

# Conditions and Trends of the Social and Economic Development of Sevastopol

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**Abstract.** Sevastopol is a region of the Russian Federation and at the same time a municipality with its own social and economic development trends. This is a city with rich military history and potential for tourism development, high level of education and culture, unique geographic location and climate. From the investment point of view, the region is attractive as it has a developed transport infrastructure, a warm-water seaport, developing wine production, ship building and maintenance, fisheries and fish processing, information technologies; green, history and wine tourism. At the same time, experts believe that Sevastopol has an insignificant investment potential and moderate investment risks. The article proposes a hypothesis on the interconnection of such performance indicators of the social and economic development as gross regional product, investment into the capital stock and annual average wages. It does not seem possible to analyse more indicators due to the statistic timeline limitations associated with the development of a model (objective data on Sevastopol is available only for 4 years). The interconnection of the variables has the multicollinearity effect that can produce false results. In order to make objective evaluations, the article presents the methodology for developing vector auto-regression equations and assess its parameters, a system of structural and reduced simultaneous autoregressive equations was presented, the unknown parameters were assessed. The simulation allowed to prove that the wages is not influenced by investments, or GRP, or vice versa. GRP grew due to investments that had been increased thanks to state support. The developed models and accumulated data can be useful for the authorities in terms of pro-active management. Further research will be aimed at assessing the feasibility of the performance indicators under the Strategy for the social and economic development of Sevastopol until 2030 taking into the account the results published in this paper.

## 1. Introduction

Sevastopol is a region of Russia and its development is important for both its residents and all Russians due to many reasons. This is a city with rich military history and potential for tourism development, high level of education and culture, unique geographic location and climate. From the investment point of view, the region is attractive as it has a developed transport infrastructure, a warm-water seaport, developing wine production, ship building and maintenance, fisheries and fish processing, information technologies; green, history and wine tourism. At the same time, from the expert point of view, Sevastopol has insignificant investment potential (the 71st among Russian

regions) and moderate investment risks (the 65th out of 85)<sup>1</sup> According the Rosstat<sup>2</sup>, in 2016 Sevastopol was the 63rd by nominal wages and the last by GRP per capita.

Medium and small businesses, town residents who invested in Sevastopol economy in 3 years not less than 3 million rubles and non-residents who invested not less than 30 million rubles can operate within the free economic zone with considerable preferences. For example, in mid-2018, 423 organizations worked under FEZ regime with the investments to the amount of 4.8 billion rubles and 7 thousand new jobs. At the same time, there are 25 000 medium and small businesses with the 46 000 jobs and only 386 of them were in the FEZ, which is 1.5% of the total number. Respondents say the main reason for not using this regime is the excessive over-regulation of the process.

Hi-tech production is supported by the Heraclides industrial park and IT Crimea, innovative IT cluster and technopark, that are under construction and that will also stimulate SME.

Regional indicators for social and economic development are interdependent variables, and this interaction can be simulated with a system of simultaneous equations. However, besides direct correlations between the indicators, their current values depend on their previous values, in other words these are autoregressive equations.

The goal of the research is to establish the interconnection between major indicators that describe the social and economic development of Sevastopol and identify the contribution of hidden factors that influence it. The article aims to analyse the mutual influence of macroeconomic indicators of the region, GRP, investments in capital stock and wages on the basis of simultaneous autoregressive equations.

The research on regional economy is quite extensive. Let us consider the publications that are relevant for this article. The article [1] proposes an alternative method that allows to forecast the social and economic development of Russian regions. Issues associated with evaluation and forecasting of GRP space configuration and convergence and divergence of Russian regions (their identification) are studied by the authors [2] from the point of view of mass distribution (volume) of GRP by growth rate. Early GRP evaluation method on the basis of relevant regional statistics [3] allows to anticipate the appearance of the indicator in press by 1-1.5 years. The pertinent challenge for Sevastopol is the attraction of investments, including foreign ones, as their flow is limited by sanctions, this is why the publications on the following topics are of interest: direct foreign investment influence on the economy of the receiving country and region [4]; measuring the contribution of direct foreign investments on production in Russian regions and interregional inequality by per capita real GRP [5]; investment attractiveness of regions as a key factor of encouraging foreign investments [6] and their impact on the Russian economy in terms of regions [7]; the interconnection between foreign investment and sanctions [8]; influence of investment process media coverage on the inflow of direct foreign investment into the region [9]; analysis of investment attractiveness evaluation methods by economic activities in regions [10]; influence of investments on social and economic development of a region [11], with options for assessment of the multiplying effects, that resulted from investments into certain industries and identification of growth focal points in regions. From our point of view, the proposed methods and simulations presented in the article will allow the regional authorities to conduct a more evidence-based development policy for the Russian regions.

## 2. Methodology and research methods

The research methods of regional social and economic groups can be divided into three large groups: sociological, geo-information technology systems and economic and mathematical ones. The sociological methods are traditionally widely used in analysis of the employment rate in regions, the creation of regional consumer markets, the development of social infrastructure in certain regions.

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<sup>1</sup> The official website of Expert RA. URL: [https://raexpert.ru/rankingtable/region\\_climat/2017/tab3/](https://raexpert.ru/rankingtable/region_climat/2017/tab3/) (date of access 02.11.2018).

<sup>2</sup> Russian regions Social and economic indicators (2017) Statistics collection / Rosstat. Moscow, 2017. 402 p.

Methods and geo-information technology systems in terms of regional economy challenges were developed when the first IT systems were designed. Their main purpose is to provide informational and analytical support for the region and decision-making on various levels of regional competences.

Despite the importance of sociological and geo-information technology methods, the economic and mathematical methods are the most popular and are aimed mainly at the formalization of economic processes in the form of mathematical models. A specific type of a math function is determined by the unique features of an economic phenomenon. The closest methods that allow to solve the established problems are the regression analysis, simultaneous equation systems and VAR multidimensional autoregression.

The regression analysis in the research of regional economies is limited to the regression equation structure with a dependent variable in the left part and a lineal combination of independent variables in the right [12, 13]. However, if we simulate the functions of a number of explained indicators with a set of other economic variables, we face the problem of simultaneity and identifiability of an equation system [14]. These problems are addressed in sufficient detail in the publications [15; 16; 17; 18].

In order to research autoregressive functions for a certain set of economic variables, a dynamic simulation for several time lines can be used with current values of several variables expressed through their previous values. This model is called a vector autoregression (VAR, Vector AutoRegression). This model was first used in practice in [19]. However, VAR-models do not have correlations with the current values of the variables. This is why the development of simultaneous autoregressive equations is relevant.

### **3. Research methodology**

Any economic system can be described in a certain way by a set of indicators that are naturally connected to each other. So that the analysis were as accurate as possible, it is preferable to include all the important by definition variables, and the analysis of the statistic qualities of the model will allow to exclude less relevant parameters. Nevertheless, such an approach requires time-series that are at least by 1.5 times longer than the number of indicators themselves. Due to obvious reasons, the Federal Service of State Statistics of the Russian Federation only provides annual aggregate data on Sevastopol for four years, namely 2014, 2015, 2016 and 2017. This is why adequate models cannot have more than three indicators.

Gross regional product, which is the main indicator for regional development, investments in capital stock as the main financial prerequisite and necessary condition for the economic development, and wages, an indicator that demonstrates the financial state of the population, are most relevant regional development indicators. These indicators may as well have objective inter-correlations, which results in multicollinearity effect, and therefore, the usual regression analysis to identify lineal functions can produce false results. This is why it is necessary to select a method that is insensitive to the multicollinearity effect and allows to produce accurate assessments of model parameters with small samples.

Such methods are also represented by auto-regression models that show the dependence of the current indicators from the previous ones in past moments or periods of time. Let us develop an auto-regression model for GRP. This is why let us assume that the level of achieved GRP this year is determined by the last year GRP. However, the GRP growth is impossible without real investments into the capital stock. At the same time, the investments influence on the regional product is time-delayed. Let us consider that this time lag is one year.

Therefore, the proposed model appears as follows:

$$E_t = \alpha_0 + \alpha_1 I_{t-1} + \alpha_2 E_{t-1} + \varepsilon_t; \quad t = 1, 2, \dots, N; \quad (1)$$

where  $E_t$  - GRP in the year  $t$ ;

$\alpha_0$  - a coefficient, a model parameter, that corresponds to GRP formed variables, which were excluded from the model,

$\alpha_1$  and  $\alpha_2$  – proportionality factor;

$I_{t-1}$  - investments into the capital stock a year before  $t-1$ ;

$E_{t-1}$  - previous year GRP;

$\varepsilon_t$  - random deviation or an disparity within the equation;

$N$  - the length of a time-series (a real number of observation period equaled  $N + 1$ );

$I_0$  and  $E_0$  is the investments and GRP in the period that was chosen as the starting one.

As such an indicator as GRP includes both parts of the equation (1), but it corresponds to different time periods, then model (1) is an autoregression equation. However, another lag variable  $I_{t-1}$  is also included into the right part (1), this is why expression (1) is an autoregression vector equation.

There are no general approaches to assess vector autoregression models, this is why it is necessary to identify a model parameter evaluation method for every specific model.

To select the evaluation method, we will make obvious assumptions:

1. Mathematical expectation of a random deviations equals zero

$$E\{\varepsilon_t\} = 0; \forall t;$$

where  $E\{\square\}$  - Mathematical expectation complement operator for a random value.

2. Dispersions of random deviations for various time periods are similar

$$E\{\varepsilon_t^2\} = \sigma^2, \forall t.$$

3. Random deviations for various time periods are independent

$$E\{\varepsilon_t \cdot \varepsilon_k\} = 0, \forall t, k (t \neq k).$$

4. Random deviations and investments are independent for any time periods

$$E\{I_{t-k} \cdot \varepsilon_t\} = 0, \forall t, k.$$

5. Random deviations and GRP are independent for various time periods

$$E\{E_{t-k} \cdot \varepsilon_t\} = 0, \forall t, k (k \neq 0).$$

Let us find the mathematical expectation for the product of GRP and a random deviation in model (1) for all real time periods  $t$  :

$$\begin{aligned} E\{E_t \cdot \varepsilon_t\} &= E\{(\alpha_0 + \alpha_1 I_{t-1} + \alpha_2 E_{t-1} + \varepsilon_t) \cdot \varepsilon_t\} = \\ &= E\{\alpha_0 \cdot \varepsilon_t + \alpha_1 I_{t-1} \cdot \varepsilon_t + \alpha_2 E_{t-1} \cdot \varepsilon_t + \varepsilon_t \cdot \varepsilon_t\} = \\ &= E\{\alpha_0 \cdot \varepsilon_t\} + E\{\alpha_1 I_{t-1} \cdot \varepsilon_t\} + E\{\alpha_2 E_{t-1} \cdot \varepsilon_t\} + E\{\varepsilon_t \cdot \varepsilon_t\} = \\ &= \alpha_0 E\{\varepsilon_t\} + \alpha_1 E\{I_{t-1} \cdot \varepsilon_t\} + \alpha_2 E\{E_{t-1} \cdot \varepsilon_t\} + E\{\varepsilon_t \cdot \varepsilon_t\} = \\ &= \alpha_0 0 + \alpha_1 0 + \alpha_2 0 + \sigma^2 = \sigma^2. \end{aligned}$$

Thus, the mathematical expectation for the product of GRP and a random deviation equals the random deviation dispersion, and this confirms that in order to assess the model parameters, in particular, coefficients, the least square method  $\alpha_0, \alpha_1, \alpha_2$  can be used. This is why we solve an extremum problem:

$$\inf_{\alpha_0, \alpha_1, \alpha_2} \left\{ \sum_{t=1}^N (E_t - \alpha_0 - \alpha_1 I_{t-1} - \alpha_2 E_{t-1})^2 \right\}. \tag{2}$$

Let us find the assessments of unknown parameters. It is best done in matrix mode. This is why we should write (1) this expression in a matrix form:

$$\mathbf{E} = \mathbf{X}\boldsymbol{\alpha} + \boldsymbol{\varepsilon}, \tag{3}$$

where the designations for the following matrices are used:

$$\mathbf{E} = \begin{pmatrix} E_1 \\ E_2 \\ \dots \\ E_N \end{pmatrix} - \text{GRP value matrix for basic time periods;}$$

$$\mathbf{X} = \begin{pmatrix} 1 & I_0 & E_0 \\ 1 & I_1 & E_1 \\ \dots & \dots & \dots \\ 1 & I_{N-1} & E_{N-1} \end{pmatrix} - \text{lag variable matrix (first column with 1s is necessary to calculate the}$$

intercept term);

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{pmatrix} - \text{model coefficient matrix;}$$

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dots \\ \varepsilon_N \end{pmatrix} - \text{random deviation matrix.}$$

Then the sum of squared deviations in the right, theoretical part of the equation (3) from the left part, the real-life one, is the actual GRP and equals

$$ss = (\mathbf{E} - \mathbf{X}\boldsymbol{\alpha})^T (\mathbf{E} - \mathbf{X}\boldsymbol{\alpha});$$

and the extremum problem (2) appears as follows:

$$\inf_{\boldsymbol{\alpha}} \{ss\}. \tag{4}$$

In order to solve problem (4), it is necessary to find the derivative of the sum of squared deviations by the coefficient matrix. Using the rules of matrix differentiation, we get:

$$\frac{\partial ss}{\partial \boldsymbol{\alpha}} = -2\mathbf{X}^T (\mathbf{E} - \mathbf{X}\boldsymbol{\alpha}). \tag{5}$$

We calculate the assessment by making the found derivative (5) equal to zero and solving the equation against the model coefficient matrix:

$$\hat{\boldsymbol{\alpha}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{E}. \tag{6}$$

The assessment (6) corresponds to the widely known type of assessment that is produced with the help of the least square method. However, the main assumption of the classic regression model is that the explanatory variables that constitute a matrix  $\mathbf{X}$  are independent, and only in this case the assessment of the type (6) are correct. In our case, the variables in the right part of the equation are lag ones, in other words, they are predetermined relative to the current variables in the left part, and, therefore, they are not random, as what happened last year is a fact that cannot be changed. This is why, the type (6) assessment is also correct for the autoregression vector model.

Further we will also consider two more autoregressive functions for investments and average wages.

**4. Models and results**

Let us analyse the capital stock investment practice in Sevastopol, the formation of GRP and the mechanism of wages generation. We will do that on the basis of statistics of the Federal Service of State Statistics of the Russian Federation that is shown in Table 1.

**Table 1.** Social and economic indicators for Sevastopol in 2014-2017, million rubles.

Exponent (million rubles)	Year			
	2014	2015	2016	2017
GRP	30 148,6	48 663,3	64 163,2	85 127,3
Investments into stock capital	3 376,6	6 557,8	12 087,0	21 538,2
Annual average wages	0.0190	0.0243	0.0269	0.0274

Source: website of the Federal Service of State Statistics of the Russian Federation

**5. GRP equation**

We choose 2014 as the initial time period for the vector autoregression model. Then the initial data matrix for the model (3) appears as follows

$$\mathbf{E} = \begin{pmatrix} 48663,3 \\ 64163,2 \\ 85127,3 \end{pmatrix}; \tag{7}$$

$$\mathbf{X} = \begin{pmatrix} 1 & 3376,6 & 30148,6 \\ 1 & 6557,8 & 48663,3 \\ 1 & 12087,0 & 64163,2 \end{pmatrix}.$$

The expression (6) produces the following result when used to calculate the coefficient assessment:

$$\hat{\mathbf{a}} = \begin{pmatrix} 28450,7 \\ 2,9 \\ 0,4 \end{pmatrix}. \tag{8}$$

Thus, the vector autoregression equation appears as follows

$$E_t = 28450,7 + 2,9I_{t-1} + 0,4E_{t-1} + \varepsilon_t. \tag{9}$$

The faithfulness of the model is established based on the disparity between the right and the left parts of the equation (9) for all periods of time, or based on the other values:

$$\hat{\mathbf{e}} = \mathbf{E} - \mathbf{X}\hat{\mathbf{a}}.$$

However, when we calculate the product  $\mathbf{X}\hat{\mathbf{a}}$ , we find that it equals

$$\mathbf{X}\hat{\mathbf{a}} = \begin{pmatrix} 48663,3 \\ 64163,2 \\ 85127,3 \end{pmatrix},$$

and to the second digit after a decimal point reproduces the matrix  $\mathbf{E}$  elements in expression (7), in other words there is no disparity in the vector autoregressive equations, and the model (9) is absolutely faithful. This can be easily explained, as in the matrix expression (3) there are three equations and three unknowns, e.g. it is a system of lineal equations, and if the range of the matrix  $\mathbf{X}$  equals three, the system is joint and has a single solution. And as the determinant from the matrix elements does  $\mathbf{X}$  not equal zero:

$$|\mathbf{X}| = \begin{vmatrix} 1 & 3376,6 & 30148,6 \\ 1 & 6557,8 & 48663,3 \\ 1 & 12087,0 & 64163,2 \end{vmatrix} = -53063197,4;$$

then solution (8) is the single one, and model (9) describes faithfully and unambiguously the function for GRP in Sevastopol that was identified on the basis of the initial data.

Model (9) demonstrates the following: there is a constant element in the GRP structure, 28450,7 million rubles that is reproduced by the factors that are not in the model every year. The current year GRP will account for 40% of the previous year GRP ( $\hat{\alpha}_3 = 0,40$ ). So, one ruble of investments in a year increases GRP by 2 rubles and 90 kopecks. In other words, the investment efficiency in Sevastopol is quite high.

**6. Equations for investments into capital stock**

The investments into capital stock in Sevastopol today are mainly determined by individual development programmes, this is why their indicators in the current year will correlate to the last year investments. We would also assume that the investments influence the last year GRP, as the success of region stimulates investors, and on vice versa, a depressive region discourages potential investors. Therefore, we propose a vector autoregression model:

$$I_t = \beta_0 + \beta_1 I_{t-1} + \beta_2 E_{t-1} + \xi_t; \quad t = 1, 2, \dots, N; \tag{10}$$

where  $I_t$  - investments into current year stock capital  $t$  ;

$\beta_0$  - intercept term, model parameter that corresponds to the constant investment element, that is formed by factor that have not been included into the model;

$\beta_1$  and  $\beta_2$  – proportionality factor;

$\xi_t$  - equation random deviation;

other denominations are the same as for equation (1).

We will assume that for model (10) the same hypothesis is true on random deviations as  $\xi_t$  for disparities of  $\varepsilon_t$  equation (1). The matrix form of equation (10) appears as follows:

$$\mathbf{I} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\xi}, \tag{11}$$

where  $\mathbf{I} = \begin{pmatrix} 6557,8 \\ 12087,0 \\ 12538,2 \end{pmatrix}$  - investment matrix  $\mathbf{I}$  ;

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$$
 - model coefficient matrix;

$$\boldsymbol{\xi} = \begin{pmatrix} \xi_0 \\ \xi_1 \\ \xi_2 \end{pmatrix}$$
 - random deviation matrix.

Then the assessment of unknown model coefficients will appears in a way similar to the unknown coefficient assessment in the first equation, in particular (6)

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{I}. \tag{12}$$

The matrix  $\mathbf{X}$  is the same as in equation (3).

Then the expression (12) produces the following result:

$$\hat{\boldsymbol{\beta}} = \begin{pmatrix} 588,9 \\ 1,7 \\ 0,01 \end{pmatrix}. \tag{13}$$

The vector autoregression equation for investments into capital stock appears as follows:

$$I_t = 588,9 + 1,7I_{t-1} + 0,01E_{t-1} + \zeta_t; \tag{14}$$

Equation (14) is similar to equation (9) , and absolutely faithful, as the assessment (13) is the only solution to (11).

According to model (14) we can say that current year investments have the constant element, which equals 588,9 million rubles. This sum will increase by 1% of GRP in the previous year. The current year investments are different also because they have grown by 1.7 in comparison to the last year investments.

**7. Annual average wages equation**

Wages level is really identified by many different factors. First of all, it is determined by the value of labour resources in labour market. The important part is the part that consists of state expenditures on wages for employees in the budget sphere. The official statistics on wages would have also been different, if it had included income of self-employed citizens who in 98% do not pay taxes.

The described aspects are difficult to include in the model, in particular, when there are no statistics, or the data does not exist at all. However, generally we would assume is that the annual average wages in the analysed year are determined by its level in the previous year, and also by the level of current GRP. So, the annual average wages can be described by a vector autoregression equation:

$$W_t = \gamma_0 + \gamma_1 W_{t-1} + \gamma_2 E_t + \zeta_t; \tag{15}$$

where  $W_t$  - annual average wages in the current year  $t$  ;

$\gamma_0$  - intercept term, model parameter that corresponds to the constant wages element, that is formed by factor that have not been included into the model;

$\gamma_1$  and  $\gamma_2$  – proportionality factor;

$\zeta_t$  - model random deviation.

For annual average wages, the assumptions formulated for equation (1) are not true, as GRP is included into the right part (15) and correlate with the random deviation of its own equation (1)  $E_t$  , and then the indicator  $W_t$  is interconnected with two random deviations. This is why the method of least squares cannot be used to calculate model parameter assessment (15).

However, models (1), (10) and (15) can be seen as a system of simultaneous equations, for which there is a two-step method of least squares to calculate equation parameter assessments. In accordance with this method, coefficients for every equation are assessed separately. The parameters for equations (1) and (10) have already been found. A two-step procedure to calculate assessment is necessary for equation (15). The first step is to change the endogenous variable  $E_t$  with the values calculated with the least squares method on the basis of regression  $E_t$  , independent, predetermined, exogenous variables. In our case, this is the equation (1), which as we have said earlier, reproduces GRP, or variable  $E_t$  to the second digit after a decimal point. Then, in order to calculate the assessments of the unknown coefficients, we use again is a method of least squares.

Let us write down equation (15) in the form of a matrix:

$$\mathbf{W} = \mathbf{X}_w \boldsymbol{\gamma} + \boldsymbol{\zeta}; \tag{16}$$

where  $\mathbf{I} = \begin{pmatrix} 0,0234 \\ 0,0269 \\ 0,0274 \end{pmatrix}$  - annual average wages matrix;

$\mathbf{X}_w$  - variable matrix in the right part of equation (15):



$$\mathbf{X}_w = \begin{pmatrix} 1 & 0,0190 & 4866,3 \\ 1 & 0,0243 & 64163,2 \\ 1 & 0,0269 & 85127,3 \end{pmatrix};$$

$$\boldsymbol{\gamma} = \begin{pmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{pmatrix} \text{ - Coefficient matrix:}$$

$\boldsymbol{\zeta}$  - Random deviation matrix.

And then the coefficient assessment will be calculated using the following expression:

$$\hat{\boldsymbol{\gamma}} = (\mathbf{X}_w^T \mathbf{X}_w)^{-1} \mathbf{X}_w^T \mathbf{I}; \tag{17}$$

and appears as follows:

$$\hat{\boldsymbol{\gamma}} = \begin{pmatrix} 0,0146 \\ 0,6603 \\ -0,0000001 \end{pmatrix}.$$

An equation for annual average wages with assessed coefficients:

$$W_t = 0,0146 + 0,6603W_{t-1} - 0,0000001E_t + \zeta_t. \tag{18}$$

The equation shows that annual average wages have a constant element that amounts to 14 600 rubles. 66.03% of annual average wages of the previous year is added to the constant element every year. Judging by the coefficient assessment, GRP influences the wages inconsiderably, as 1 million rubbles in  $E_t$  GRP reduces annual average wages by 10 kopeiks. So, the wages development processes in Sevastopol are positively effected by gross regional products.

### 8. System of simultaneous autoregressive equations

As equations (1), (10) and (15) are analysed for the same time periods and explain three indicators that can be considered endogenous variables, these equations constitute the simultaneous equations system. There were three endogenous variables in the system:  $E$  gross regional product,  $I$  investments into stock capital and  $W$  annual average wages. The vector (a row matrix) of endogenous variables should be described in the following way:

$$\mathbf{y}_t = (E_t \quad I_t \quad W_t).$$

There are exogenous elements in simultaneous autoregressive equations, a certain constant and some pre-determined variables:  $I_{t-1}$ ,  $E_{t-1}$  and  $W_{t-1}$ . The vector of exogenous variables appears as follows:

$$\mathbf{x}_t = (1 \quad I_{t-1} \quad E_{t-1} \quad W_{t-1}).$$

Then the structural form of the equation system is determined by the following matrix equation:

$$\mathbf{y}_t \mathbf{B} + \mathbf{x}_t \boldsymbol{\Gamma} = \mathbf{u}_t; \tag{19}$$

where  $\mathbf{B} = \begin{pmatrix} 1 & 0 & -\gamma_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  - Coefficient matrix with endogenous variables;

$$\Gamma = \begin{pmatrix} -\alpha_0 & -\beta_0 & -\gamma_0 \\ -\alpha_1 & -\beta_1 & 0 \\ -\alpha_2 & -\beta_2 & 0 \\ 0 & 0 & -\gamma_1 \end{pmatrix} \text{ - Coefficients matrix was exogenous variables;}$$

$\mathbf{u}_t = (\varepsilon_t \quad \xi_t \quad \zeta_t)$  - Random deviation vector.

The structural form of simultaneous equations can be written in the so-called reduced form, when endogenous variables are described by exogenous ones:

$$\mathbf{y}_t = \mathbf{x}_t \mathbf{\Pi} + \mathbf{v}_t; \tag{20}$$

where  $\mathbf{\Pi} = -\mathbf{\Gamma} \mathbf{B}^{-1}$  - reduced form coefficient matrix;

$\mathbf{v}_t = \mathbf{u}_t \mathbf{B}^{-1}$  - reduced random deviation vector.

If we perform all matrix operations for calculating the matrix  $\mathbf{\Pi}$ , we will receive the following result:

$$\mathbf{\Pi} = \begin{pmatrix} 28450,7 & 588,9 & 0,01173 \\ 2,9 & 1,7 & -0,0000003 \\ 0,4 & 0,01 & -0,00000004 \\ 0 & 0 & 0,66031 \end{pmatrix}.$$

Thus the presented system of simultaneous autoregressive equations appears as follows:

$$\begin{cases} E_t = 28450,7 + 2,9I_{t-1} + 0,4E_{t-1} + v_{Et}; \\ I_t = 588,9 + 1,7I_{t-1} + 0,01E_{t-1} + v_{It}; \\ W_t = 0,01173 - 0,0000003I_{t-1} - 0,00000004E_{t-1} + 0,66031W_{t-1} + v_{Wt}. \end{cases} \tag{21}$$

The first two equations in system (21) are different respectively from (1) and (10) only by random deviations. Annual average wages in the previous period do not influence the gross regional product and investments. There are certain differences from the model (18) for the third endogenous variable in system (21). First, the constant element of the annual average wages reduced from 14,600 rubles to 11,730 rubles. Percentage from wages of the previous years that goes to the next one, remained the same as in (18). Also, the third equation under system (21) shows that neither investments, nor gross regional product of the previous year do not positively influence the annual average wages.

### 9. Conclusion

On the basis of the presented calculations and developed models, the results are received that show the trends in the creation and inter-dependency of such social and economic development indicators of a town, as GRP, investment into capital stock, average monthly wages in Sevastopol.

Every equation has an intercept term that considerably affects the studied indicator and shows hidden processes in the economy of the town.

In other words, GRP is in quantitative terms by 28450,7 developed on the basis of factors, that have not been included in the model. GRP for every next year will account for 40% of the previous year GRP. GRP is positively impacted by investments into stock capital (one ruble of investments generates every year the GRP growth of 2 rubles and 90 kopeiks, this is why we can say that the efficiency of additional financial resources attracted to the economy of Sevastopol is quite high. At the same time, GRP is not an indicator that positively affects wages, which can be considered a negative phenomenon.

The current year investments increase considerably by 1.7 times in comparison the last year investments. The sum of investments is positively dependent on the last year GRP (1% growth), which shows the attractiveness of the region for investors has improved. The constant in the current year investment equation equals 588,9 million rubles.

As for the annual average wages, we should say that to the constant element of 11,730 rubles is annually supplemented by 66.03% of the annual average wages of the previous year, and the GRP exerts inconsiderable but negative Influence (1 million rubles of the real GOP reduces the annual average wages by 10 kopeiks).

Further research will be aimed at: 1) establishment of interconnection and mutual influence of other social and economic development indicators in the analysed region; 2) Forecast of the analysed indicators; 3) Establishment of factors that constituted intercept terms in the developed models, but exerts considerable influence on the analysed indicators. At the same time we should understand that these factors are hidden, and there is a possibility that they can not be taken into account in principle, for example the influence of the shadow economy on the open economy, including the so called black salary, proceeds of crime, counterfeit products sales, etc.

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