

The Flexibility of Students' Mathematical Creative Thinking in Solving Mathematical Problems

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Abstract—The ability to think creatively is needed to live in the era of the 21st industrial revolution and the work field, but learning that is oriented towards creative thinking skills is still lack. This ability can be given to students through mathematics learning so that the characteristics of mathematical creative thinking need to be understood by the teacher. One indicator of the emergence of this ability can be seen from the aspect of flexibility, that is the ability to solve mathematical problems using a variety of different methods. This study aims to describe the flexibility of students' mathematical creative thinking in solving mathematical problems. The research subjects are senior high school students who have studied systems of linear equations. This research is a qualitative descriptive study with assistive devices in the tests and interviews. The results showed that students already have the flexibility of mathematical creative thinking in solving mathematical problems using a variety of different settlement methods. The methods used are elimination, substitution, and combination methods. The flexibility of students' thinking is still influenced by the understanding and explanation of the teacher. Students have not given their own ideas in the process of solving. Therefore, teachers should provide training through the use of Multiple Solution Tasks and applying the Problem Based Learning model with Self Regulation Learning approach so that students are accustomed to thinking flexibly.

Keywords: *creative thinking, flexibility, mathematical problems*

I. INTRODUCTION

Creative thinking is a life skill that must be possessed by students. Students must be accustomed to thinking creatively in order to face challenges in the 21st industrial revolution era and the work field. For example, there is competition between industrial companies that require the ability to think creatively. Over the past 25 years, industrial companies must introduce new tools in the production process and require new skills from employees to compete [1]. This shows that the ability to think creatively is needed in the 21st industrial era.

In the 21st century industrial era science and technology are developing very rapidly. If the students do not have the ability to think creatively, then there is a tendency to experience difficulties in facing life and work field challenges. Basically, everyone has the potential to be creative [2]. Munandar said that this ability could be

developed by providing opportunities for students to express ideas [3]. This statement is also in accordance with Adams and Hamm who stated that the ability to think creatively is a natural potential and can be developed through awareness or practice (Nasution, Surya, Asmin, & Sinaga, 2017)[4]. That is, the ability to think creatively can be trained. One of them is through learning mathematics. The teacher must have an understanding of students' mathematical creative thinking abilities, so that stimulus to get accustomed to mathematical creative thinking can be given through learning. Thus, there needs to be an analysis of mathematical creative thinking abilities that are already possessed by students. One indicator of mathematical creative thinking ability that must be understood and trained through the learning process is the aspect of flexibility.

Flexibility is one of the important components needed in the career domain and life skills. Flexibility is the ability of students to solve a problem using a variety of different ways [5-7]. Then, do students already have aspects of mathematical creative thinking flexibility? What are the characteristics of mathematical creative thinking flexibility that students currently have? What must be done by the teacher so that students' ability to think flexibly can be trained and developed?

To answer the questions above, this paper will present a description of the flexibility of students' mathematical creative thinking abilities. The description presented is based on students' flexibility in solving mathematical problems. Next, a number of suggestions that can be made by the teacher regarding training and the development of students' mathematical creative thinking flexibility will be discussed.

II. METHOD

The approach of this research is descriptive qualitative. This study aims to describe the flexibility of students' mathematical creative thinking abilities in solving mathematical problems. The flexibility of students' mathematical creative thinking is determined based on differences between the completion methods. Data collected through tests and interviews. The test was conducted for 56 students of class X high school who have knowledge of the two-variable linear equation system material. Furthermore, the subjects who were interviewed were determined using the

purposive sampling method. The subjects chosen were students who were able to provide a variety of different settlement methods and were able to communicate well. There are four subjects, namely FK, SHN, NTM, and MA.

III. RESULTS AND DISCUSSION

A) Results

The results of this study describe the flexibility of students' mathematical creative thinking in solving mathematical problems. There are four subjects who are able to provide a variety of different completion methods on the test, namely FK, SHN, NTM, and MA. The following will be presented answers from each subject.

1) FK's Answer

misal K = kaos dan B = buku maka misal psm
 ana = $1k + 2b = \dots$
 bunga = $2k + 1b = \dots$
 Uang ana utk membeli 1 kaos dan 2 buku adl Rp 200.000 - Rp 8000
 = Rp 192.000
 Uang bunga utk membeli 2 kaos dan 1 buku adl Rp 250.000 - Rp 58.000
 = Rp 192.000

$$\begin{array}{rcl} \text{ana} & = & 1k + 2b = 192 \\ \text{bunga} & = & 2k + 1b = 192 \end{array} \quad \begin{array}{l} \times 2 \\ \times 1 \end{array} \quad \begin{array}{l} 2k + 4b = 384 \\ 2k + 1b = 192 \end{array}$$

$$\begin{array}{r} 3b = 192 \\ b = 64 \text{ (setelah diskon)} \end{array}$$

$$\begin{array}{rcl} \text{ana} & = & 1k + 2b = 192 \\ \text{bunga} & = & 2k + 1b = 192 \end{array} \quad \begin{array}{l} \times 1 \\ \times 2 \end{array} \quad \begin{array}{l} 1k + 2b = 192 \\ 4k + 2b = 384 \end{array}$$

$$\begin{array}{r} -3k = -192 \\ k = \frac{-192}{-3} \\ k = 64 \text{ (setelah diskon)} \end{array}$$

Substitusi

$$\begin{array}{l} 1k + 2b = 192 \text{ (i)} \\ 2k + 1b = 192 \text{ (ii)} \end{array}$$

ambil mikanya.
 $1k + 2b = 192$
 $1k = 192 - 2b \text{ (iii)}$

Pers iii disubs ke Pers ii

$$2k + 1b = 192$$

$$2(192 - 2b) + 1b = 192$$

$$384 - 4b + 1b = 192$$

$$-4b + 1b = 192 - 384$$

$$-3b = -192$$

$$b = \frac{-192}{-3}$$

$$b = 64 \text{ (setelah diskon)}$$

Setelah diket $b = 64$

$$1k + 2b = 192$$

$$1k + 2(64) = 192$$

$$1k + 128 = 192$$

$$1k = 192 - 128$$

$$1k = 64 \text{ (setelah diskon)}$$

Campuran

$$\begin{array}{l} 1k + 2b = 192 \\ 2k + 1b = 192 \end{array}$$

eliminasi salah satu variabel.

$$\begin{array}{rcl} 1k + 2b = 192 & \times 2 & 2k + 4b = 384 \\ 2k + 1b = 192 & \times 1 & 2k + 1b = 192 \end{array}$$

$$\begin{array}{r} 3b = 192 \\ b = 64 \end{array}$$

Setelah eliminasi hasil eliminasi disubstitusikan ke salah satu persamaan.

$$\begin{array}{l} 2k + 1b = 192 \\ 2k + 64 = 192 \\ 2k = 192 - 64 \\ 2k = 128 \\ k = 64 \end{array}$$

harga asli buku = $\frac{100}{80} \times 80k = 80k$

harga asli kaos = $\frac{100}{80} \times 128k = 128k$

Jadi harga buku dan kaos adalah Rp 80.000 dan Rp 128.000

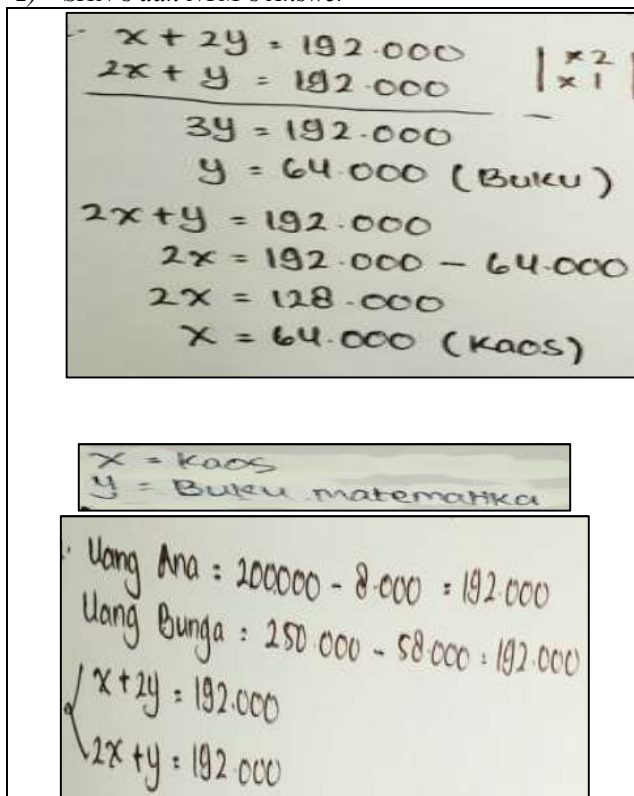
Fig.1. FK's Answer

Figure 1 shows that FK is able to determine the price of t-shirts and math books before discounting correctly. The step used is to suppose a shirt with variable k and a math book with variable b . Next, FK forms a mathematical model from the illustration of the problem. FK uses three different (flexible) methods to determine the price of t-shirts and math books after discounting. Then, FK determines the price of t-

shirts and books before discounting correctly. Based on the description, FK is able to understand the problem, plan a solution to the problem, and use three different solutions (elimination, substitution, elimination-substitution).

Based on the results of the interview, FK was able to explain smoothly. FK only uses methods that are understood and taught by teachers. FK said that there was still one more method, namely determinants, but FK was unable to write this method because FK did not understand. FK added that the easiest method for this problem is elimination because there are only two variables whose values must be determined. From the results of tests and interviews, it can be concluded that FK is able to provide three different methods and has the flexibility of mathematical creative thinking.

2) SHN's dan NTM's Answer



Handwritten mathematical work for SHN's answer using the elimination method:

$$\begin{array}{r} x + 2y = 192.000 \quad | \times 2 \\ 2x + y = 192.000 \quad | \times 1 \\ \hline 3y = 192.000 \\ y = 64.000 \text{ (Buku)} \\ 2x + y = 192.000 \\ 2x = 192.000 - 64.000 \\ 2x = 128.000 \\ x = 64.000 \text{ (Kaos)} \end{array}$$

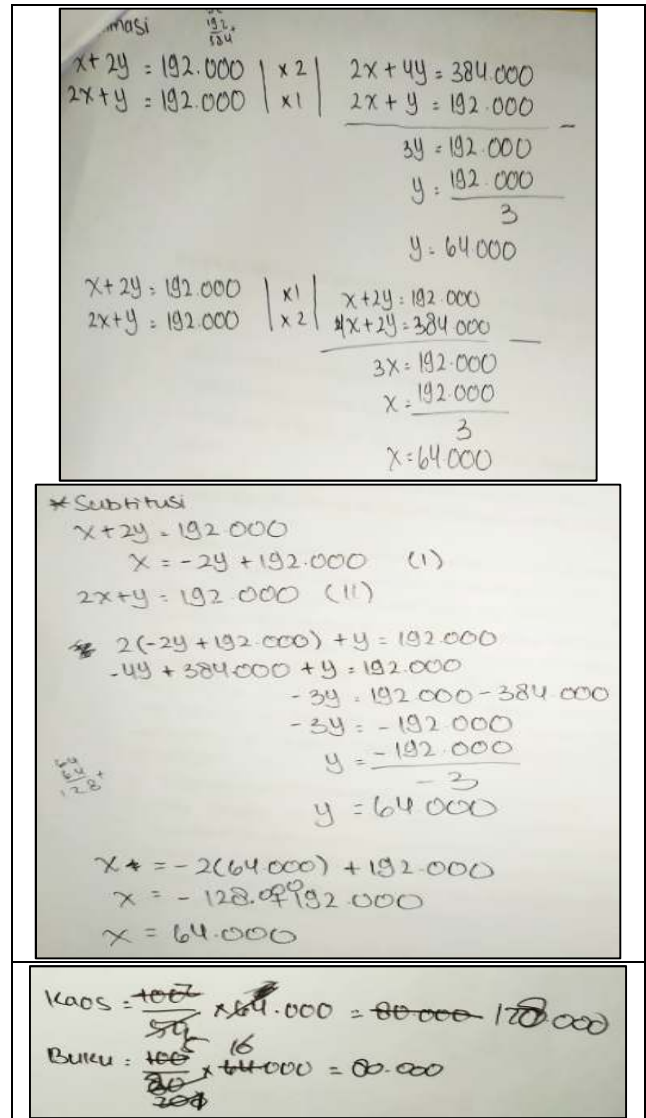
Below the main work, there is a small box with:

$$x = \text{Kaos}$$

$$y = \text{Buku matematika}$$

At the bottom, there is another box with calculations for original prices:

$$\begin{array}{l} \text{Uang Ana} : 200.000 - 8.000 = 192.000 \\ \text{Uang Bunga} : 250.000 - 58.000 = 192.000 \\ \begin{array}{l} x + 2y = 192.000 \\ 2x + y = 192.000 \end{array} \end{array}$$



Handwritten mathematical work for SHN's answer using the elimination and substitution methods:

Elimination:

$$\begin{array}{r} x + 2y = 192.000 \quad | \times 2 \\ 2x + y = 192.000 \quad | \times 1 \\ \hline 3y = 192.000 \\ y = 64.000 \\ x + 2y = 192.000 \\ x + 128.000 = 192.000 \\ x = 64.000 \end{array}$$

***Substitusi:**

$$\begin{array}{l} x + 2y = 192.000 \\ x = -2y + 192.000 \quad (I) \\ 2x + y = 192.000 \quad (II) \\ 2(-2y + 192.000) + y = 192.000 \\ -4y + 384.000 + y = 192.000 \\ -3y = 192.000 - 384.000 \\ -3y = -192.000 \\ y = \frac{-192.000}{-3} \\ y = 64.000 \\ x = -2(64.000) + 192.000 \\ x = -128.000 + 192.000 \\ x = 64.000 \end{array}$$

Final calculations:

$$\begin{array}{l} \text{Kaos} = \frac{192.000}{2} \times \frac{16}{16} = 64.000 \\ \text{Buku} = \frac{192.000}{2} \times \frac{16}{16} = 64.000 \end{array}$$

Fig.2. SHN's answer

SHN and NTM provide almost the same answer so that presented in Figure 2 is only the SHN answer. SHN and NTM showed being able to determine the price of t-shirts and math books before discounting correctly. SHN and NTM consider t-shirts with variable x and math books with variable b . Next, a mathematical model is formed according to the illustration in the problem. The method used to determine the price of t-shirts and math books after the discount. There are three (flexible) methods, namely a combination (elimination-substitution), elimination, and substitution. Figure 2 shows that SHN uses a formula to look for prices before a discount if it is known after the discount price and discount amount. In contrast to SHN, the method used to determine prices before discounting is to add the price of t-shirts with the same value (Rp. 64,000, -) and to prioritize the price of a math book before discounting with A. Furthermore, the calculation is carried out in order to obtain the price of the book before discounting. The SHN and NTM answers are correct even though the method is different. How NTM determines the price of t-shirts and math books before discounting in Figure 3.

Handwritten mathematical work for Figure 3. It shows two columns of equations. The left column, titled 'Harga sebelum diskon', contains:

$$Kaos = 69.000 + 69.000$$

$$= 128.000$$
The right column, titled 'buku', contains:

$$A = 69.000 + \frac{20}{100} \times A$$

$$A = \frac{320.000}{5} + \frac{1}{5} \times A$$

$$A - \frac{1}{5}A = 69.000$$

$$\frac{5}{5} - \frac{1}{5} = \frac{4}{5} = 69.000$$

$$A = 80.000$$

Fig.3. How NTM Determined Prices Before Discounts

Based on the description, SHN and NTM were able to understand the problem, plan a solution to the problem, and use three different solutions (elimination-substitution, elimination, substitution). Through interviews, SHN and NTM were able to explain the solutions they used. SHN and NTM use all three methods in accordance with the understanding and knowledge obtained from the teacher. From the results of tests and interviews, it can be concluded that SHN and NTM show the flexibility of mathematical creative thinking.

3) MA's Answer

Handwritten mathematical work for Figure 4. It shows two methods. The first method, titled 'Gabungan', shows:

$$K : Kaos$$

$$B : Buku Matematika$$

$$angk : K + 2B = Rp. 200.000$$

$$bunga : 2K + 1B = Rp. 200.000$$

$$: K + 2(Rp. 80.000) = Rp. 200.000$$

$$K = Rp. 100.000$$

$$K - 80\% = Rp. 100.000 \times$$
The second method, titled 'Eliminasi', shows:

$$K + 2B = Rp. 200.000$$

$$2K + B = Rp. 200.000$$

$$K + 2B = Rp. 200.000$$

$$2K + B = Rp. 200.000$$

$$K = Rp. 100.000$$

Handwritten mathematical work for Figure 5. It shows two methods. The first method, titled 'Substitusi', shows:

$$K + 2B = Rp. 200.000$$

$$2K + B = Rp. 200.000$$

$$\rightarrow 2(Rp. 200.000 - 2B) + B =$$

$$Rp. 400.000 - 4B + B =$$

$$- 3B =$$

$$3B =$$

$$B =$$

$$K + 2B = Rp. 200.000$$

$$2K + B = Rp. 200.000$$

$$\rightarrow K + 2(Rp. 200.000 - 2K)$$

$$K - 4K + Rp. 400.000$$

$$- 3K$$

$$K =$$
The second method, titled 'Eliminasi', shows:

$$K + 2B = Rp. 200.000$$

$$2K + B = Rp. 200.000$$

$$K = Rp. 100.000$$

$$K - 80\% = Rp. 100.000 \times$$

Fig.4. MA's Answers

Figure 4 shows that MA uses three methods of completion, namely combination (elimination-substitution), elimination, and substitution even though the answer given is wrong. Firstly, MA gives the T-shirt with the K variable and the math book with the B variable. The completion steps given by the MA are correct, but the mathematical model is arranged incorrectly. As a result, the MA's answer is wrong.

Based on the results of the interview, the MA said that the answers written were wrong. Then the Supreme Court tries to write down the answer improvement with the combined method (elimination-substitution). The repairment of MA's answer is in the figure 5.

Handwritten mathematical work for Figure 5. It shows two methods. The first method, titled 'Gabungan', shows:

$$K + 2B = Rp. 200.000$$

$$2K + B = Rp. 200.000$$

$$K + 2B = Rp. 200.000$$

$$2K + B = Rp. 200.000$$

$$K = Rp. 100.000$$

$$K - 80\% = Rp. 100.000 \times$$
The second method, titled 'Eliminasi', shows:

$$K + 2B = Rp. 200.000$$

$$2K + B = Rp. 200.000$$

$$K = Rp. 100.000$$

$$K - 80\% = Rp. 100.000 \times$$

Fig.5. The repairment of MA's answer

Figure 5 shows that the mathematical model developed by MA is correct, but there is an error in determining the price before the discount. Through interviews, the MA is able to explain the answers given. The MA explained that initially had difficulty in understanding the problem. MA also said that he did not understand how to determine the price before the discount if what was known was the price after the discount and the amount of the discount. MA chose to use the combined method (elimination-substitution) because it has been accustomed since junior high school to use that method.

MA also never thought of other completion methods. MA only uses the completion method which is in accordance with the teacher's explanation. MA feels lazy and is not interested in thinking of other methods and using commonly used methods. Based on the results of tests and interviews, it can be concluded that the MA has been able to demonstrate the flexibility of creative thinking. Mistakes were made due to difficulty understanding the problem and not understanding the concept of discount.

B. Discussion

Berpikir kreatif sangat diperlukan dalam proses pemecahan masalah [8]. Flexibility is one aspect of creative thinking that is closely related to the ability of students to provide a variety of different ways in solving problems. Test and interview results show that students have demonstrated the flexibility of mathematical creative thinking. Students are able to provide three different methods for solving mathematical problems. The completion methods used are elimination, substitution, and combination (elimination-substitution). Students use various methods that are already understood and explained by the teacher. That is, the flexibility of students' mathematical creative thinking is still influenced by an understanding of a mathematical concept. A good understanding of a concept will stimulate the brain to think flexibly [9]. This stimulus will make students able to produce a variety of different ideas.

Even though students have demonstrated the flexibility of mathematical creative thinking, this ability must still be trained and developed. The teacher can get students used to completing Multiple Solution Tasks (MST). MST is a task that indirectly asks students to solve mathematical problems in a variety of different ways of solving [10]. The flexibility of mathematical creative thinking students are expected to be trained and developed using MST because this task refers to three components of creative thinking, namely fluency, flexibility, and novelty. So, students who often complete MST will get used to thinking flexibly.

In addition, learning that is oriented towards the development and training of student flexibility also needed to be considered by the teacher. One learning model that can be applied is Problem Based Learning (PBL) with the Self Regulation Learning (SRL) approach. This has been proven through the results of research which states that this model is more effective than expository learning [11]. Students involved in the PBL learning model with the SRL approach are able to determine the many possible ideas in solving mathematical problems [11]. That is, flexibility of students can be trained and developed by applying PBL with SRL approach.

IV. CONCLUSION

Students have been able to demonstrate the flexibility of mathematical creative thinking, namely using a variety of different methods in solving mathematical problems. Students use three methods, namely elimination, substitution, and combination (elimination-substitution). The method used by students is still limited to methods that have been understood and explained by the teacher. Students are not interested in finding methods according to their own ideas.

The flexibility of mathematical creative thinking students must continue to be trained and developed. One way is to provide a Multiple Solution Task (MST). Student's flexibility is still influenced by understanding of a mathematical concept. Therefore, it would be better to conduct further research on the relationship between students' flexibility and understanding of mathematics and the relationship to the other two aspects of mathematical creative thinking.

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