

# The Importance of Regions on Debt Networks

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**Keywords:** networks, vital nodes, centrality, epidemic model

**Abstract.** This paper researches the important regions identification on real-world debt networks via epidemic dynamic model. The authors observe that regions with lower betweenness centrality and higher clustering coefficient are unstable on the networks from the perspective of network topology. Moreover, excessive debt is not the only reason of the risk infection. Network topological structure is also a significant factor. The findings can deepen the understanding of the spread process of risk in real financial systems.

## 1. Introduction

With the development of globalization, the dependency between government, banks, enterprises and other organizations has become increasingly complex. Such complex interdependency increases the probability and the extent of risk spread. The recent economic researches have raised a broad awareness that the financial systems should be regarded as complex networks whose nodes typically represent financial institutions and edges represent financial dependency [1]. Thus applying the complex networks analysis to financial networks is crucial to designing incentives and regulatory responses when worldwide systems risk happens. Thanks for the challenge and significance, the vital nodes identification on financial networks attracts increasing attentions recently. A variety of measures have been proposed to determine the centrality of a node in real social networks [2,3,4]. Similar to social networks, when the central nodes are affected, it would lead to a financial cascade and serious consequences [5]. As for risk propagation process, most work adopt a simulation approach to examine how risk propagate through different network structure based on debt holdings or interbank lending [6,7]. It can be quantified and measured from the analysis of the dynamical evolution of the nodes and the structure of the network [8,9,10,11,12].

Comparing with previous works, our main contribution is that we set up the model on real-world debt networks rather than synthetic networks and we use the Susceptible-Infected epidemic dynamic model with economic meanings. We first illustrate a modified Susceptible-Infected epidemic dynamic model, and then we set up the debt networks based on real data. Finally, combining several commonly used networks topology indicators, we analyze the simulation results on the networks among the 193 regions.

## 2. Epidemic dynamic model

Susceptible-Infected model played an important role in population structure in determining properties of disease invasion and spread in heterogeneous populations. Assume an infinite population, and for each individual there are two states, (1) susceptible state, (2) infected state [13,14].

In order to assess the regions importance on real-world debt networks, we set up the weighted directed networks with data on the crossholdings of debt and GDP (data from World Bank) among 193 regions. Then we develop a general model regarding financial contagions among nodes linked through a network of financial interdependency. This is equal to the value of institution  $i$ 's primitive assets plus the value of its claims on other institutions [15,16]:

$$V_i = \sum_k D_{ik} p_k + \sum_j C_{ij} V_j \quad (1)$$

Written in matrix notation and solved to yield, we can get  $V=(I-C)^{-1}Dp$ . The matrix  $C$  is a crossholdings network and the matrix  $D$  denotes direct-holdings. According to previous research, the ultimate value of an organization is well-captured by the equity value of that organization that is held by its outside investors [15,16]. Therefore, the market value  $v=\hat{C}v=\hat{C}(I-C)^{-1}Dp=ADp$ . Thus we get the dependency matrix:

$$A=\hat{C}(I-C)^{-1} \quad (2)$$

$A_{ij}$  describes the dependence of  $i$ 's value on  $j$ 's proprietary asset. For all  $j \in N$ ,  $\sum_{i \in N} A_{ij}=1$ .  $\hat{C}$  is a share of organization not owned by any organization in the system.

With the dependency matrix above, we then combine it with the basic SI epidemic dynamic model via Eq. 2. We set the ratio of total debt held by outside regions  $c = 1/3$  [17]. After that, we select a node randomly and reverse its state into infected. Then we start the epidemic-like dynamical evolution. We find all of the infected nodes and list their uninfected neighbor nodes. If a node is infected, we reduce the value of it. When the node loses half of its value, we change the state of it into infected.

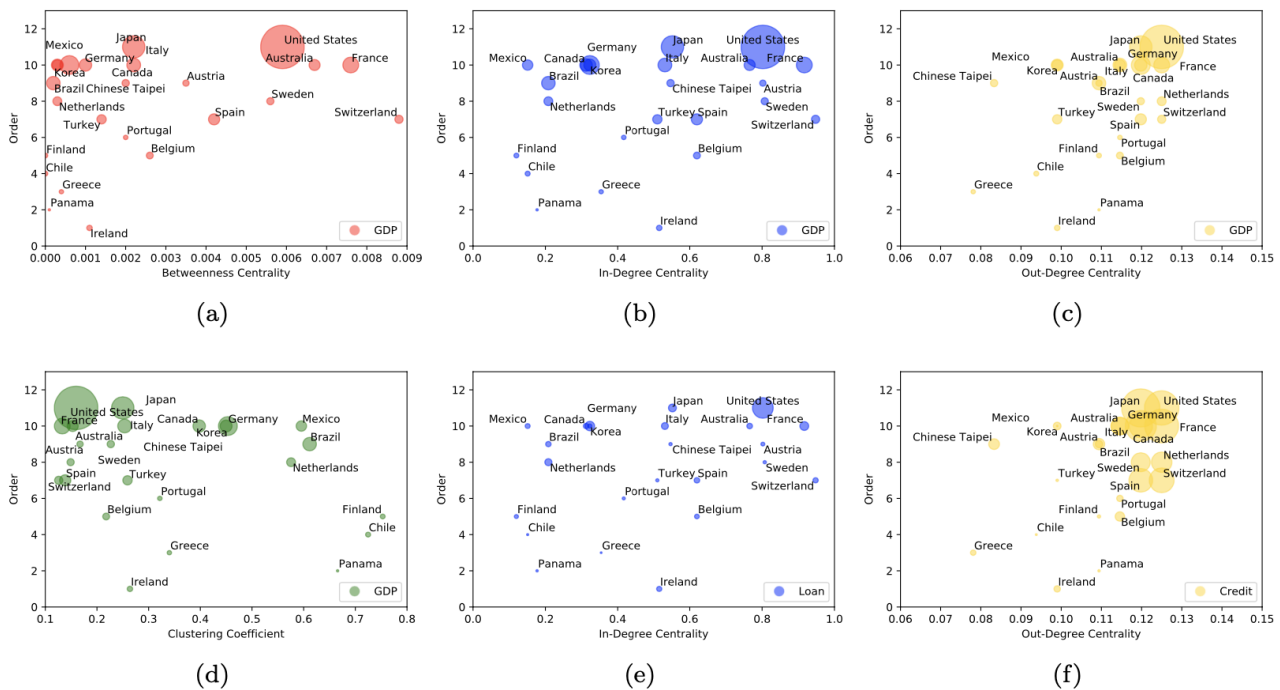


Fig. 1. Order of risk infection and network indicators. Scatter plot of risk infection sequence versus betweenness centrality, clustering coefficient, in-degree centrality, out-degree centrality. For sake of simplicity, in the simulation, asset size was assumed constant during the time span of the data. The risk infection sequence happens when we set the UK as the source node. The four network indicators are calculated based on the 2016 consolidated foreign claims from BIS. The size of bubble is proportional to the GDP of 2016 in (a),(b),(c),(d). And the bubble size is proportional to the total number of loan and credit respectively in (d),(f).

In the propagation of infection, we set the loss value of successor susceptible node  $S_j$  in the following form  $L_{ij}=W_{ij} \times N_i \times M_j$ , in which  $L_{ij}$  is the loss value of node  $S_j$  from infected node  $I_i$ ,  $W_{ij}$  is the link weight in dependency matrix,  $N_i$  represents the total neighbors' amount of infected nodes  $I_i$ . The loss value  $L_{ij}$  is proportional to the total amount of connections the infected node has. So it will spread the risk dependent on the link weight and the number of neighbors. Another aspect we need to consider is that the higher GDP a node has the less loss it will take.

### 3. Analysis of Results

#### 3.1 The data

On June 23, 2016, the United Kingdom held a referendum on leaving the European Union. The Brexit was an impact for the European Union and even the whole world. Many experts and scholars took

the Brexit event compared with the Greek debt crisis and worried whether it will trigger a European or global crisis. Thus we set up the debt networks based on the 2016 dataset from BIS (<https://www.bis.org>). The debt data we used to set up are the consolidated foreign claims of banks from one region on debt obligations of another region, which focus on the immediate borrower rather than the final borrower. And we use the GDP of each regions for that year as the node size. The GDP data are obtained from World Bank (<https://data.worldbank.org>).

### 3.2 Simulation results

We give the UK a shock on the network to observe the position of each region on the network from the perspective of the ability to be infected among the 193 regions. Because most regions are at the end of the power-law distribution, only 24 of the regions in Fig. 1 are infected due to tight dependencies when shocking the UK. The Y axis in Fig. 1 shows the infected sequence. The higher the ranking, the more stable the region, and vice versa. We find that Ireland, Panama and Greece are most susceptible to infection, Japan and the United States are the most stable regions.

### 3.3 Ranking based on stability and spread speed

Next we mainly discuss the topological properties of the network to figure out the ranking factors. The value of a node in the networks depends on the location of the node in the network. The more centrally located, the greater the value of the node. We use several commonly used indicators: betweenness centrality, clustering coefficient and degree centrality to explore the risk infection sequence on the financial network.

Fig. 1 shows the values of some indicators about network structure on the real networks. According to the calculation results, the following conclusions are obtained:

(1) Region with smaller betweenness centrality usually have lower influence on others. Betweenness centrality represents the degree to which nodes stand between each other. A node with higher betweenness centrality would have more control over the network, because more risk will pass through that node. We observe that in Fig. 1(a) the betweenness centrality of Finland, Chile, Panama, Greece, Ireland are smaller than United States and France, so they have lower influence on others than the latter.

(2) Region with bigger clustering coefficient are more likely to be infected. The clustering coefficient of a node in a graph quantifies how close its neighbors are to be a clique. The clustering coefficient is 1 if every neighbor connected  $v_i$  is also connected to every other node within the neighborhood, and 0 otherwise. We find that, in the debt network, the bigger the clustering coefficient, the more likely the region to be infected (Fig. 1(d) Finland, Chile, Panama).

Combining the two conclusions above, we can explain why Panama, Ireland and Greece do worse when they are forced into crisis. From our results, they have lower betweenness centrality and higher clustering coefficient on the network. That is to say, they have lower influence on other regions but they can easily be infected by others.

(3) Degree is a basic indicator to study networks [18]. For a weighted directed network, in-degree is the number of in-coming links (the number of predecessor nodes); out-degree is the number of outgoing links (the number of successor nodes). Thus in our network, in-degree is interpreted as a popular creditor, and out-degree as debtor. Degree centrality is historically first and conceptually simplest indicator to study the centrality of a node [19].

Considering in-degree as a popular creditor and out-degree as debtor [20], we also present the total number of loan and credit in Fig. 1(e), (f). In classical economy, too much debt will lead to bankruptcy. For example, Greece and Ireland both have debt crisis due to their high debt in 2009 and 2010. But an interesting phenomenon is that unlike the pessimistic attitude of the excessive debt, the United States, Japan, France and Germany all have higher debt than Greece and Ireland, while they are still able to maintain good situation (showing in Fig. 1(f)). Due to the balanced connections to others, the United States, Japan, France and Germany not only have higher GDP to absorb the loss but also have more stable situation in the debt network comparing with others. Thus when we estimate the debt crisis infection, the complex network topology is also need to be considered.

#### 4. Summary

In this paper, we identify the vital regions on the real-world debt networks. Vital node identification is very significant to link prediction and networks control in the field of complex networks. The financial systems can be regarded as complex networks. Thus network science can contribute to the identification of vital regions under the financial risk infection from the analysis of the epidemic-like dynamical evolution.

In order to adapt the research on financial networks and investigate the important regions on debt networks, the classical epidemic-like model is modified. It can be seen that Ireland, Panama and Greece are most susceptible to infection while Japan and the United States are the most stable regions when UK is given a shock. To find out more, we use several commonly used indicators: betweenness centrality, clustering coefficient and degree centrality to explore the risk infection sequence on the debt network. In our results, Panama, Ireland and Greece perform worse when they are forced into crisis. From the perspective of network topology, they have lower betweenness centrality and higher clustering coefficient. That is to say, they have lower influence on others, but they can easily be infected by others. On the contrary, the United States and Japan have higher betweenness centrality and lower clustering coefficient, so they are more stable when they are forced into crisis.

#### Acknowledgment

This research was financially supported by the National Natural Science Foundation of China (Grant Nos.61573065 and 71731002).

#### References

- [1] Stefania Vitali, James B Glattfelder, and Stefano Battiston. The network of global corporate control. *PloS one*, 6(10):e25995, 2011.
- [2] Linyuan Lü, Duanbing Chen, Xiao-Long Ren, Qian-Ming Zhang, Yi-Cheng Zhang, and Tao Zhou. Vital nodes identification in complex networks. *Physics Reports*, 650:1–63, 2016.
- [3] Stephen P Borgatti. Centrality and network flow. *Social networks*, 27(1):55–71, 2005.
- [4] Linton C Freeman, Douglas Roeder, and Robert R Mulholland. Centrality in social networks: Ii. experimental results. *Social networks*, 2(2):119–141, 1979.
- [5] Giulia Iori, Saqib Jafarey, and Francisco G Padilla. Systemic risk on the interbank market. *Journal of Economic Behavior & Organization*, 61(4):525–542, 2006.
- [6] Daron Acemoglu, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. The network origins of aggregate fluctuations. *Econometrica*, 80(5):1977–2016, 2012.
- [7] Prasanna Gai and Sujit Kapadia. Contagion in financial networks. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 466(2120):2401–2423, 2010.
- [8] Andrew G Haldane and Robert M May. Systemic risk in banking ecosystems. *Nature*, 469(7330):351, 2011.
- [9] Giulia De Masi, Giulia Iori, and Guido Caldarelli. Fitness model for the italian interbank money market. *Physical Review E*, 74(6):066112, 2006.
- [10] Stefano Battiston, James B Glattfelder, Diego Garlaschelli, Fabrizio Lillo, and Guido Caldarelli. The structure of financial networks. In *Network Science*, pages 131–163. Springer, 2010.
- [11] Stefano Battiston, Domenico Delli Gatti, Mauro Gallegati, Bruce Greenwald, and Joseph E Stiglitz. Liaisons dangereuses: Increasing connectivity, risk sharing, and systemic risk. *Journal of economic dynamics and control*, 36(8):1121–1141, 2012.

- [12] Yang-Yu Liu, Jean-Jacques Slotine, and Albert-László Barabási. Controllability of complex networks. *nature*, 473(7346):167, 2011.
- [13] Roy M Anderson, B Anderson, and Robert M May. *Infectious diseases of humans: dynamics and control*. Oxford university press, 1991.
- [14] Steven Riley. Large-scale spatial-transmission models of infectious disease. *Science*, 316(5829):1298–1301, 2007.
- [15] Francesco Brioschi, Luigi Buzzacchi, and Massimo G Colombo. Risk capital financing and the separation of ownership and control in business groups. *Journal of Banking & Finance*, 13(4-5):747–772, 1989.
- [16] Matthew Elliott, Benjamin Golub, and Matthew O Jackson. Financial networks and contagion. *American Economic Review*, 104(10):3115–53, 2014.
- [17] Carmen M Reinhart and Kenneth S Rogoff. From financial crash to debt crisis. *American Economic Review*, 101(5):1676–1706, 2011.
- [18] Stanley Wasserman, Katherine Faust, et al. *Social network analysis: Methods and applications*, volume 8. Cambridge university press, 1994.
- [19] Tore Opsahl, Filip Agneessens, and John Skvoretz. Node centrality in weighted networks: Generalizing degree and shortest paths. *Social networks*, 32(3):245–251, 2010.
- [20] Coseteng, M. Y., and C. Y. Lee. "Changes in apple polyphenoloxidase and polyphenol concentrations in relation to degree of browning." *Journal of Food Science* 52.4 (1987): 985-989.