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Research Article

Data-based Analysis Methods for the State Controllability and State Observability of Discrete-time LTI Systems with Time-delays

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ABSTRACT

We present a couple of data-based methods to analyze the state controllability and state observability of discrete-time Linear time-Invariant (LTI) systems with time-delays, which have unknown parameter matrices. They first augment the system into a high dimensional LTI model, then apply the measured state and output data to directly build the controllability and observability matrices respectively of this high dimensional model, whose ranks are used as the criteria of the corresponding properties of the system before augmentation. These data-based methods have low computational load and calculation complexity.

1. INTRODUCTION

In the control theory and engineering area, time delay universally and frequently happens in daily life and practical industries, which makes the time-delay systems a class of important research objects [1,2]. There are some typical examples of time-delay systems, such as the artificial neural networks [2], the multi-agent systems [3], the electronic and optical systems [4], etc. As an essential property of the real world, even by using the state-of-the-art technology, the impact of time delays cannot be completely eliminated. Hence, the extensive research on characteristics of systems with time delays, for instance, the stability and robustness [5], the controllability and observability [6], etc. is both theoretically and practically significant.

But when it is put into practice, people always encounter difficulties in both theoretical and practical areas. One typical difficulty is about the system dimension: when it is in the continuous-time domain, it can be regarded as an infinite dimensional system, which has a transcendental equation with infinite solutions as its characteristic equation [7]; when the system is discrete-time, its dimension increases dramatically with the increase of the time-delay terms [1,3,8]. In addition, nowadays modern industries have become so large-scale and complex that correctly building their dynamic models has become more and more difficult and even impractical [9,10]. All these problems have prevented analyzing the characteristics of time-delay systems.

Nevertheless, it is fortunate that the modern sensor technologies, the wireless communication techniques, the computer and internet science and technologies, etc. have been developed rapidly, and have been widely used in more and more fields. People acquire all kinds of data from industrial production and daily life in more effective and systematic ways [10]. The information contained in the measured data usually reflects the intrinsic nature of the systems [11]. In the big data era, this situation is very obvious, so the idea of developing data-based methods is naturally generated, which only use the measured data to analyze the characteristics of time-delay systems.

In most of the published research reports, the data-based analysis methods for system characteristics could only deal with particular delay model structures, but may not be able to analyze the systems with a general form of time delays. For the sake of overcoming these problems, in this paper we will develop a couple of data-based methods to analyze the state controllability and state observability of a general class of discrete-time delayed Linear time-Invariant (LTI) systems. They can directly determine the system characteristics just by utilizing a simple augmented state-space model together with the measured historical and current testing data, without identifying the unknown parameter matrices. Their advantages are discussed from the aspects of identification workload and computational complexity.

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2. DESCRIPTION OF THE PROBLEM

This study concerns the analysis problem of the state controllability and state observability of the general discrete-time LTI systems with time delays in the following form:

$$\begin{cases} x(k+1) = \sum_{i=0}^{N_x} A_i x(k-i) + \sum_{j=0}^{N_u} B_j u(k-j), \\ y(k) = \sum_{l=0}^{N_y} C_l x(k-l), \end{cases} \tag{1}$$

where $u \in \mathbb{R}^m$, $x \in \mathbb{R}^n$, $y \in \mathbb{R}^q$ are the input, the state and the output of system (1), respectively; the time delays $1 \le N_u, N_y \le N_x < \infty$ are integer constants and the time index $k \ge -N_x$; among each group of the parameter matrices $A_1,\ldots,A_{N_x} \in \mathbb{R}^{n\times n}$, $B_1,\ldots,B_{N_u} \in \mathbb{R}^{n\times m}$ and $C_1,\ldots,C_{N_y} \in \mathbb{R}^{q\times n}$, there is at least one nonzero matrix.

This paper supposes that these A_i, B_j and C_l are all unknown and none of them has random elements. Next below, we will propose a couple of analysis methods to determine the above system characteristics without the necessity of identifying the parameter matrices, but just by utilizing the measured data. The advantage of these data-based characteristics determination methods will also be illustrated.

To begin with, system (1) is first expanded into a high dimensional LTI model by state-space augmentation:

$$\begin{cases} X(k+1) = \hat{A}X(k) + \hat{B}U(k) \\ y(k) = \hat{C}X(k) \end{cases} \quad (k \ge 0), \tag{2}$$

where $U(k) = \begin{bmatrix} u(k) \\ u(k-1) \\ \vdots \\ u(k-N_u) \end{bmatrix}$, $X(k) = \begin{bmatrix} x(k) \\ x(k-1) \\ \vdots \\ x(k-N_x) \end{bmatrix}$ are the input and the state of system (2), respectively; and the augmented parameter matrices are defined as

$$\hat{A} = \begin{bmatrix} A_0 & A_1 & A_2 & \cdots & A_{N_x} \\ I_n & 0 & 0 & \cdots & 0 \\ 0 & I_n & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & I_n & 0 \end{bmatrix},$$
$$\hat{B} = \begin{bmatrix} B_0 & B_1 & \cdots & B_{N_u} \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{3}$$
$$\hat{C} = \begin{bmatrix} C_0 & C_1 & \cdots & C_{N_y} & 0 & \cdots & 0 \end{bmatrix}.$$

where $\hat{A} \in \mathbb{R}^{n(N_x+1)\times n(N_x+1)}$, $\hat{B} \in \mathbb{R}^{n(N_x+1)\times m(N_u+1)}$, $\hat{C} \in \mathbb{R}^{q\times n(N_x+1)}$ and $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix.

Obviously, the state controllability as well as the state observability of both systems (1) and (2), are equivalent.

2.1. State Controllability Analysis with Data-based Criterion

We start with the state controllability in this subsection, where a commonly used criterion is first given below.

Lemma 1. *The necessary and sufficient condition for LTI system $z(k + 1) = Hz(k) + Gv(k)$ $(k \ge 0)$ with the input $v \in \mathbb{R}^m$ and the state $z \in \mathbb{R}^n$, to be completely state controllable, is*

$$\mathrm{rank}([H^{n-1}G, H^{n-2}G, \ldots, HG, G]) = n.$$

Because the state controllability of system (2) is equivalent to that of system (1), based on Lemma 1, we can determine this characteristic by checking the rank of

$$W_C = [\hat{A}^{n(N_x+1)-1}\hat{B}, \ldots, \hat{A}\hat{B}, \hat{B}]. \tag{4}$$

In order to obtain the augmented controllability matrix W_C, the traditional analyzing approaches need to identify the unknown A_0, A_1, ..., A_{Nx}, B_0, B_1, ..., B_{Nu} in system (1) first, and construct matrices $\hat{A}, \hat{B}$ as in (3), then calculate $\hat{A}\hat{B}$, $\hat{A}^2\hat{B}$, ..., $\hat{A}^{n(N_x+1)-1}\hat{B}$, and build W_C with these identification and calculation results. It is natural that the identification errors are often introduced in the above process, and the calculation workload is really heavy especially when the state dimension n and the time delay N_x in state are large.

Due to this situation, to avoid the identification errors and to reduce the workload effectively, it is intended to find a more simple and direct method to construct W_C. This goal can be achieved by utilizing the data-based analysis method given below.

This method starts with a set of $n(N_x + 1) + 1$ tests on system (2). In the first $n(N_x + 1)$ tests, we design the input sequences as $U^{\{p\}}(0)$, ..., $U^{\{p\}}(n(N_x + 1) - 1)$ $(p = 1, 2, \ldots, n(N_x + 1))$, which can make the following $n(N_x + 1)$ vectors

$$V^{\{p\}} = \begin{bmatrix} U^{\{p\}}(0) \\ U^{\{p\}}(1) \\ \vdots \\ U^{\{p\}}(n(N_x+1)-1) \end{bmatrix} (1 \le p \le n(N_x+1))$$

linearly independent. Therefore, the matrix

$$\bar{V} = [V^{\{1\}}, \ldots, V^{\{n(N_x+1)\}}] \in \mathbb{R}^{mn(N_u+1)(N_x+1)\times n(N_x+1)} \tag{5}$$

has the rank of $n(N_x + 1)$. On the other hand, the last test is done on system (2), where the input sequence is set as $U^{\{n(N_x+1)+1\}}(k) \equiv 0$.

Next, let all the corresponding states of system (2) have the same initial value:

$$X^{\{1\}}(0) = X^{\{2\}}(0) = \cdots = X^{\{n(N_x+1)+1\}}(0) = X_0.$$

Measure and record the values of $X^{\{1\}}(n(N_x + 1))$, ..., $X^{\{n(N_x+1)\}}(n(N_x + 1))$ and $X^{\{n(N_x+1)+1\}}(n(N_x + 1))$. With the above particular augmented states, we present the following data-based state controllability criterion.

Theorem 1. *Define*

$$Z_X^{\{p\}} = X^{\{p\}}(n(N_x+1)) - X^{\{n(N_x+1)+1\}}(n(N_x+1)),\ (1 \le p \le n(N_x+1)),$$
$$Z_X = [Z_X^{\{1\}}, Z_X^{\{2\}}, \ldots, Z_X^{\{n(N_x+1)\}}]. \tag{6}$$

Then, both system (1) and system (2) are completely state controllable, if and only if rank $(Z_X) = n(N_x + 1)$.

Proof. With the particular $n(N_x + 1) + 1$ testing input sequences, the same initial state X_0, and the measured augmented state data, we should obtain

$$\begin{aligned} X^{\{n(N_x+1)+1\}}(n(N_x+1)) &= \hat{A}^{n(N_x+1)} X_0,\ X^{\{p\}}(n(N_x+1)) \\ &= \sum_{k=0}^{n(N_x+1)-1} \hat{A}^k \hat{B} U^{\{p\}}(n(N_x+1)-1-k) \\ &\quad + \hat{A}^{n(N_x+1)} X_0, \\ Z_X^{\{p\}} &= \sum_{k=0}^{n(N_x+1)-1} \hat{A}^k \hat{B} U^{\{p\}}(n(N_x+1)-1-k), \end{aligned} \tag{7}$$

for all $1 \le p \le n(N_x + 1)$. Based on Equations (4)–(7),

$$\begin{aligned} Z_X &= [\hat{A}^{n(N_x+1)-1} \hat{B}, \ldots, \hat{A}\hat{B}, \hat{B}] \\ &\times \begin{bmatrix} U^{\{1\}}(0) & \cdots & U^{\{n(N_x+1)\}}(0) \\ \vdots & \ddots & \vdots \\ U^{\{1\}}(n(N_x+1)-1) & \cdots & U^{\{n(N_x+1)\}}(n(N_x+1)-1) \end{bmatrix} \\ &= W_C \bar{V}. \end{aligned}$$

Since $\bar{V}$ has the rank of $n(N_x + 1)$ because of the linear independence of $V^{\{1\}}, V^{\{2\}}, \ldots, V^{\{n(N_x+1)\}}$, then

$$\begin{aligned} \mathrm{rank}(W_C) + \mathrm{rank}(\bar{V}) - n(N_x+1) &\le \mathrm{rank}(W_C\bar{V}) \\ &\le \min\{\mathrm{rank}(W_C), \mathrm{rank}(\bar{V})\} \\ &= \mathrm{rank}(W_C) \end{aligned} \tag{8}$$

Consequently, $\mathrm{rank}(Z_X) = \mathrm{rank}(W_C\bar{V}) = \mathrm{rank}(W_C)$.

By Lemma 1, the sufficient and necessary condition for system (2) to be completely state controllable is rank $(Z_X) = n(N_x + 1)$, and the same holds true for system (1). **q.e.d.**

2.2. State Observability Analysis with Data-based Criterion

After the introduction of data-based state controllability criterion, this subsection will propose the method to determine the state observability of both system (1) and system (2), only by utilizing the measured data. Here, a relevant lemma is given first to make a start.

Lemma 2. *The LTI system*

$$\begin{cases} z(k+1) = Hz(k) + Gv(k) \\ w(k) = Fz(k) \end{cases} \quad (k \ge 0),$$

with the input $v \in \mathbb{R}^m$, the state $z \in \mathbb{R}^n$ and the output $w \in \mathbb{R}^q$, is completely state observable, if and only if

$$\mathrm{rank}([F^T, H^T F^T, \ldots, (H^{n-1})^T F^T]^T) = n.$$

As aforementioned, the state observability of system (1) and system (2) are equivalent. According to Lemma 2, the rank of the following matrix can be used as the criterion of the state observability of system (2):

$$W_{OB} = [\hat{C}^T, \hat{A}^T \hat{C}^T, \ldots, (\hat{A}^{n(N_x+1)-1})^T \hat{C}^T]^T. \tag{9}$$

Similar to the state controllability case, it is necessary to find a new criterion of the state observability only based on the measured data, if the identification errors and a large amount of calculations are to be avoided.

We still start with some tests on system (2), where $n(N_x + 1)$ linearly independent initial states $X^{\{1\}}(0), X^{\{2\}}(0), \ldots, X^{\{n(N_x+1)\}}(0)$ are particularly set first. Such that the following matrix

$$\Phi_X = [X^{\{1\}}(0), X^{\{2\}}(0), \ldots, X^{\{n(N_x+1)\}}(0)] \tag{10}$$

has rank of $n(N_x + 1)$. Then, set corresponding inputs as

$$U^{\{1\}}(k) = U^{\{2\}}(k) = \cdots = U^{\{n(N_x+1)\}}(k) \equiv 0 \tag{11}$$

At the same time, sample the corresponding output data $y^{\{1\}}(k)$, $y^{\{2\}}(k), \ldots, y^{\{n(N_x+1)\}}(k)$ at the sampling instants $k = 0, 1, \ldots, n(N_x + 1) - 1$, with which we let

$$\begin{aligned} Y_p &= \begin{bmatrix} y^{\{p\}}(0) \\ y^{\{p\}}(1) \\ \vdots \\ y^{\{p\}}(n(N_x+1)-1) \end{bmatrix} (1 \le p \le n(N_x+1)), \\ Y &= [Y_1, Y_2, \ldots, Y_{n(N_x+1)}] \in \mathbb{R}^{qn(N_x+1)\times n(N_x+1)} \end{aligned} \tag{12}$$

Next, a data-based criterion on the state observability will be presented below.

Theorem 2. *Suppose that the initial states of system (2) can be set as in (10). Then, system (1) and system (2) are both completely state observable, if and only if*

$$\mathrm{rank}(Y) = n(N_x+1),$$

where Y is defined in (12).

Proof. Since the proving process is similar to that of Theorem 1, this proof is omitted here for brevity. **q.e.d.**

Similar to the previously introduced data-based state controllability criterion, the data-based criterion of the state observability introduced here has some advantages:

(i) it can determine the state observability of discrete-time delayed LTI systems either with (at least one of $A_1, \ldots, A_{N_x}, C_1, \ldots, C_{N_y}$ in system (1) is nonzero) or without time delay;

(ii) it does not need to identify these parameter matrices, such that the identification errors can be avoided;

(iii) it does not need to calculate $\hat{C}\hat{A}, \ldots, \hat{C}\hat{A}^{n(N_x+1)-1}$ for the construction of the observability matrix W_{OB} in Equation (9), which greatly reduces the workload.

3. COMPUTATIONAL COMPLEXITY ANALYSIS

This section will take the data-based state controllability analysis algorithm given in Theorem 1 as a representative, to show the merit of lower computational complexity. In order to make the discussing process simpler, this paper considers both a summation and a multiplication of two matrix elements as one operation. For the discrete-time delayed LTI systems described by (1), when we multiply $\hat{A}^i$ $(1 \le i \le n(N_x + 1) - 2)$ with $\hat{A}$, there should be $n(N_x + 1)$ multiplication operations and $n(N_x + 1) - 1$ summation operations for calculating each element, so that there are totally $n^2(N_x + 1)^2 (2nN_x + 2n - 1)$ operations to obtain $\hat{A}^{i+1}$, since the number of elements in $\hat{A}^{i+1}$ is $n^2 (N_x + 1)^2$.

When $\hat{A}^k$ $(1 \le k \le n(N_x + 1) - 1)$ multiplies $\hat{B}$, there are $n(N_x + 1)$ multiplications and $n(N_x + 1) - 1$ summations for each calculated element, while there are $mn(N_u + 1)(N_x + 1)$ elements in the obtained $\hat{A}^k\hat{B}$. Thus, there are $mn(N_u + 1)(N_x + 1)(2nN_x + 2n - 1)$ operations. And it is needed to do $(nN_x + n - 1)$ such kind of matrix multiplications in total. For computing $W_C = [\hat{A}^{n(N_x+1)-1}\hat{B}, \ldots, \hat{A}\hat{B}, \hat{B}]$, it is needed to perform

$$mn(N_u+1)(N_x+1)(2nN_x+2n-1)(nN_x+n-1) + \sum_{i=1}^{n(N_x+1)-2} i n^2(N_x+1)^2(2nN_x+2n-1)$$

operations. In summary, the computational complexity of the traditional model-based state controllability analysis algorithms is $\mathcal{O}(n^5 N_x^{\,5})$.

As a comparison, by observing (5) and (6), our data-based state controllability analysis algorithm performs only $n(N_x + 1) + 1$ groups of experiments as well as $n^2 (N_x + 1)^2$ element subtractions. So that the developed data-based analyzing method only has the computational complexity of $\mathcal{O}(n^2 N_x^{\,2})$.

By extension, we may conclude that our data-based analysis methods have lower computational complexity than the traditional analysis algorithms.

4. CONCLUSION AND FUTURE WORK

In this paper, some new data-based methods are proposed to analyze and determine the state controllability and the state observability of a general class of discrete-time delayed LTI systems.

These data-based analyzing methods conducted some specific input tests, using the measured historical and current data to directly construct the system matrix, the state controllability matrix and the state observability matrix, which are utilized to determine the corresponding characteristics. Generally speaking, they have merits in two major aspects over the traditional model-based analysis approaches: the less identification work and the lower computational complexity.

Our data-based methods provided a new perspective for the analysis of the time-delay systems, but there are still some shortcomings to be improved. For the current stage, they can only analyze discrete-time LTI systems, but cannot deal with continuous-time, nonlinear or time-variant systems. Besides, if the system is stochastic, we have no other way but to identify all the parameter matrices and the developed data-based analysis methods are no longer feasible. In the future researches, we would like to study these problems and hope to find appropriate methods to overcome these difficulties.

CONFLICTS OF INTEREST

The authors declare they have no conflicts interest.

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