



Editorial

The name of the new journal *Growth and Form* is based on one of the most important science books of the 20th century. *On Growth and Form* by D'Arcy Wentworth Thompson was published in 1917 and it combined a wide range of models used in applied mathematics and physics with biological phenomena and growth, linking biological growth to mathematical models and optimization [1]. For example, chapters on allometry, the relation of biological shapes to constant mean curvature (CMC) surfaces, and the transformation theory [2] have inspired generations of biologists and mathematicians [2]. D'Arcy Thompson's central message is: *'The living and the dead, things animate and inanimate, we dwellers in the world and this world wherein we dwell, - πάντα γὰρ μὲν τὰ γιγνώσκόμενα¹, - are bound alike by physical and mathematical law.'*

In the 21st century, many of the same geometric ideas will continue to play a major role in understanding biology, in which the notion of curvature defined as deviation from what is straight, is crucial. Straight lines are the hallmark of allometric relationships in biology and physics. Peter L. Antonelli and Solange Rutz argue in *Allometric Laws in Modular Dynamics: The Bauplan of Ontogenesis* that allometry as a collection of straight lines can be thought of as a blueprint of intrinsic and extrinsic processes in living and non-living natural organisms, objects and phenomena. Interestingly, they connect this to Hilbert's 4th Problem of classifying Finsler geometries, having straight lines as the shortest distance between two points [3].

Curvature of curves was codified by Newton, in Oresme's tradition. In *Curvature against the grain*, Jan Koenderink conjectures that curvature has been around from the dawn of human civilisation, using different notions of geometry. For ancient hunters, the curvedness of a bow is directly connected to the tension. The opposite are the straight arrows. For the curvature of curves, as deviations of going straight, or in the case of surfaces, deviations from planarity [4] and beyond, there is still plenty of room for further research.

Curvature in plane geometry has been a major topic since the 17th century in geometry, but mainly from an isotropic point of view. New insights and extensions of classic curves and polynomials generate new approaches to study natural shapes and phenomena. Bennett Palmer and Álvaro Pámpano ask (and answer) the question of how Euler's classification of *elasticae* (including lemniscates) is affected if the rod's energy is anisotropic in *Classification of planar anisotropic elasticae*. Previously, a similar question of extending CMC surfaces to the anisotropic case, led to constant anisotropic mean curvature (CAMC) surfaces [5].

In combining geometry and natural phenomena, there is a close and fascinating relationship between shape description and polynomials [6]. Starting from the complex Bernoulli spiral and Grandi curves (a.k.a. rose curves describing the shape of flowers) Paolo Emilio Ricci defines a set of irrational functions, which extend Chebyshev polynomials to fractional degree. *Chebyshev polynomials, Rhodonea curves and pseudo-Chebyshev functions. A Survey* focuses on mathematical aspects, but in *A Note on Spirals and Curvature* the usefulness of pseudo-Chebyshevs of half-integer degree to model natural flowers is shown. Starting from three classes of spirals, this *Note* links classical power law curves with polynomials and different metrics to study natural phenomena. The focus is on a very general understanding of spirals and curvature. A broader notion of anisotropic curvature is introduced based on Pythagorean compact shape descriptors.

This first issue of *Growth and Form* is thus a collection of authoritative articles outlining the scope of Growth and Form, namely a geometrical approach to model and understand natural shapes, their development and evolution. All articles focus on a better understanding of what is already there, in other words, what was known before the advent of computers.

The authors of this first issue of *Growth and Form* are all experts with a long and broad experience in their field. This is also the rationale behind the editorial board. Unfortunately very recently P.L. Antonelli (1941–2020) passed away; he is first author of *Allometric Laws in Modular Dynamics: The Bauplan of Ontogenesis*, and member of the editorial board. He was an expert in mathematical biology [7] and in Finsler geometry [8] and its applications to biology and physics [9].

Growth and Form owes its birth to the enthusiasm of Arjen Sevenster (1946–2019). After a long career as main editor with Elsevier for mathematics, he became a driving force with Atlantis Press in the successful transition from the classical publication business model to the modern Open Access models. *Growth and Form* was one of his favourite projects, and we can only hope that it will become a major source of inspiration for biologists, physicists and mathematicians in modelling natural phenomena, as he envisaged it.

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¹Everything we can ever know

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