Between Timbres of Different Instruments

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Abstract—The definition of consonance is the level of pleasant and stable people feel when two tones played simultaneously. For example, octave pitches are considered as consonant and adjacent pitches are considered as dissonant. The prevalent theory to explain this phenomenon is Pythagoras's theory that the simpler the frequency ratio between two tones is, the more harmonious the sound is perceived. This quantitative representation of pitches implies that the timbres of instruments can also be calculated, which helps grasp the characteristics of timbres and therefore deepen the explanation of the formation of musical styles. This paper makes a comparison of the dissonance distribution of the spectrum aspect of timbres of different instruments. By applying the Discrete Fourier transform to decompose sounds, the dissonance levels of 250 sounds of 15 kinds of instruments are calculated and plotted on a statistical line diagram representing the dissonance distribution of instruments. Then, the experiment analyzes the characteristics of dissonance distributions of instruments in different music genres. Results admitted that there may be potential links in dissonance distributions, and further investigation is needed.

Keywords: *dissonance, statistics, music genres, the Fourier transform*

I. INTRODUCTION

Any sound consists of three elements: timbre, pitch, and loudness. Chords, two or more notes played together, can be simplified as combinations of the pitch. We generally know that some chords are relatively attractive (consonance), while others are less attractive (dissonance)[1]. Musicians and theorists have long believed that levels of consonance and dissonance depend on pitch's ratio (the degree of overlap of sound waves) between tones[2][3][4][5]. The subsequent studies will confirm this conclusion. Moreover, some scientists have proposed that biological evolution also has an impact in this area after synthesizing researches in mathematics, physics, music, psychology, and philosophy[6][7][8].

But apart from the pitch's research, the research on timbre is relatively rare. Therefore, in this experiment, we will combine the physical method of measuring consonance level between pitches to graphically represent the relationship between different musical instruments' timbre by summarizing the timbre graph of each instruments and analyzing timbres of instruments in different types of music (rock, jazz, classical, country) and classes of instruments (stringed, wind), and finally get the characteristics of the timbre of different musical instruments.

II. METHODOLOGY

A. Definition of Timbre

Timbre is a way for the human to distinguish different instruments when pitch, loudness, and duration are identical [9]. However, several pieces of researches proved that this definition is insufficient. For example, when a recorded piano tone is played backward, even the spectra is the same with the forward one, the perceived timbre is completely different from it [Berger, 1964]. From this, scientists concluded that musical timbre does not only depend on the physical dimension. From past researches, scientists found other factors, such as the amplitude and phase patterns of components and particularly the temporal characteristics of a tone, may influence timbre perception as well[9]. Therefore, timbre not only represents the spectrum of note but also represents temporal information about the transient regimes of a note.

B. The Fourier Transform

The Fourier transform can express a function satisfying certain conditions as several trigonometric functions (sine and cosine functions) or a linear combination of their integrals [9].

Every sound in life is composed of frequencies and loudness, which varies with time. Most of the time, the audio file only displayed the time vs. loudness graph. By applying the Fourier transform, we can map the original graph (time vs. loudness) into multiple loudness and frequency relations.

However, when people use the Fourier transform to transfer sounds, the information of temporal characteristics, one of the factors that influence the timbre of instruments, is lost. Therefore, multi-dimensional analysis is needed. In this experiment, I only focus on the spectrum extent of the timbre, and the temporal characteristics will be discussed in the Discussion section.

C. Consonance Level of Pitches

If we look at the fundamental wave of a single sound, we can analyze the harmony of different pitches. When a person hears two pitches at the same time (for simplification, imagine that you hear a piano chord), what enters the brain is two fundamental waves, and the brain will analyze their consonance level by calculating the ratio between wavelengths of them.

For example, assume the wavelength of one frequency is one basic unit and that of the other one is two-thirds of the basic unit. Subsequently, their waves will coincide every 2 basic units, which can be easily captured by the brain, so the two sounds are very harmonious. Similarly, assume the wavelength of one sound is $13/3$ basic units and the other is $307/405$ basic units, they will overlap every $3991/1215$ basic units, and this overlap is difficult for the brain to capture, so they sound dissonance [3][10].

Since the speed of sound is the same, the frequency of a wave is inversely proportional to its wavelength. From this point of view, the ratio of two frequencies is an indirect representation of the degree of consonance of two sounds. The smaller the denominator of that ratio, the easier the coincidence interval of the two frequencies can be captured, and therefore the more consonant they are. However, there are exceptions. For example, when the ratio of two waves equals to $(e^{\pi} - \pi)/15$, the value is close enough to $4/3$, and this little difference is difficult to be recognized by the brain, which means that the two sounds are also consonant [3][10][11][12].

Thus, the original conclusion can be modified to that when the ratio is close enough to a rational number with a smaller denominator, and the two pitches are consonant [10][11][12]. Each pitch has its specific frequency in real life, as shown in Table I.

TABLE I. FREQUENCIES OF 12 PITCHES (/HERTZ)

C4	C#4	D4	D#4	E4	F4
261.5	276.4	293.4	310.9	329.5	349.1
F#4	G4	G#4	A4	A#4	B4
369.8	391.7	415	439.8	466	493.7

III. EXPLANATION OF PROCEDURES OF THE EXPERIMENT

A. Audio Input

In order to compare the consonance level between instruments, we need lots of samples sounds at different pitches from different instruments. By sampling the recorded sounds online from the website <http://www.freeecardgreeting.net>, Applying artificial sounds can prevent actual external factors like environmental conditions, types of materials used in instruments. Furthermore, artificial sounds avoid the unstable nature of the timbre of real sounds and enable me to control the variable. Moreover, to avoid the influence of external noise on the experimental results, this experiment adopts the method of internal recording of the computer, storing the internal sound of the computer directly in the form of electromagnetic waves and eliminating the external recording process. A software, ApowerREC, can achieve the goal of internal recording.

This experiment recorded sounds from 15 instruments: guitar, electric guitar, bass, violin, viola, cello, banjo, cornet, trumpet, soprano saxophone, alto saxophone, clarinet, flute, piano, and French horn. All the stringed instruments imitate the sounds played by using bows only. In addition, this experiment only used part of notes of the tessitura of instruments (250 audio files in total) due to the limitation of digital resources. The specific data are shown in the Table II.

TABLE II. DIFFERENT AUDIOS FOR DIFFERENT INSTRUMENTS

Instrument	Audio Files ^a
Guitar	C4, C#4, D4, D#4, E4, F4, F#4, G4, G#4, A4, A#4, B4, C5
Electrical guitar	C3, C#3, D3, D#3, E3, F3, F#3, G3, G#3, A3, A#3, B3, C4, D4, E4, F4, E4, A4, B4, C5, D5, F5, G5, A5, B5
Bass	A2, A#2, C3, D3, D#3, E3, F3, F#3, G3,
Violin	G#3, F4, F#4, G4, G#4, A4, B4, A5
Viola	A#2, B2, C3, C#3, D3, D#3, E3, F3, F#3, G3, G#3, A3, A#3, B3, C4
Cello	B2, C3, D3, D#3, E3, F3, F#3, G#3, A3, A#3, B3, C4
Banjo	C4, C#4, D4, D#4, E4, F4, F#4, G4, G#4, A4
Cornet	B3, C4, C#4, D4, D#4, E5, F5, F#5, G5, G#5, A5, A#5, B5, C6
Trumpet	F3, F#3, G3, G#3, A3, A#3, B3, C4, C#4, D4, D#4, F4, F#4, G4, B4, C5, D#5, E5, F5, F#5, G5, G#5, A#5, C6, A6, B7
Soprano	G2, A2, A#2, C3, E3, G3, A3, A#3, C4, E4, G4, A4, B4,
Saxophone	C5, C#5, D5, D#5, G5, G#5, A5, A#6
Alto saxophone	E2, F2, F#2, G2, G#2, A2, B2, C3, C#3, D3, D#3, E3, F3, F#3, A3, C4, D4, D#4, E4, F4, F#4, A4, B4, C5, C#5, D5, D#5, E5, F5, F#5, A#5, D6
Clarinet	C4, C#4, D4, D#4, E4, F4, F#4, G4, G#4, A4, A#4, B4, C5
Flute	C4, C#4, D4, D#4, E4, F4, F#4, G4, G#4, A4, A#4, B4, C5
Piano	G3, B3, D4, D#4, E4, F4, F#4, G4, G#4, A4, A#4, C5, C#5, D5
French horn	C3, C#3, D3, D#3, E3, F3, F#3, G3, G#3, A3, A#3, B3, C4, C#4, D4, D#4, E4, F4, G4, G#4, A#4, B4, C5, D5, D#5

^a Naming is based on the experiment's requirement. For more information, please refer to the Naming of Audio File below.

B. The Naming of Audio Files

Since the basic tonality of each instrument is slightly different, two pitches that have the same fundamental waves in two different instruments may have different pitches' names. Therefore, in order to make the analysis process more convenient and improve the accuracy of the results, in this experiment, the original pitches' names of instruments are adjusted to the "concert-pitch" notation. For example, the frequency of the fundamental wave of G in a B-flat clarinet is the same as that of the central F in a piano. In this experiment, the pitch will be uniformly named F4.

In this experiment, the number will be used to distinguish the unified phonetic names of different octaves. The number "4" will represent central octave, "3" will represent a group of the first octave below the central octave, "5" will represent a group of the first octave above the central octave, "6" will represent a groups of the second octave above of central octave, and so on. For example, G4 represents central Sol, and F5 represents Fa in the first octave above the central one.

C. Calculating the Consonance Level Between Timbres

In data processing, all audio files are converted into the frequency domains in the application of a fast Fourier transform (FFT) algorithm of MATLAB. Then the findpeaks function of MATLAB chooses the local maximum in frequency domain. The intensities and the corresponding frequencies of the selected peaks are recorded. The recorded intensities and the corresponding frequencies are sorted in descending order. This experiment preserves all the frequency beans (waves), the intensity of which are larger than ten percent of the maximum intensity of the frequency. The intensity of all frequencies obtained in the above experiments will be normalized by divide with the intensity of the highest peak to set all loudness equal. In this way, the loudness dependence to audio for each are eliminated to ensure the accuracy of the magnitude of intensity. Finally, we calculates the discordance between two sounds in MATLAB program. According to Plomp and Levelt's work [10], when considering sounds with spectra that are more complex, dissonance can be calculated by summing the dissonance of all the partials, and weighting them according to their relative amplitude. To be concrete, the dissonance between a sinusoid of frequency f1 and a sinusoid of frequency f2 can be calculated as [10],[12]

$$D(f_1, f_2, v_1, v_2) = v_1 v_2 \{ \exp(-a s |f_1 - f_2|) - \exp(-b s |f_2 - f_1|) \} \tag{1}$$

where

$$s = d^* / (s_1 \times \min(f_1, f_2) + s_2) \tag{2}$$

Here a=3.51, b=5.75, d*=0.24, s1=0.0207, and s2=18.89 are determined parameters in Sethares's work. v1 and v2 are the amplitude of f1 and f2.

Since couple of different frequencies were used for each musical note, it is necessary to consider musical sounds comprehensively in order to make a complete comparison between two sounds. Therefore, two sets of frequencies of two musical sounds are compared pair by pair to calculate the dissonance level between them. Due to unequal number of applied frequencies from each sound, the resulting magnitude is the average dissonance levels rather than sum of all dissonance levels.

$$D(i,j) = \text{avg}(\sum f_i \sum f_j (d(f_i, f_j, v_i, v_j))) \tag{3}$$

(i≠j and note of i and j are not the same)

Correspondingly, the average dissonance level between every two sounds in the same instrument but from different notes are determined, and the statistical value is expressed by a three-dimensional bar chart, showing the distribution of the level of dissonance. Fig. 1 shows an example of this comparison.

Finally, the values of average dissonance levels are collectively converted into histogram. For example, there are nine vertices, 20% of the total number of vertices, in the distribution graph have intensity 1, then the vertices (1, 0.2) are presented in the final line chart. In this way, the line chart can accurately reflect the timbre of the instrument to a certain extent, and the line charts of different instruments are used to

compare the timbres between instruments, and therefore to compare the dissonance level between instruments.

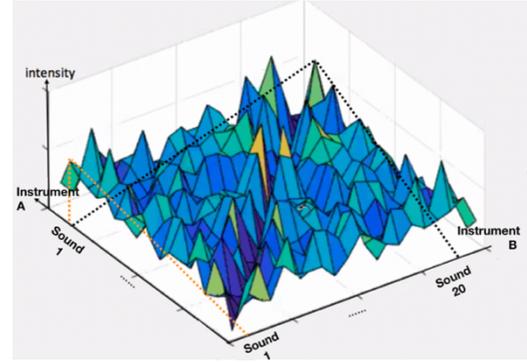


Fig. 1. The distribution of the dissonance level in an instrument.

IV. RESULTS

A. Distribution of Dissonance Levels of Different Instruments

Firstly, this experiment will categorize the instruments into two different classes: stringed instruments and wind instruments.

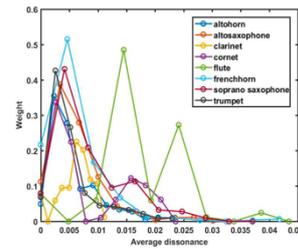


Fig. 2. The distributions of dissonance levels of wind instruments.

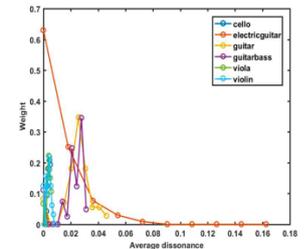


Fig. 3. The distributions of dissonance levels of stringed instruments.

From Fig. 2 and Fig. 3, wind instruments have a clearer overall distribution trend, a decreasing trend, than stringed one, and dissonance distributions of the same type of instruments (e.g. alto saxophone and soprano saxophone) have more similar pattern than that of different types of instruments. In order to make a clear description, I categorize all distributions into three categories: long tailed descending (e.g. alto horn), short symmetrical (clarinet), long symmetrical (guitar), and exponential descending (electric guitar).

In the distribution of stringed instruments, dark blue, light blue, and green represent cello, violin, and viola, which belong to the violin category and they have similar short symmetrical dissonance distribution. Moreover, the dissonance distributions of guitar and guitar bass are similar too: they both have long symmetrical dissonance distributions. However, the electric guitar has the unexpected exponential descending trend that is not similar with the guitar and any other stringed instruments. This is the reason why the overall distribution of stringed instruments is incongruent, which means there are noticeable differences in the dissonance distributions among them.

In contrast, the overall distributions of wind instruments are similar: long-tailed descending distributions. Particularly, soprano saxophone and alto saxophone have very similar dissonance distributions: their tails decreasing in a similar slop compared with other long-tailed distributions. The distributions of alto horn and trumpet are also similar, which is unexpected since they sound completely different in life. In addition, the clarinet and flute have distinct distributions in wind instruments: the clarinet has a short symmetrical distribution, which is similar to violin family, and the flute has a long symmetrical distribution, which is similar to guitar and guitar bass.

Fig. 2 and Fig. 3 explain that the timbre distributions of musical instruments are diverse, and there is no distinct pattern for each trend, but the overall distributions are mostly similar, which can be roughly divided into four main distribution trends introduced above.

B. Dissonance Distribution of Timbres in Different Musical Styles

From the above conclusions, it is easy to see that there exist internal connections between patterns of dissonance distributions of musical instruments. In this experiment, we took classical music, country music, rock music, and jazz as examples.

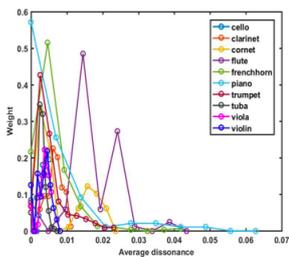


Fig. 4. Timbre distribution of common instruments in classical music.

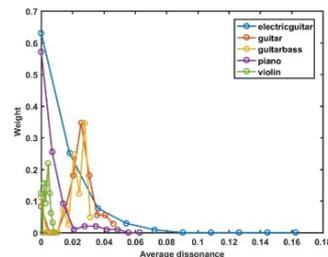


Fig. 5. Timbre distribution of common instruments in country music.

In classical music, as shown in Fig. 4, cello, clarinet, violin, cornet, trumpet, horn, trumpet, flute, and piano are selected as common instruments and analyzed. Although symphonies have various instruments, they uniformly have descending patterns, and most of the distributions concentrate on the left corner of the graph. Most of them belong to heavy-tailed and symmetrical shapes and the piano has the similar distribution as electric guitar.

In country music, as showed in Fig. 5, electric guitar, guitar, bass, piano, and violin are chosen as common instruments and analyzed. The timbre distributions of country music are different from that of classical music: it contains all 4 types of distributions. Compared with classical music, country music has a incongruent overall distribution of dissonance.

Alto horn, alto saxophone, soprano saxophone, banjo, trumpet, cornet and tuba were chosen as examples for analysis in jazz music shown in Fig. 6 and Fig. 7. Dissonance distributions of instruments in jazz music are downward trend. Unlike classical music, trends of instruments in jazz mainly have a long-tailed and short symmetrical pattern. The overall

distribution also concentrate on the left bottom corner of the graph.

Finally, in the analysis of rock music, electric guitar, guitar, bass, and piano are used as examples. Instruments in rock music show a descending and symmetrical trend. Compare with jazz music, rock music has a less clear overall pattern.

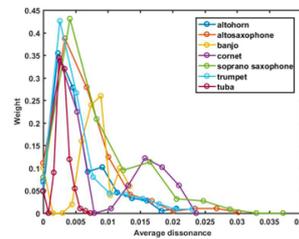


Fig. 6. Timbre distribution of common instruments in jazz.

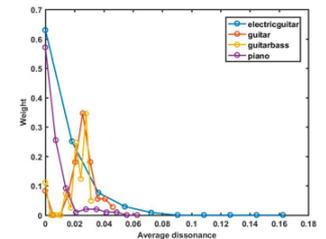


Fig. 7. Timbre distribution of common instruments in rock.

V. DISCUSSION

Most of the results calculated in this experiment are not unexpected, showing the connections between the spectra aspect of timbre of instruments. However, the distribution of electric guitar is the only unexpected result, and this exponential descending trend may be caused by its electrical nature, which is different from guitar. In addition, classical music and jazz music have clearer overall distribution trends, and they were also formed earlier than the other types. Therefore it may indicate that music genres will become more and more unique, composing of more diverse instruments in the future. Further research is needed.

However, this experiment only focuses on the spectrum extent of the timbre, thus the resulting pattern can only show partial characteristics of timbres. That why instruments with similar patterns of dissonance distribution, like electric guitar and guitar, may sound similar in real life and instruments with similar curves may sound completely different in the human ear. In order to obtain a multi-dimensional analysis that can represent the timbres of instruments completely, further investigations in this area are needed. Moreover, since I normalized all the sounds taken in the experiment, I didn't take the playing intensities (dynamics) of instruments into account. However, in real life, different playing intensities may also affect the dissonance level of the timbres. Further research is needed too. For the sounds recording part, an improvement in method of finding the more accurate way to represent the real sounds is needed. The generalizability of the results is somehow limited, since not all the notes of the instruments are considered.

VI. CONCLUSIONS

In this experiment, ApowerREC is used to record 250 music sounds from 15 kinds of musical instruments. MATLAB computer programming is used on data processing, including the Discrete Fourier Transform, decomposition and normalization of sounds, and estimation of dissonance between sounds. Then, the experiment collectively captured the different values of the same instrument in order to draw a statistical line diagram (dissonance versus weight) representing

the timbre distribution of that instrument. Finally, I tested the feasibility of the timbre distribution and analyzed its application in different music types. As a result, the experiment concludes that music types are related to the instruments used in it.

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