

A Stochastic Approach in Modeling of Regional Atmospheric CO₂ in the United States

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ABSTRACT

Global warming is a function of two main contributable entities in the atmosphere, carbon dioxide, and atmospheric temperature. The objective of this study is to develop a statistical model using actual fossil fuel carbon dioxide emissions data from the United States to predict relative probability of rate of change in fossil fuels carbon dioxide emissions from nine US climate regions using transition modeling. The sensitivity of these transition probabilities to five sectors, that are the commercial, industrial, residential, transportation, and electric power sector, is also investigated for all nine US climate regions. The present study also suggests that the US government should be developing regional policies to control fossil fuel carbon dioxide emissions that will be more effective in addressing the subject problem.

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1. INTRODUCTION

One of the main issues in our planet is the climate change problem; rising atmospheric temperature, the shifted patterns of snow and rainfall, and much more extreme climate changes are daily features in our media. Scientists speak with confidence that all these problems are related to climbing levels of the atmospheric carbon dioxide (CO₂) emission along with other growing greenhouse gases such as methane (CH₄), nitrous oxide (N₂O), and fluorinated gases in the atmosphere. These greenhouse gases absorb the thermal radiation from the surface of the earth and radiate again to the surface, and this repeating process elevates the atmospheric temperature [1,2].

The CO₂ in our atmosphere has increased dramatically after the industrial revolution (1760). The risk of increasing CO₂ emission is not only on the amount of the CO₂ in the atmosphere but also on the survival time of the CO₂ in the atmosphere; it remains in our atmosphere for thousands of years. Before the industrial revolution, the CO₂ level never increased more than 30 ppm in any period; however, it has increased more than 30 ppm within the past two decades alone. Also the proportion of the CO₂ among all greenhouse gases emission in the United States reached 82% in 2012, and this speaks of the importance of controlling the CO₂ emission and, in fact, we are able to reduce the level of the CO₂ emission by controlling related human activities [3].

The world's top polluter of CO₂, China, pledged to peak the CO₂ emissions around 2030 after a remarkably rapid increase of the CO₂ emission in the 21st century, whereas the world's second CO₂ polluter, the United States, already reached the peak prior to 2010 and promised to try to cut the CO₂ emission by at least 26% from 2005 levels by 2025 [4]. In order for the United States to carry out this promise, more efficient regulations must be established. The present study provides a rough sketch of the CO₂ problem in the United States and recommends that regional policies based on our findings will be more effective. Also, additional interesting research on the subject area can be found in the references [5–16].

2. STATISTICAL MODELING

2.1. The Data

The original data used in the present study is obtained from the US Environmental Protection Agency (EPA) and contains state CO₂ emission inventories from fossil fuel combustion by end-use sectors; the commercial, electric power, industrial, residential, and

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transportation sector, in million metric tons of CO₂ from 1992 through 2012 for all 50 states in the United States. The structure of the data with sample size in each level is displayed in Figure 1.

Figure 2 shows the data structure based on nine US climate regions with five end-use sectors, and the following data modification enables us to perform a transitional modeling of the data.

$$I_{ij} = \begin{cases} 0, & \text{if } r_{ij}^y \leq \overline{r_j^y} \\ 1, & \text{otherwise} \end{cases} \quad \text{and} \quad S_{kij} = \begin{cases} 0, & \text{if } r_{kij}^x \leq \overline{r_{kj}^x} \\ 1, & \text{otherwise} \end{cases},$$

where $i(= 1, 2, \dots, 51)$ is a state index, $j(= 1993, 1994, \dots, 2012)$ is a year index, $k(= 1, 2, \dots, 5)$ is a sector index (1: commercial sector, 2: electric power sector, 3: industrial sector, 4: residential sector, 5: transportation sector), y_{ij} is the CO₂ emission for the state i in year j , $r_{ij}^y = \frac{y_{ij} - y_{i,j-1}}{y_{i,j-1}}$ for all i and j , x_{kij} is the CO₂ emission due to the sector k for the state i in year j , and $r_{kij}^x = \frac{x_{kij} - x_{ki,j-1}}{x_{ki,j-1}}$ for all j in each k .

2.2. Transitional Modeling

The key idea of the present study is predicting the probability that the changing rate of the CO₂ emission in a specific region is higher than the average changing rate based on values of attributable variables in the past over all climate regions. While we use the past response values

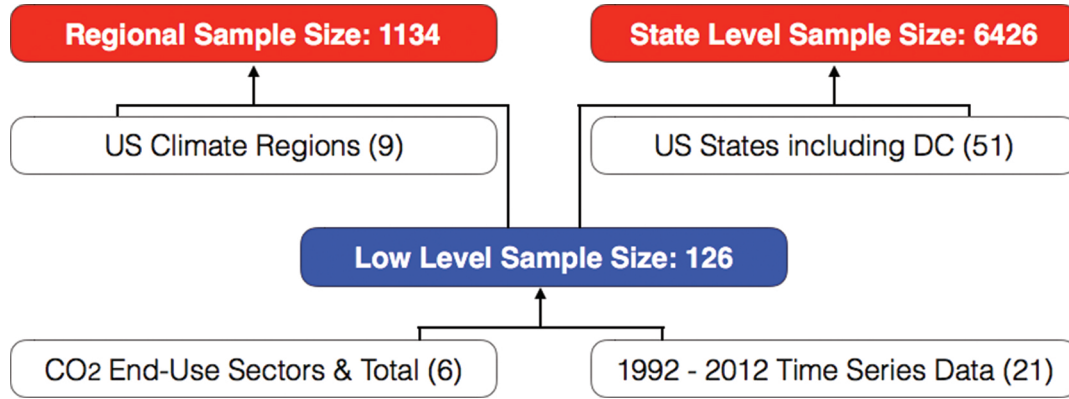


Figure 1 | Structure of data with sample size in each level.

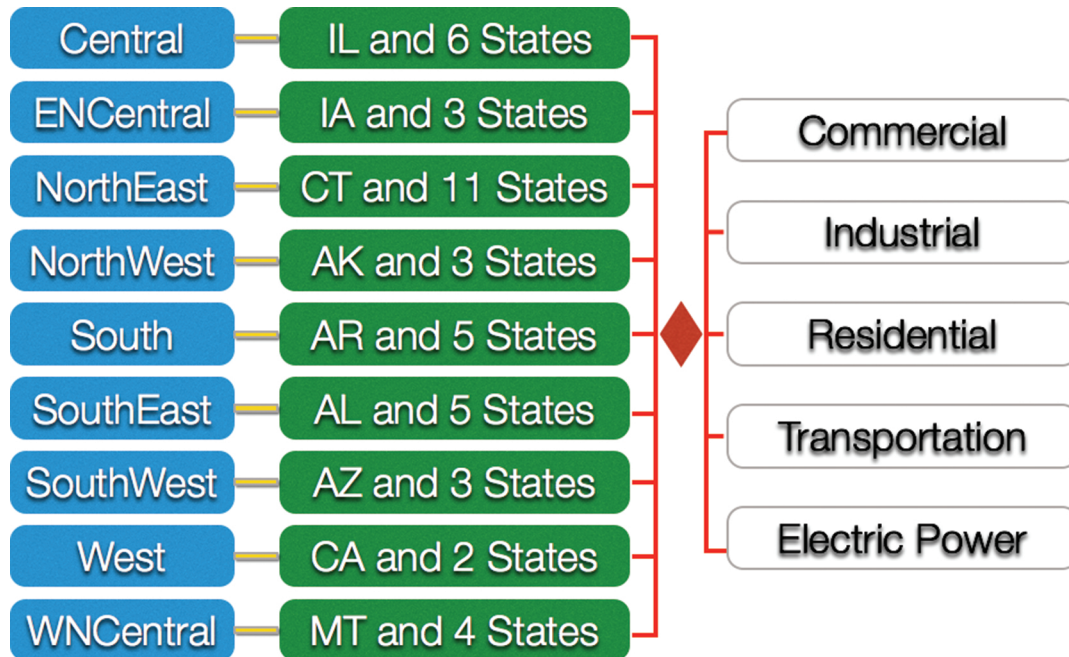


Figure 2 | Illustration of US climate regions with CO₂ emission sectors.

as independent variables in direct transitions for the ordinary transitional modeling [17,18], the indirect transition method has been applied in the present study. In other words, our interest in this modeling procedure is on the statistical modeling of

$$Pr(I_{ij} = 1 \mid S_{k \ i \ j-1}, \text{ where } k = 1, 2, \dots, 5).$$

The Equation (1) below represents the theoretical indirect transition model of the regional data, and Equation (2) shows the fitted probability models for all nine US climate regions along with the table of the estimated coefficients.

$$\text{logit} \left(E[I_{ij}] \right) = \beta_0 + \sum_{r=1}^9 \left[\beta_r(t_{ij} \cdot g_{ir}) + \sum_{k=1}^5 \beta_{9k+r}(S_{k \ i \ j-1} \cdot g_{ir}) \right], \quad (1)$$

where r is a categorical variable indicating below regions:

r	1	2	3	4	5	6	7	8	9
Region	C	ENC	NE	NW	S	SE	SW	W	WNC

and where $t_{ij} = j - 1990$ for all 50 states, and $g_{ir} = 1$, if state i belongs to region r , $g_{ir} = 0$, otherwise.

$$\hat{\pi}_r = \frac{\exp \left(\hat{\beta}_0 + \hat{\beta}_1 t_{ij} + \sum_{k=1}^5 \hat{\beta}_{k+1} S_{k \ i \ j-1} \right)}{1 + \exp \left(\hat{\beta}_0 + \hat{\beta}_1 t_{ij} + \sum_{k=1}^5 \hat{\beta}_{k+1} S_{k \ i \ j-1} \right)} \quad (2)$$

Estimated coefficients in the Equations (2).

Region	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$
C	-0.3848	0.0317	0.1005	-0.9000	0.6631	-0.2748	0.5202
ENC	-0.3848	0.0774	-0.5266	-1.8590	0.3162	1.6961	-0.5375
NE	-0.3848	0.0593	0.1332	-1.2377	0.4604	1.0280	0.0271
NW	-0.3848	0.0092	-0.3751	-0.3450	0.6133	0.4202	-0.4589
S	-0.3848	0.0199	-0.2813	-0.4035	0.9975	-0.4903	-0.5316
SE	-0.3848	0.0421	-0.5519	-0.9326	0.2614	0.2175	0.2121
SW	-0.3848	-0.0296	-0.3225	-0.6208	0.3235	1.0365	-0.3863
W	-0.3848	0.0361	-0.5263	-0.5422	-0.4334	0.5534	0.5709
WNC	-0.3848	-0.0326	0.2560	1.3760	-0.2732	-0.4342	-0.0973

Probabilities that the CO₂ emission in each region is more than the average US CO₂ emission at t_{ij} based on all possible combinations of $S_{k \ i \ j-1}$, $k = 1, 2, 3, 4, 5$, are displayed in Table 1. For example,

$$\textcircled{1} Pr(I_{ij} = 1 \mid S_{1 \ i \ j-1} = S_{3 \ i \ j-1} = S_{5 \ i \ j-1} = 1) = 0.8359$$

and

$$\textcircled{2} Pr(I_{ij} = 1 \mid S_{3 \ i \ j-1} = S_{5 \ i \ j-1} = 1) = 0.8217$$

in the central region, and this implies that the contributions of the sector 3 and the sector 5 to the regional CO₂ emissions are statistically significant in the central region. Accordingly, the main key factors causing CO₂ emission in the central region are the industrial sector and the transportation sector. Although the sector 1, the commercial sector, also contributes to the highest probability, the marginal contribution to the highest probability is only 0.0142 ($0.8359 - 0.8217$) and it is not as significant as two other sectors; the industrial and transportation sector. When we look into the east north central region, the contribution of the sector 4, the residential sector, to the atmospheric CO₂ emission in this region is remarkably obvious while the sector 3, the industrial sector, has merely small effects on the CO₂ emission compare to the residential sector, because

$$Pr(I_{ij} = 1 \mid S_{3 \ i \ j-1} = S_{4 \ i \ j-1} = 1) = 0.9679$$

and

$$Pr(I_{ij} = 1 \mid S_{4 \ i \ j-1} = 1) = 0.9565$$

with 0.0114 ($0.9679 - 0.9565$) as a marginal contribution of the industrial sector.

Table 1 Probabilities for all possible cases.

S1	S2	S3	S4	S5	C	ENC	NE	NW	S	SE	SW	W	WNC
0	0	0	0	0	0.5852	0.8015	0.7269	0.4568	0.5182	0.6419	0.2562	0.6096	0.2433
1	0	0	0	0	0.6094	0.7045	0.7526	0.3663	0.4481	0.5079	0.1997	0.4798	0.2935
0	1	0	0	0	0.3645	0.3861	0.4357	0.3733	0.4181	0.4136	0.1562	0.4758	0.5601
0	0	1	0	0	0.7325	0.8470	0.8084	0.6083	0.7447	0.6995	0.3225	0.5030	0.1966
0	0	0	1	0	0.5174	0.9565	0.8815	0.5614	0.3971	0.6902	0.4927	0.7308	0.1724
0	0	0	0	1	0.7036	0.7022	0.7323	0.3470	0.3873	0.6890	0.1897	0.7343	0.2258
1	1	0	0	0	0.3881	0.2709	0.4687	0.2904	0.3516	0.2888	0.1183	0.3491	0.6218
1	0	1	0	0	0.7517	0.7658	0.8282	0.5162	0.6876	0.5727	0.2564	0.3742	0.2402
1	0	0	1	0	0.5424	0.9286	0.8948	0.4680	0.3321	0.5620	0.4130	0.6160	0.2120
1	0	0	0	1	0.7241	0.5821	0.7576	0.2675	0.3230	0.5606	0.1450	0.6201	0.2737
0	1	1	0	0	0.5268	0.4632	0.5503	0.5238	0.6608	0.4781	0.2038	0.3705	0.4921
0	1	0	1	0	0.3035	0.7743	0.6834	0.4755	0.3056	0.4671	0.3430	0.6122	0.4519
0	1	0	0	1	0.4911	0.2687	0.4424	0.2735	0.2969	0.4658	0.1118	0.6164	0.5360
0	0	1	1	0	0.6754	0.9679	0.9218	0.7027	0.6411	0.7431	0.5731	0.6377	0.1368
0	0	1	0	1	0.8217	0.7639	0.8125	0.4953	0.6315	0.7421	0.2445	0.6418	0.1817
0	0	0	1	1	0.6433	0.9278	0.8843	0.4472	0.2791	0.7336	0.3976	0.8278	0.1589
1	1	1	0	0	0.5518	0.3376	0.5830	0.4305	0.5952	0.3453	0.1564	0.2580	0.5558
1	1	0	1	0	0.3252	0.6695	0.7115	0.3839	0.2493	0.3355	0.2744	0.4826	0.5158
1	1	0	0	1	0.5162	0.1783	0.4754	0.2055	0.2417	0.3343	0.0835	0.4870	0.5987
1	0	1	1	0	0.6970	0.9469	0.9309	0.6190	0.5741	0.6249	0.4930	0.5098	0.1699
1	0	1	0	1	0.8359	0.6564	0.8320	0.4028	0.5640	0.6237	0.1899	0.5142	0.2229
1	0	0	1	1	0.6660	0.8836	0.8973	0.3573	0.2261	0.6133	0.3235	0.7395	0.1962
0	1	1	1	0	0.4582	0.8247	0.7738	0.6260	0.5440	0.5324	0.4191	0.5058	0.3856
0	1	1	0	1	0.6519	0.3352	0.5570	0.4100	0.5338	0.5311	0.1481	0.5102	0.4678
0	1	0	1	1	0.4230	0.6671	0.6892	0.3643	0.2055	0.5201	0.2619	0.7365	0.4280
0	0	1	1	1	0.7778	0.9464	0.9238	0.5990	0.5121	0.7815	0.4770	0.7570	0.1257
1	1	1	1	0	0.4833	0.7354	0.7962	0.5350	0.4738	0.3960	0.3432	0.3768	0.4477
1	1	1	0	1	0.6744	0.2294	0.5895	0.3232	0.4636	0.3947	0.1119	0.3810	0.5317
1	1	0	1	1	0.4477	0.5420	0.7170	0.2825	0.1633	0.3843	0.2045	0.6228	0.4915
1	0	1	1	1	0.7947	0.9124	0.9326	0.5066	0.4421	0.6732	0.3979	0.6480	0.1567
0	1	1	1	1	0.5873	0.7333	0.7785	0.5141	0.4122	0.5847	0.3290	0.6443	0.3628
1	1	1	1	1	0.6114	0.6188	0.8006	0.4210	0.3461	0.4477	0.2621	0.5170	0.4238

3. CLUSTER ANALYSIS

We develop six cluster maps showing the atmospheric CO₂ emission regions in the United States based on effects of the total CO₂ emission and all five end-use sectors; the commercial, electric power, industrial, residential, and transportation sector. After normalizing the probability data in Table 1 using Johnson's transformation [19] and [20], the hierarchical clustering procedure has been performed using Ward's method, in which we consider a clustering problem as a problem of minimizing within-cluster sum of squares in each cluster rather than a distance problem [21–23]. In Ward's method, we begin with nine clusters of size 1 and combine two clusters that render the minimum error sum of squares in Equation (3), or yield maximum R^2 in Equation (4) equivalently, repeating this procedure until we reach the optimal R^2 value with the number of clusters we desired.

$$\textcircled{1} SSE = \sum_l \sum_r \sum_m \left(P_{lrm} - \bar{P}_{l..m} \right)^2, \quad (3)$$

$$\textcircled{2} R^2 = \frac{SST - SSE}{SST}, \quad (4)$$

and

$$\textcircled{3} SST = \sum_l \sum_r \sum_m \left(P_{lrm} - \bar{P}_{..m} \right)^2, \quad (5)$$

where P_{lrm} denotes the probability after Johnson's transformation for the m^{th} combination of $S_{k \ i \ j-1}$ ($k = 1, 2, 3, 4, 5$) in region r belonging to the cluster l .

Table 2 illustrates the results of the hierarchical clustering using Ward's method based on effects of total CO₂ emission, the commercial, electric power, industrial, residential, and transportation sector denoted by *Total*, S_1 , S_2 , S_3 , S_4 , S_5 , respectively along with R^2 values for all clustering criteria.

3.1. Clustering Based on the Effect of the Total CO₂ Emissions

In Figure 3, we see that nine US climate regions are combined into three CO₂ emission clusters based on the effect of the total CO₂ emission.

Table 2 | Clustering based on different factors.

Region	Clustering Based on					
	Total	S ₁	S ₂	S ₃	S ₄	S ₅
C	C 1	C 1	C 2	C 2	C 2	C 2
ENC	C 2	C 2	C 2	C 1	C 3	C 3
NE	C 2	C 1	C 2	C 1	C 1	C 1
NW	C 1	C 2	C 1	C 2	C 1	C 3
S	C 1	C 2	C 1	C 2	C 2	C 3
SE	C 1	C 3	C 2	C 1	C 1	C 1
SW	C 3	C 2	C 1	C 1	C 3	C 3
W	C 1	C 3	C 1	C 3	C 1	C 2
WNC	C 3	C 1	C 3	C 3	C 2	C 1
R ²	0.566	0.869	0.838	0.898	0.853	0.822

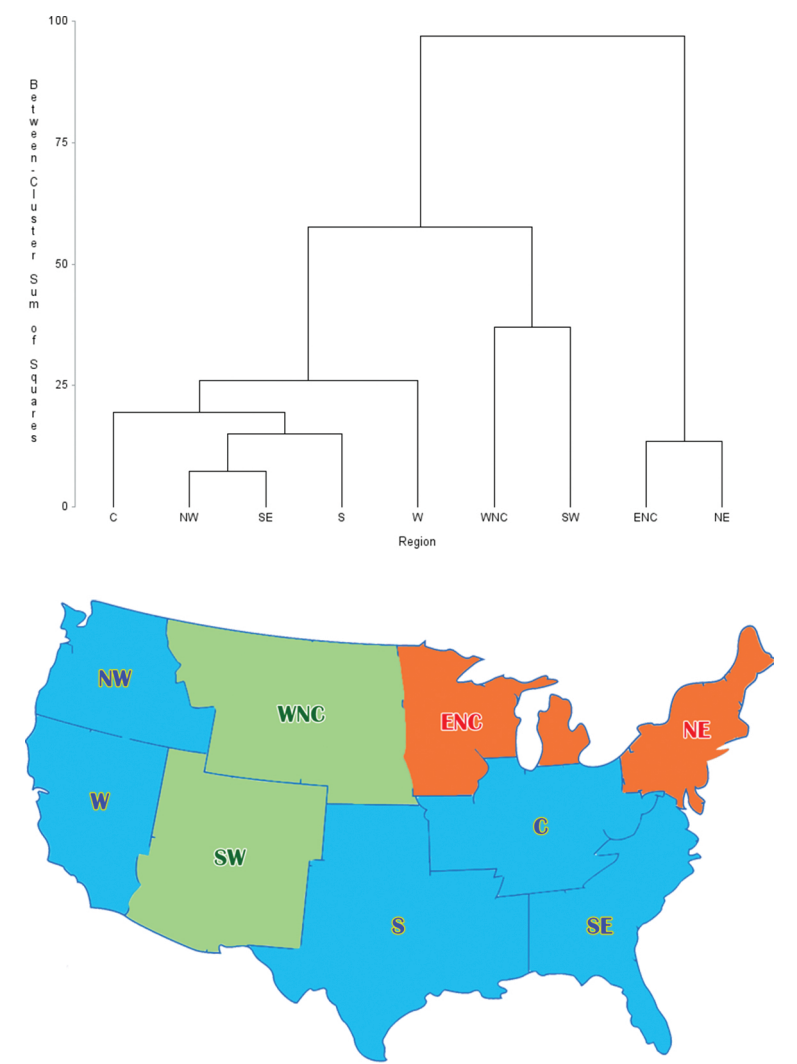


Figure 3 | Dendrogram and cluster map based on the effect of the total CO₂ emission.

The cluster 1 consists of five US climate regions; the central, northwest, south, southeast, and west regions, and the cluster 2 is composed of the east north central and northeast regions. Finally the remaining two regions; the west north central and southwest regions, build the cluster 3. Regions in the same cluster share common characteristics with respect to the clustering criterion, and the total CO₂ emission criterion yields geographical clustering results and this tells that the total CO₂ emission is highly related to the geographic climate condition.

3.2. Clustering Based on the Effect of the Commercial Sector

The commercial sector includes all businesses except manufacturing and transportation, and any CO₂ emissions from related fossil fuels combustion such as heating, driving, and other activities within business purposes are counted to the commercial CO₂ emissions.

Figure 4 displays three-cluster solution by the commercial sector criterion. The west region and the southeast region have similar characteristics with respect to the commercial aspect and this seems to be proved by The Walt Disney Company because two Disney resorts, Disney World and Disney Land, are located in these two regions. The other two clusters are also comprised of regions with similarity upon the commercial sector criterion.

3.3. Clustering Based on the Effect of the Electric Power Sector

The electric power sector not only involves the generation of the electricity but also includes transmission and distribution of the electricity. The CO₂ emission from the electric power sector makes up about 32% of the total amount of CO₂ emission in the United States and this is the top contributing sector among all five sectors to the CO₂ emission in the United States.

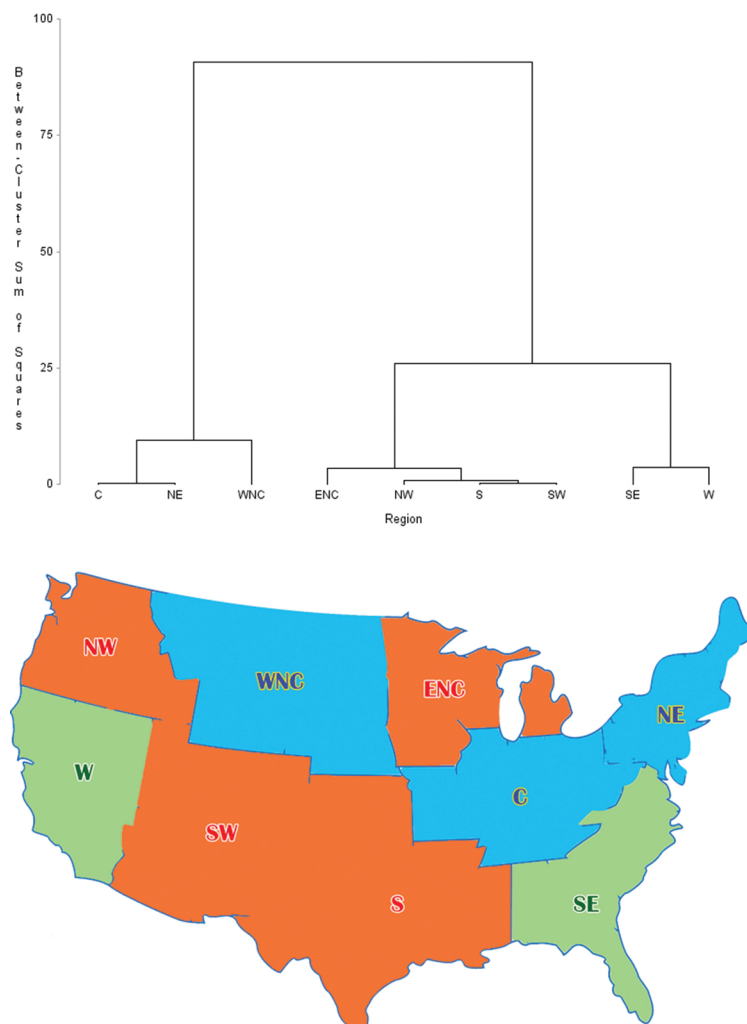


Figure 4 | Dendrogram and cluster map based on the effect of the commercial sector.

The electric power sector criterion also highlights three-cluster solution with reasonably well combined cluster map in Figure 5. The relationship between the electric power sector and the CO₂ emission can be found in the source of the electric power and the amount of the electric usage. The geographical distribution of the type of the major power plants in Figure 6 proves such a relationship. Most of the steam and nuclear power plants are located in the cluster with the southeast, central, east north central, and northeast regions, whereas we find major hydroelectric power plants in the cluster with the northwest, west, southwest, and south regions.

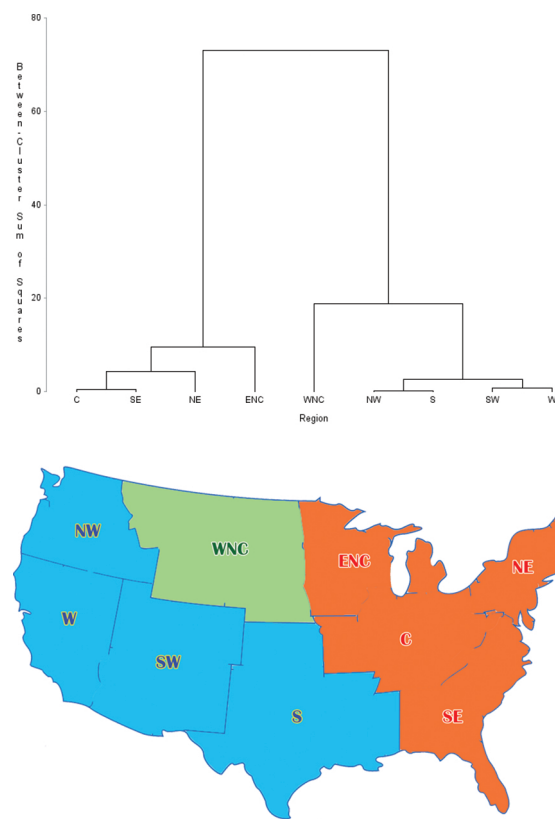


Figure 5 | Dendrogram and cluster map based on the effect of the electric power sector.

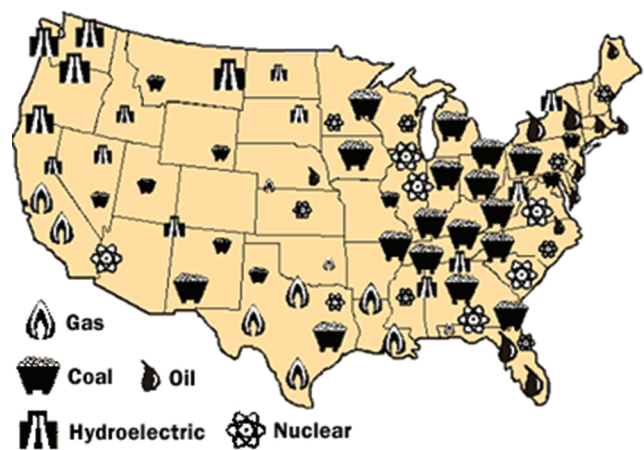


Figure 6 | A breakdown of the major power plants in the United States, by type.

3.4. Clustering Based on the Effect of the Industrial Sector

The industrial sector emits the CO₂ directly and indirectly to our atmosphere. The direct way of emissions involves burning fossil fuels to produce commercial goods, and the CO₂ emission at a power plant to generate electricity to use in industrial facilities is categorized to the indirect way of emissions. The industrial sector occupies around 20% of total CO₂ emissions in the United States.

Figure 7 shows similarities between the west and west north central regions, among the northwest, south, and central regions, and among the other four regions. In order to control the CO₂ emission from the industrial aspect in each cluster, we need further scientific research, which may suggest re-location of chemical plants that may have significant interaction effects, as a solution of reducing CO₂ emission nationwide.

3.5. Clustering Based on the Effect of the Residential Sector

The residential sector increases the atmospheric CO₂ concentration through heating, cooking, and other home maintaining activities. Although the contribution of the residential sector to the total CO₂ emission is less than 10% in the United States, it is very important to control the emission due to the residential sector because every individual is a member of this residential sector and the effect of a campaign against the CO₂ emission may reach all other sectors.

Clustering based on the residential sector criterion also provides three-cluster solution in Figure 8. One remarkable feature of this clustering is that the cluster, comprised of the northwest, west, northeast, and southeast regions, includes all the Pacific and Atlantic seaside regions, while other inland regions form the other two clusters. It is very interesting that the effect of the residential aspect to the CO₂ emission is related to the human lifestyle founded on the geographic characteristics.

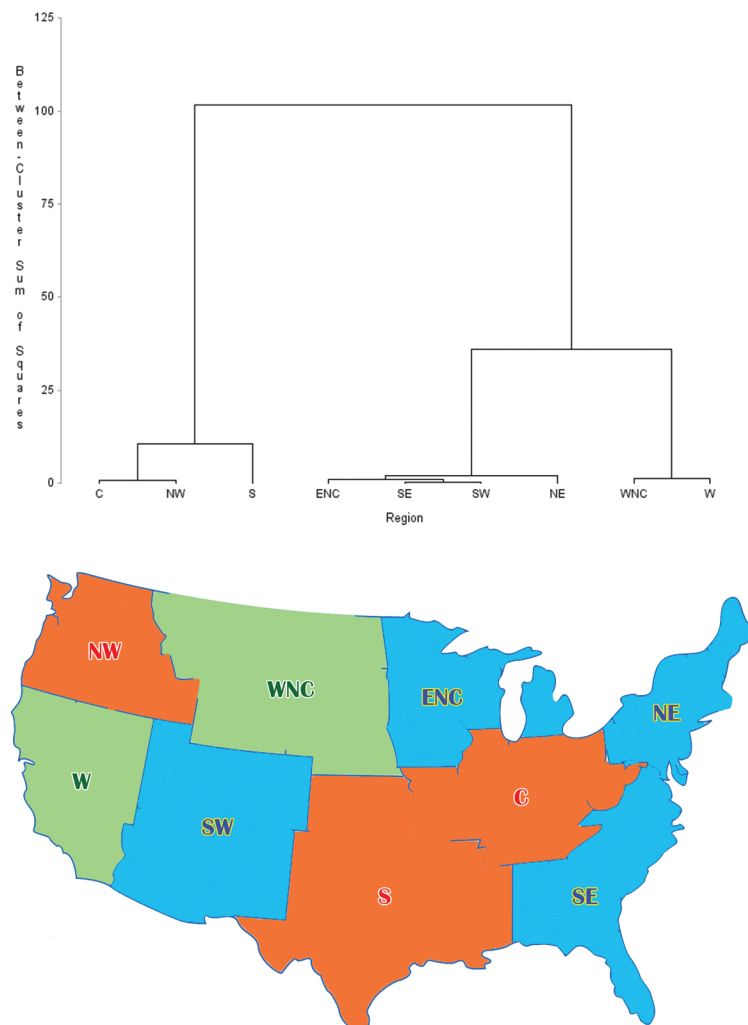


Figure 7 | Dendrogram and cluster map based on the effect of the industrial sector.

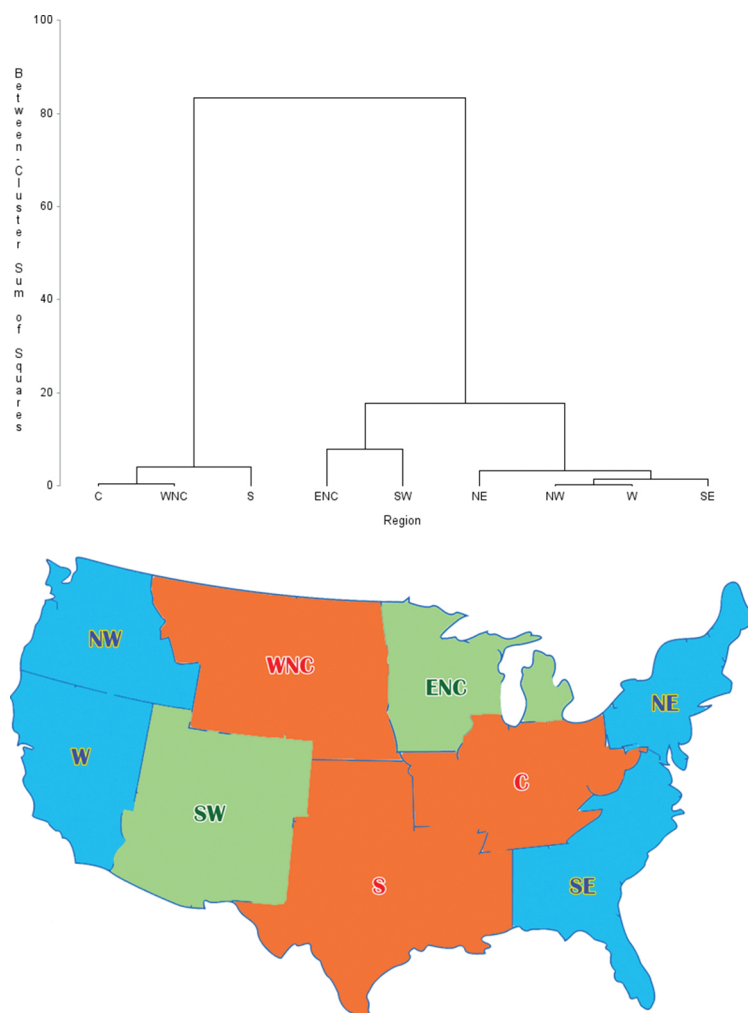


Figure 8 | Dendrogram and cluster map based on the effect of the residential sector.

3.6. Clustering Based on the Effect of the Transportation Sector

The transportation sector produces the atmospheric CO₂ through the movement of merchandise and people by the combustion of petroleum-based products such as gasoline, diesel, and bunker fuels. This sector has the second greatest contribution to the total CO₂ emissions, about 28%, in the United States.

Figure 9 shows similarities between the west and central regions, among the southeast, northeast, and west north central regions, and all remaining regions. This three-cluster solution provides reasonable evidence to share regulations to reduce the CO₂ emission in a transportation aspect for regions within the same cluster.

4. CONCLUSION

The present study provides several guidelines, for policy makers, to effectively control the level of the carbon dioxide emissions in each US climate region. Firstly, fitted regional probability models driven by Equation (2), derived from transitional models in Equation (1), that allow us to calculate the probabilities of the CO₂ emission at risk in each region based on all possible combinations of by-sector CO₂ emission behaviors in the previous year. Ranks of the effect of by-sector behaviors to the level of the CO₂ emissions in each climate region are displayed in Table 3, below.

We can conclude that the number one risk sector in the central region is S3, the industrial sector, the number two risk sector is S5, the transportation sector, and the rank three sector is S1, the commercial sector. Accordingly, the industrial sector CO₂ emission has a role of a preceding index when we predict how the CO₂ emission changes in the following year for the central region. Similarly, we consider the residential sector CO₂ emission as a preceding index for the east north central region, the transportation sector CO₂ emission as a leading index for the west region, and so on. Moreover, ranks in Table 3 are assigned under consideration of interaction effects among all possible combinations of five sectors. Secondly, we can effectively control the total CO₂ emission using CO₂ clusters by the effect of each sector

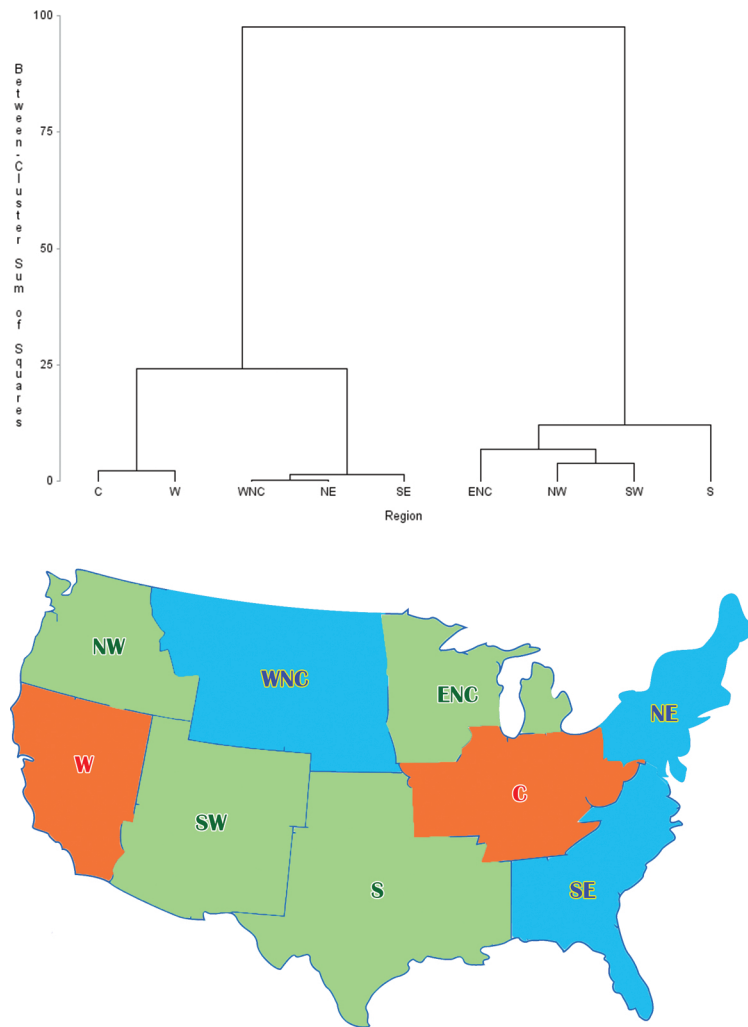


Figure 9 | Dendrogram and Cluster Map based on the Effect of the Transportation Sector.

Table 3 | Ranks of sectors with maximum probabilities in each region.

Region	Ranks with Maximum Probability			
	Rank 1	Rank 2	Rank 3	Max. Prob.
C	S3	S5	S1	0.8359
ENC	S4	S3	S1	0.9679
NE	S4	S3	S1	0.9326
NW	S3	S4	S2	0.7027
S	S3	S1	S2	0.7447
SE	S3	S4	S5	0.7815
SW	S4	S3	S1	0.5731
W	S5	S4	S3	0.8278
WNC	S2	S1	S5	0.6218

shown in Table 2. For instance, we may apply the same policy regarding the residential sector to all west and east coastal regions because they share similar properties within residential related problems as shown in Figure 8.

Providing a solution to an environmental problem is not so simple because most environmental problems are due to human activities that are not predictable. However, these statistical models would be a strong background for our government to legislate more effective regulations to control the optimal level of the CO₂ emission in the United States on regional basis.

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