

Picturing Textbook on Exponent Equations Based on Praxeology Organization

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Abstract—In studying the exponent equations, high school students are confronted with the concept of exponent equations and applications in doing exercises on exponent equations. This is also reflected in the textbook students used. At the same time, textbook is an important factor in learning. Thus, what about the state of the textbook that contains the material exponential equations? The textbook used in this study is a student textbook. To find out the state of the textbook, researchers will use the theory of praxeological organization. In analyzing the textbook, the researcher uses a reference epistemological model which is then compared with the praxeological organization in the textbook. The results of this study indicate that theory of exponent equations in textbook is not given a detailed explanation why and how a formula can be obtained. In practice the questions, it is only contain application problems from the formulas that have been provided, not problems that are related to the theory of exponent equations. From the results of the research it is expected that the teacher as a mediator between students and textbook needs to provide explanations or additional links with concept and/or questions that are not in the textbook.

Keywords: *textbooks, praxeology organization, exponent equation*

I. INTRODUCTION

Textbooks are important resources not only for students but also for teachers [1]. In mathematics, Robitaille and Travers argued that a great dependence upon textbooks in teaching of mathematics than of any other subject [2]. At the sametime, textbook play an important role as a translator of curriculum policy into operations that teachers and students can carry out [3]. For example, In Indonesian curriculum, there is a term saying that it needed to connect between materials. Then, how exactly a textbook translate this term in to a theory and practice in a textbooks? especially for a theory that is learned gradually sunch as exponent equation.

In order to understand and solve exponent equation, students need to understand the exponential number and properties of exponent. Many research pointed that there is student difficulty to work with property of exponent. For example, where she found that student had a lower success rate when they are ask to compare exponent using decimal numbers as a base and a natural number as a power (e.g. $(0,5)^{21} > (0,5)^{17}$) [4]. From the task, students neglected the basis and

only considered the power, as a result they only compared $21 > 17$. On the other hand, Ulusoy found that students overgeneralized the rule of exponent expression (multiplication and division operation) [5]. For example, students made error when they multiply exponential expression with the different basis e.g. $5^4 \cdot 6^4 = \dots$. In this case, students multiplied the basis and add the power $5 \cdot 6^{4+4}$. Additionally, students tend to multiplay question: $5^2 \cdot 5^4 = \dots$ instead of adding them. Ramazan [4] and Ulusoy [5] research show us that there still many thing left to fix students error on exponential number and exponential properties. However, students are also challenged by time where they need to master the next level of exponent where they are asked to solve exponent equation e.g. $a^{f(x)} = 1$, $a^{f(x)} = a^p$, and etc. Does the textbooks porivide more opportunity for students to learn exponenet equation and connect it with exponential number and exponential properties?

To answer this question, we will use a praxeology organization, a notion developed in Anthropological theory of the didactic [6]. ATD (anthropological theory of didactics) see mathematics and how it's taught belong to numerous institutions. And it is not easy to model because- in a way- we are the stake holder that sometime difficult for us to make a distance and define those didactic phenomena. One of the ways to construct mathematics in different institution such as textbooks is to capture those didactic phenomena using praxelogy organization.

Research on mathematics textbooks analysis using praxeology organization, proposed by Chevallard has become a major attention among researchers [6]. For example, Barbe et al [7] and González-Martín et al [8] describe a very detail analysis both textbook and how students learn in a class using the notion of praxeology organization. Furthermore, a research using praxeology organization can also use to capture how a sector in mathematics (e.g proportion) changes from time to time [9,10]. This notion can also use for comparing textbooks from different countries, in this case a textbook comaprisson between Japan and England [11]. A minimal study from textbooks analysis based on praxeology organization can be used to see a condition of a domain both in theory and practice in one sector, e.g proportion [12,13]. Not only that, praxeology organization can also be used to analyse a connection between two domains for example [14-16]. In this case, Wijayanti found a disconnection in a textbook between proportion, linear

function, and similarity. She argued that eventought these three ‘sectors’ share a common technique, it is very rare that textbooks mentioned the connection among them [15]. Then, how is the condition of a textbooks (theoritically and practically) that discuss connection within sector, for example exponent equation?

II. METHODS

As we mentioned before that we will use the theory anthropology of didactic. In this research we will discuss three notions in this theory namely didactic level of codetermination, epistemological reference model, praxeology organization. Then, the methodolgy section will follow after these three notions discussion.

A. Didactic Level of Co-Determination

A model to determine hierarchy of institutional levels which contribute to “determine” what happens in the classroom has proposed by Chevallard [17] and this model is called didactic level of co-determination (figure 1).

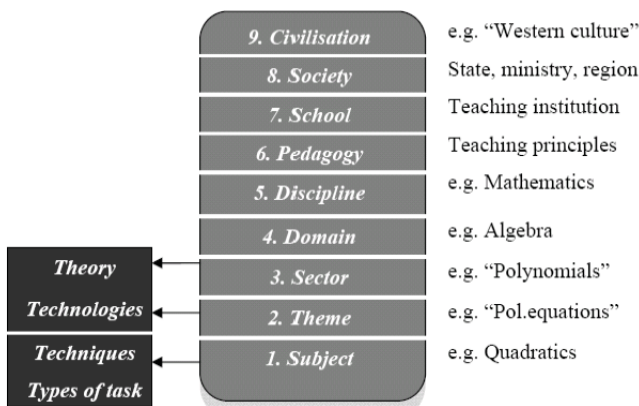


Fig. 1. Levels of didactic co-determination adapted from Chevallard [17] related to the components of (mathematical) organisations [18].

The model in figure 1 discuss a wider unit of didactic called *civilisation*. It is identify cultural norms and traditions shared by a number of societies. Then, an *educational system* establish and regulate society in different institution for example *school institution*. Finally, the *pedagogy* is norm and other conditions for teaching which is common for all dicipline within a given school institution.

B. Epistemological Reference Model

Epistemology can be defined as a branch of philosophy that studies the origin or source, structure, method and validity of knowledge. It is talks about the theory of science. This branch tries to find answers to the question of how it exists. The process is from the side of science certainly has clear theoretical principles [19]. Additionally, epistemology is assumed to be knowledge obtained through a strategy that respects differences between people and objects of natural science and therefore requires social scientists to understand the subjective meaning of social action [20]. The social action

in question is the action of giving an answer to the facts of the differences that occurred. The action described in this condition is called epistemological reference model.

The epistemological reference model of a body of knowledge is an alternative description of the body of knowledge described by the researcher to be questioned and provide answers to didactic facts and problematic aspects that occurred in certain institutions [21]. Epistemological reference models describe and analyse specific contents of the core teaching and learning process, general models in terms of praxeology arranged in language articulate a series of specific models from different areas of mathematical activity at stake [7,22].

The epistemological emancipation of scientific institutions and schools requires researchers to make their own perspectives on various types of interventional knowledge in didactic transposition processes, including their own ways of describing their knowledge and practice, their own epistemology [23]. In a sense, there is no special referral system to observe phenomena that occur in the institutions involved in the teaching process. Researchers must build their own epistemological reference model [7]. The researcher makes epistemological reference model based on knowledge, several sources of books, and questions in textbooks that are often used by educational institutions. The epistemological reference model in this study is the view of the researcher on knowledge. This model is also the eyes of the researcher to find out the transposition didactic knowledge that will be taught in the school institution. This model was compiled using praxeology organization elements to find out the answer how exponent equation should be learned in a textbook.

C. Praxeology Organization

Praxeology organization is used to determine didactic transpositions that occurred in mathematical knowledge. Barbe et al “states that praxeology organization has four elements including types of questions, techniques, technology and theory” [7]. These four element are also located in the basis of didactic co-determination (figure 1).

TABLE I. EXAMPLE OF PRAXELOGY ORGANIZATION

Discipline: Mathematics		
Domain		Arithmetic
Sector	Theory	Proportions
Theme	Technology	Direct proportion: Ratio and scale
Subject	Technique	$r = x_2/x_1$
	Type of task	Given x_1 and x_2 , find r so that $(x_1, x_2) \sim (1, r)$.

Praxeology Organization comes from the words praxis (practice) and logos (theory), which indicates that praxeology is a mixture of models from several theories and practices in everyday human life [7]. Praxeology contains four elements consisting of Type of Task "T" (Types of Questions), Technique "τ" (Completion Technique), Technology "θ" (Technology), and Theory "θ" (Theory). The type of question is a collection of tasks that can be completed with several

techniques. In many contexts, it is very important to justify techniques with praxeological elements. Finally, a theory is needed to describe technology [7]. The example of praxeology organization can be seen on table 1. From table 1 we can see an example where mathematics is categorized as discipline. It covers all domain for example arithmetic, algebra and geometry. Arithmetic can also be divided into some sectors (theory); proportion, division, addition. Then, a sector (theory) can be explained with a theme (technology), e.g. direct proportion. Additionally, a theme can be justified by subject (technique and type of task). In this discussion we will focus on exponent equation.

D. Methodology

This study is a descriptive qualitative research. We used a textbook that was used by teachers of 10th grade students at SMA Negeri 1 Cepiring 2018/2019 written by Noormandiri [24]. On the theory of exponent equation, there are (at least) four categories that we can focus on. They are characteristics of exponent, exponent graph, application of exponent, and exponent equation. However, the focus of this study is praxeology organization of exponent equation on theory and practice. It is because we found that students have more difficulty on exponent equation. Firstly, we categorized epistemological model reference on exponent equation. Secondly, we compare that reference epistemological model with praxeology organization in the textbook. As a result, we can have a picture of what theory and practice that is located or missed in the textbook.

III. RESULTS AND DISCUSSION

We will divide the result into two main discussions. First, how is the reference epistemological model on exponent equation. Here we focus on theory and technology. Second, we focus on how praxeology organization of exponent equation on student textbook. In this discussion we analyze textbook based on the reference model we had. In this subchapter, we also focus on discussion why such differences appear (if any).

A. Epistemological Reference Model on Exponent Equation

Theoretically, an exponent equation is an equation in which there are exponents with principal numbers, whereas the solution of an equation is the substitute values of the equation variable which causes the equation to be true. Thus, determining the set of solutions of an exponent equation is to determine all the substitute values for the variables that satisfy the equation.

A theory can be explained by technology. In this case we propose six dominant exponent equations that can be discussed by students. For example:

- $a^{f(x)} = 1$
- $a^{f(x)} = p$
- $a^{f(x)} = a^{g(x)}$
- $a^{f(x)} = b^{f(x)}$
- $h(x)^{f(x)} = g(x)^{g(x)}$

- $A(a^{f(x)})^2 + B(a^{f(x)}) + C = 0$

Furthermore, we also propose a technology where students can discuss the explanation of those six exponent equations. For example, given the equation $a^{f(x)} = 1$ with $a > 0$ and $a \neq 1$. The set of solutions to the equation can be determined as follows. For every $a > 0$ and $a \neq 1$, $a^0 = 1$ applies. As a result, if $a^{f(x)} = 1$ with $a > 0$ and $a \neq 1$ then $f(x) = 0$. The set of solutions of the equation $a^{f(x)} = 1$ with $a > 0$ and $a \neq 1$ is the set which members are x so that $f(x) = 0$.

In the technique and type of task, we propose two types of task. Firstly, it's all about application and secondly, a task that focuses on theory. In order to make a type of task, we consider a task first (Table II). Additionally, the task in table II can be named in terms of type of task as we can see in table III.

TABLE II. EXAMPLE OF TASK FOCUSING ON APPLICATION

No	Task	answer
1	Find the answer from this exponent equation $5^x = 1$	$5^x = 1$ $5^x = 5^0$ $x = 0$
n	...	

The task in table II is from the first exponent equation $a^{f(x)} = 1$. The variety of task can be based on characteristics of exponent, e.g. $a^n \times a^m = a^{n+m}$. As a result there will be more tasks in one exponent equation. In short, we divide tasks into six groups. Each group is divided into different tasks that are 9 types of task, 13 types of task, 10 types of task, 10 types of task, 5 types of task, and 5 types of task.

TABLE III. TYPES OF TASK

No	Task	answer
T ₁	Given an exponent equation $a^{f(x)} = 1$ that contains an exponent equation $a^0 = 1$, $a \neq 0$. The student is asked to find the value of x	To find the value of x , students need to find the solution of x with the solution $f(x) = 0$. Students can use $a^0 = 1$ to find the solution of x
T ₂
T ₃
T _{...}
T _{S2}

Regarding a task that focuses on theory, we propose that students are asked to find the answer on each exponent equation theoretically. For example: given an exponent equation, students are asked to find the value of x . To sum up, table IV will explain the type of our reference epistemological model:

TABLE IV. REFERENCE EPISTEMOLOGICAL MODEL

No	Praxeology organization	Example
1	Theory	Definition of exponent equation
2	Technology	Proof of exponent equation
3	Technique	Application of exponent equation (technically and theoretically)
4	Types of task	

B. Praxeology Organization on Textbook

Definition of exponent equation does not appear in textbook. Furthermore, the textbook only provide the type of exponent equation without explaining the prove behind exponent equation. Regarding technique and type of task, the textbooks only provide a practical application of exponent equation (20 tasks). Moreover, it only provides three exponent equations that are first, second, and third exponent equation (Figure 2).

Type of task	Task in textbook																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
T ₁																				
T ₂																				
T ₃																				
T ₄																				
T ₅																				
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T ₃₁													√							
T ₃₂																		√		

Fig. 2. Type of tasks in the textbook.

Furthermore, based on the table v, it is captured that type of tasks mostly appear only in the second and third exponent. Thus, teacher need to consider more (textbooks resources) to fill the gap on the non existing of definition of exponent equations, prove of exponent equations and theoretical application type of task. When teacher only depend on the textbook to teach, students will be lose an opportunity to learn definition of exponent equations and prove of exponent equations. Moreover, students also do not have an opportunity to learn variation of type of task, especially the first, fourth, fifth and sixth exponent. This condition is also indicate there is no disconnection within sector. As Wijayanti mentioned this disconnection is also appear among three sectors [16].

IV. CONCLUSION

The textbook that teacher used have provide component of exponent equation. However, it does not provide definition and ‘the reason behind’ type of exponent equation. As a consequence, teacher need to explain by her/himself regrading theory and technology of exponent equation. Furthermore, the re is no theoretical type of task. We also can not find type of task on the first, forth, fifth and sixth exponent equation. Again, teacher need to check and explain to students some task that aro not located in textbooks. For further research, we need to check how teacher explain exponent equations to student.

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