

The Dual Production Frontiers: A New Preliminary Framework for Productivity Research

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ABSTRACT. The mainstream measurement framework for total factor productivity is the original production function defined in the "quantity" space, which often leads to the lack of measurement of allocation efficiency and distortion of other factors. Based on production theory, this paper incorporates the price factors of input and output, uses duality theory to derive the concept of dual production function, and constructs a dual production frontier to explore a feasible new paradigm for productivity research. In contrast, the new framework can clarify the impact of market price changes on production decisions, accurately measure allocation efficiency, and other factors that cause productivity changes, extend the measurement of total factor productivity to the "quantity-price" joint space.

1. INTRODUCTION

Total factor productivity (TFP) is one of the important sources of macroeconomic growth. However, we still can't measure and decompose the TFP accurately and comprehensively [1][2].

Compared with the index method, frontier analysis methods including stochastic frontier analysis (SFA) and data envelopment analysis (DEA) are widely used because of their reasonableness and scientificity[3][4]. However, at present, there are obvious shortcomings when using the frontier method to study the TFP: mainstream research generally uses the original production function to characterize production behavior, estimates the original production frontier function to estimate the technical efficiency of production decision units, to measure and decompose TFP growth rates. Because the original production function can't clarify the input and output price factors, failed to examine the production decision process affected by the market[5][6], especially the adjustment of the input factor density and output structure based on price changes, the TFP measurement based on the original production function is only a study of the "quantity" dimension, which often leads to the lack of measurement of allocation efficiency and distortion of the other production characteristics.

In fact, in addition to the quantity that has a significant impact on input-output efficiency, the price factor is also a crucial factor in the production decision-making process[7] [8][9]. These production and management adjustment processes will directly or indirectly affect technological progress and technological efficiency, and will also affect the scale of production, and will have a significant impact on the allocation efficiency of inputs and outputs[10][11].

Therefore, it is significant to explore and build a new framework that includes both the quantity and price of inputs and outputs, so as to measure TFP comprehensively and accurately[12][13][2]. Used the constrained profit function which including input and output price information, we establish a relationship between the constrained profit function and the dual production function, and try to study the TFP by replacing it with the original production function. The stochastic frontier measurement framework in the "quantity" space is extended to the dual frontier framework defined in the "quantity-price" joint space. Theoretically, the effects of various factors on TFP can be revealed in more detail and accurately, so as to make up for the lack of measurement of allocation efficiency in the current research, to avoid distortion of the description of production technology structure by the measurement results of technical efficiency, technological progress and



returns to scale, and to build a new The TFP measurement method which provides a whole new theoretical paradigm.

2. Defects of the original production frontier approach in the TFP measurement

2.1. The Meaning of the original production frontier

In 1928, the mathematician Cobb and the economist Douglas proposed to use inputs and outputs to describe the production function. The general nature of the technical structure is used to reflect the basic characteristics of inputs and outputs, and to study the law of returns followed by inputs and outputs[14]. Later, after nearly a century of development, the production function has gone from macro to micro, and from two inputs and one output to a multi-input and multi-output model, which has been widely used in neoclassical economics. In 1967, Arrow and others proposed a two-factor CES production function model; in 1968, Sato and Hoffman proposed a VES production function; in 1973, Christensen and Jorgenson built a translog production function model. However, it must be clear that no matter what form of production function is, its essence is to describe the "physical and technical structure" based on the quantitative relationship between input and output, without examining the impact of price factors on the decision-making and technology in the production process.

The original production frontier is a concept based on the original production function. According to the assumptions of neoclassical economics, the original aggregate production function is generally expressed as:

$$y(t) = f(x,t) \tag{1}$$

Where, ^y represents the total output of the production decision unit; ^x represents the total input of the production decision unit. $f(\cdot)$ describes the input-output "technical structure" of the production decision unit, which reflects the "material quantity" relationship between various inputs and outputs.

If we consider the output performance of different producers at the same input level at a given time, we define the optimal performance as "frontier". In the set of producers H which examined here, the input-output relationship of producers h can be expressed as:

$$y_h(t) = f_h(x_h, t) \tag{2}$$

In formula (2), y_h represents the scalar of the composite output, that is, the total output under the condition of Hicks-Leontief; $x_h = (x_{h1}, x_{h2}, \dots, x_{hm})'$ represents the quantity vector of the multiple inputs; and $f(\cdot)$ represents the multiple synthetic relationships of inputs and output.

According to the general definition of production economics, the frontier is the boundary formed by the minimum input cost or the maximum output corresponding to different input costs under a given technical level. The maximized production at the input level is:

$$y_{h}^{*}(t) = f_{h}^{*}(x_{h}, t) = \max_{k \in H} \left\{ f_{k}(x_{h}, t) \right\}$$
(3)

Among them, y_h^* indicates the output level of the unit h with the best performance in the producer set H under the total input level of the production decision unit, which is the largest possible output. This is also the maximum output under static conditions. In the long run, the output after adjusting for the input of different elements forms a curve (a curved surface when there are multiple inputs), which is named the production frontier.

It is easy to understand that those producers who produce along the production frontier are technically effective, while production at any point below the production boundary is technically ineffective, and there are multiple factors in the production of the producer. The loss of output efficiency in the quantity dimension, according to the radial measurement methods of Debreu (1951) and Farrell (1957), measures the output-oriented technical efficiency by the distance between a single decision unit and the frontier of production[15][16]. The distance function is:



$$D_{\rho}(x, y) = \min\left\{\mu : y/\mu \in f(x)\right\}$$

$$\tag{4}$$

Equation (4) uses the method of output growth to measure the radial distance from the producer's output to the boundary of the possibility of production, which indicates the possibility of the largest

increase under the given input when the output vector increases. Since y/μ still belongs to the set of production possibilities, therefore, $\mu < 1$, represents the production efficiency of the production decision unit.

With a certain function forms and computer software, we can estimate function form of the production frontier and obtain the technical efficiency of each decision unit. And on this basis, the growth equations of the frontier production functions can be calculated, and their technological progress rates can be calculated to examine the impact of time on the production frontier; their returns to scale can be calculated to examine the impact of changes in production scale on productivity; If the price and quantity of various inputs can be obtained, and the cost share of different inputs can be calculated, the change in the allocation efficiency can be calculated in combination with the change in the number of inputs, which partially reflects the efficiency brought by the producer to make production adjustments based on market prices Variety.

2.2. The two defects of the original production frontier

The production function method has been widely used to measure the material quantity relationship between input and output to reveal the elasticity and contribution of various inputs to output. However, it is flawed to study the process and effect of production decision using the production function framework.

On the one hand, the original production function cannot examine the most basic homo economicus assumption in economics. That is, in economic activities, the goal is to achieve "optimal", as consumers want to maximize utility; as a producer, regardless of cost factors, the pursuit of maximum output, in the market behavior to maximize income and minimize costs And profit maximization; as an economy, it is also necessary to maximize profit from the perspective of production. Although the number of inputs and outputs is already the result of producers' "rational" decisions, the more important goals in economic research are how much the producers are rational, how to implement their rational decision-making process, and how exertion of rationality? These are not reflected by the original production function itself.

On the other hand, the exogenous nature of certain fixed input factors is not considered in the original production function model, which is essentially a constraint rather than an endogenous variable. In the short term, exogenous variables themselves will not be changed by the influence of the model. Its impact on independent and dependent variables is more suitable for comparative static analysis rather than direct analysis as endogenous variables for dynamic analysis. The disposable income held by consumers is limited; the factors of production held by producers are also limited; as an economy, the amount of resources it owns is also limited. Therefore, it is more suitable to conduct short-term analysis in the study of economic activity decisions, that is, to convert exogenous variables into constraints for optimization analysis. Only in this way can the resource scarcity hypothesis be better reflected[11][17][1].

3. The Meaning and Superiority of Dual Production Frontiers

In recent decades, with the deepening of research on production theory, the limitations of the traditional original production function as "describe the characteristics of physical production" have continued to emerge, and economists have strived to find more scientific and reasonable methods to characterize the entire process of production. Therefore, based on the production function, Shepard et al. Studied the relationship between the cost function and input demand and proposed Shepard's Lemma; Hotelling used the profit function to sort out the relationship between output supply and the prices which is proposed by Hotelling's Lemma. They further deepen and concrete the "resource"



scarcity" and " homo economicus" of the neo-classical hypotheses[9]. Later, McFadden, Jorgenson, Diewert, Lau, and others rigorously proved the dual relationship between the production function and the profit function, that is, the production behavior described by the profit function can analyze the production with exactly the same original production function[11][16]. The technical characteristics can also study the production decision-making process that the original production function function function function a complete, comprehensive, scientific and reasonable important paradigm for the study of production behavior.

3.1. From the original production frontier to the dual production frontier

Starting from the original production function, according to formula (1), a producer can be set to use a variety of N variable inputs in a given period, expressed as a vector $x = (x_1, x_2, ..., x_N)$, and have M fixed resource, expressed as a vector $v = (v_1, v_2, ..., v_M)$. The function is described as:

$$y = f(x_1, ..., x_N; v_1, ..., v_M)$$
(5)

Among them, x and v represent the variable and fixed inputs. Therefore, the short-term profit, defined as the benefit minus variable costs, that is:

$$\pi = pf(x_1, ..., x_N; v_1, ..., v_M) - \sum_{n=1}^{N} q_n^* x_n$$
(6)

Among them, p represents the price of the output y, and q_i^* represents the price of the i variable input. Using the Hotelling's lemma, the relationship between the normalized constrained profit function and the dual output can be obtained by derivation transformation:

$$\hat{y} = F(q; v) = \hat{\pi}(q; v) - q' x^{*}(q; v)$$
(7)

Here, \hat{y} is a dual output composed of a standardized and constrained profit function and an input counter-demand function, $\hat{\pi}$ is a standardized and constrained profit function, \hat{q} is a standardized variable input price transposition matrix, and x^* is a standardized and constrained profit. The function calculates the input inverse demand function. Equation (7) shows that the dual production frontier is a function of the price of variable inputs and the number of fixed resource constraints. As a result, the dual production frontier expands the "quantity" space of the original production frontier into a "quantity-price" joint space. With reference to equations (3) and (4), we can carry out the estimation of the dual production frontier and measure the technical efficiency, and use equation (7) to conduct growth analysis, which can further decompose and measure productivity growth.

3.2. Significance of Measuring Productivity in Dual Production Frontiers

It is not difficult to understand that the measurement of productivity can rely on two different reference frames: the original production function and the dual production function. The latter uses the duality between the original production function and the normal restricted profit function. The latter can describe the efficiency of produce which can be measured together from the two dimensions of quantity and price, making the decomposition of productivity more specific and in-depth[15].

There are at least three important points to productivity research in using dual frontier framework. Firstly, it has redefined the concept of total factor productivity. Total factor productivity is a well-known but misleading concept. Mainstream economics has not always put the importance of total factor productivity in place, and usually regards technological changes as a secondary factor in analyzing research issues[10]. So, productivity has so far been mainly defined by technology-based input-output analysis in the "quantity" space. The dual production frontier framework proposed in this paper is an accurate concept defined by the combination of technological improvement and configuration efficiency improvement in the "quantity-price" joint space. Secondly, the research methods of total factor productivity have been expanded. In general, total factor productivity wants to examine the ratio of output and input by using the ratio of effective



input and total input of the same dimension in engineering. The input-output measurement has different meanings due to different ratio calculation methods [10]. Therefore, different research methods directly determine the connotation of total factor productivity and herald the interpretation of economic policy. So far, the most important research methods of total factor productivity are probably the following four: the least square method (LS) of the econometric production model; the total factor productivity index (TFP INDEX) method, including the Tornqvist and Fisher index Data Envelopment Analysis (DEA); Stochastic Frontier Analysis Method (SFA). There may be new methods in recent years, but in the final analysis they are all extensions of these four methods. However, these methods did not examine the price factors of input and output or the impact of market conditions on production decisions, and could not clarify the effect of price on the resource allocation process. The dual production frontier framework provides a feasible method for us to examine the mechanism of the impact of input and output prices on resource allocation. Thirdly, it has enriched the connotation of total factor productivity. Productivity is a concept defined on the basis of the production function. Initially, it was the ratio of the output and input of the production unit in terms of material quantity at the micro level [18] [19]. The growth of total factor productivity, which is defined on the basis of the production function, can be decomposed into measures of technical efficiency, technological progress, and economies of scale, but the analysis of allocation efficiency cannot be in-depth because it is not included in the price factor. Therefore, the dual production frontier defined in the "quantity-price" joint space can study the allocation efficiency more deeply and enrich the connotation of total factor productivity.

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