Research of Investment Processes with a Limit Capacity

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Abstract. Currently, there are many approaches to determining the function of accumulation of the investment project participant. Some do not correspond to the axioms formulated earlier in financial mathematics, others correspond to them, but have a number of shortcomings .This work is devoted to the study of the mathematical model of the investment process, the rate of accumulation in which over time tends to zero, that is, the process with the limit capacity. We analyze the case with multiple investors. The graphs of the dynamics of the accumulated values for the case of two investors are presented. The influence of the accumulations of the parties involved on each other is shown. The case of optimal distribution of capital between several investment projects is also investigated. The problem is reduced to the problem of mathematical programming (search for the conditional extremum of the function of several variables). The optimal capital distribution is found by the method of uncertain Lagrange multipliers. The graph of dynamics of the maximum value of the accumulation function for a set of investment projects is constructed.

1. Introduction

The active development of investment activity in recent years has stimulated the expansion of the class of mathematical methods used for modeling investment projects. Dynamic models of investments, described by differential equations and their systems, are currently devoted to a lot of work [1-3]. The approach using the compound interest scheme does not always adequately describe reality. Continuous function of accumulation of deterministic investment process with participation of one investor, which is the solution of Cauchy problem

$$\dot{y} = \delta y, \; y(t_0) = c , \tag{1}$$

where δ - the intensity of savings, c - the initial capital, also has a significant drawback-an unlimited increase in the accumulated value. Therefore, for modeling the investment process requires the use of another function, the rate of savings which over time would be reduced, and the accumulation function sought to a certain limit value.

Based on the General financial and economic principles, Basharin G P [4], Bocharov P P and Kasimov Yu F [5] formulated the axioms of normalization, transitivity, uniformity in initial capital, stationarity [4], which should correspond to the accumulation function, which made it possible to determine the whole class of such functions. The axiomatic approach to the definition of financial parameters is used in the theory of utility, the theory of efficiency. Currently, there are a large number of approaches to the description of the accumulation function [6,7]. For many of them, the above axioms are not fulfilled. On the other hand, classical models that meet the axiomatic approach also have many disadvantages. In reality, the behavior of investors in one project has a direct impact on the savings of

other participants in this project. The above disadvantages are excluded in the case of the model with the limit capacity, which will be studied in this paper.

2. Investment project of maximum capacity

2.1. One-dimensional investment process with the limit capacity

The study of the investment project with the limit capacity Δ is the solution of an ordinary differential equation

$$\dot{y} = \delta y \left(1 - \frac{y}{\Delta} \right), \; y(t_0) = c. \tag{2}$$

The solution to this equation is a function

$$y(c,t) = \frac{\Delta c}{c + (\Delta - c)e^{-\delta(t-t_0)}}. \tag{3}$$

Figure 1 shows a graph of the function (1) in the region $D = \{(c,t) | 0 \leq c \leq 1, 0 \leq t \leq 50\}$ at $\Delta = 10$, $\delta = 0.3$. At small values t the accumulated amount is determined by the intensity of interest accrual, with increasing t the accumulated amount tends to the limit value Δ.
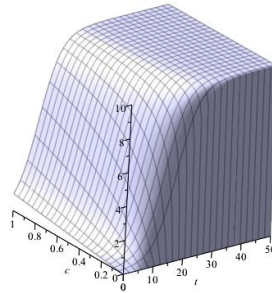

Figure 1. Deterministic function of the accumulation.

2.2. N-dimensional investment process with the limit capacity

As part of an investment project with the limit capacity, it is possible to consider a process involving n investors (the so–called n-dimensional project) carrying out continuous investments with intensity ρ_i, $i = \overline{1, n}$. Parameters ρ_i can be constant values, and can be functions of time. The time investment is $t = 0$.

In General, the investment process will be determined by a system of ordinary differential equations

$$\dot{\mathbf{y}} = \delta y_i \left(1 - \frac{\sum\limits_{k=1}^{n} y_k}{\Delta} \right) + \boldsymbol{\rho}, \; \mathbf{y}(0) = \mathbf{y}^0, \tag{4}$$

where $\mathbf{y}$ is the vector of accumulated values of investors, $\boldsymbol{\rho}$ is the vector of intensity of continuous investments, $\mathbf{y}^0$ is the vector of initial values of capital. In the case where the vector $\boldsymbol{\rho}$ components are constant, the system (4) satisfies the existence and uniqueness theorem of the solution and is integrable. For example, for the case, when $i = 1,2$, the analytical solution of the problem (4) has the form

$$y_i = \frac{\rho_i}{ch(at-b)}\left(\frac{1}{2a-\delta}e^{at-b} - \frac{1}{2a+\delta}e^{-at+b} + c_i e^{\frac{\delta}{2}t}\right),$$

$$a = \sqrt{\frac{\delta^2}{4} + \frac{\delta(\rho_1+\rho_2)}{\Delta}}, \quad b = arcth\left[\frac{\delta}{2\Delta a}(\Delta - 2(y_1^0 + y_2^0))\right], \tag{5}$$

$$c_i = y_i^0 \cdot ch(b) - \rho_i\left(\frac{1}{2a-\delta}e^{-b} - \frac{1}{2a+\delta}e^b\right).$$

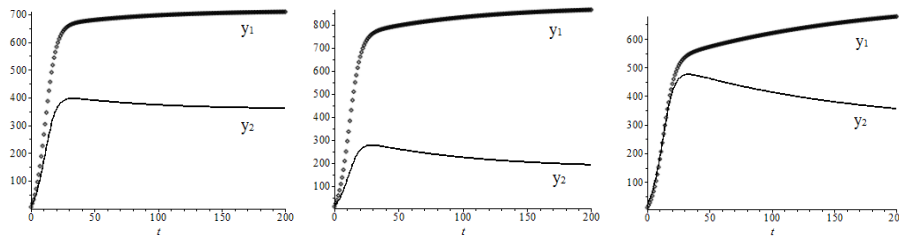Figures 2 (a, b, c) show graphs with different values of economic parameters.

Figure 2. Graphs of accumulation functions at.

a) $y_1^0 = 20, y_2^0 = 10, \rho_1 = 10,$ *b)* $y_1^0 = 10, y_2^0 = 15, \rho_1 = 10,$ *c)* $y_1^0 = 10, y_2^0 = 25, \rho_1 = 5,$

    $\rho_2 = 5, \Delta = 1000, \delta = 0.2$ $\rho_2 = 2, \Delta = 1000, \delta = 0.2$ $\rho_2 = 2, \Delta = 1000, \delta = 0.2$

It is seen that the investor with the predominant savings affects the accumulation of the investor with a smaller accumulated amount. Moreover, the accumulation function of the second investor has an extremum, after which the amount of savings begins to asymptotically decrease. It is advisable to withdraw money from the investment process at the maximum point, that is, at the time, which is the solution of the equation $y_2'(t) = 0$. Due to the bulkiness, we will not give a General formula in the article.

3. The problem of optimal capital allocation

The problem of an investment project with the participation of several investors was discussed above. Now consider the problem of participation of one investor in several investment projects. In this case, there is a problem of the most optimal distribution of the invested capital among the available investment projects, taking into account their intensity of accumulation and limit capacity.

Suppose that at time $t=0$ the initial capital Y_0 is invested in n investment projects with known accumulation intensities δ_i, limit capacities Δ_i in fractions equal x_i , $i=1,2,...,n$. The accumulation functions will be solutions of differential equations (2) and have the form similar to (3). It is necessary to determine the optimal allocation of capital for projects so that the total accumulated value for the set of projects takes the greatest value.

Mathematically, the problem takes the form:

$$y(\mathbf{x},t) = \sum_{i=1}^{n} \frac{\Delta_i x_i Y_0}{x_i Y_0 + (\Delta_i - x_i Y_0)e^{-\delta_i t}} \to \max,$$

$$\sum_{i=1}^{n} x_i = 1. \tag{6}$$

where $\mathbf{x}=(x_1, x_2, ..., x_n)$. The problem allows negative values of fractions x_i. Economically, this means that a loan in the amount of $x_i Y_0$ is taken in the corresponding i-th investment project.

We have a problem on a conditional extremum for which the method of undefined Lagrange multipliers is used [8]. Make the Lagrange function

$$L(\mathbf{x},t) = \sum_{i=1}^{n} \frac{\Delta_i x_i Y_0}{x_i Y_0 + (\Delta_i - x_i Y_0)e^{-\delta_i t}} - \lambda \left(\sum_{i=1}^{n} x_i - 1 \right). \tag{7}$$

Solving a system of equations

$$\begin{cases} \dfrac{\partial L(\mathbf{x},t)}{\partial x_i} = 0, \\[2mm] \dfrac{\partial L(\mathbf{x},t)}{\partial \lambda} = 0, \end{cases} \tag{8}$$

get the solution in the form

$$x_i = \frac{\Delta_i e^{-0.5\delta_i t}}{Y_0\left(1 - e^{-\delta_i t}\right)} \cdot \left[\frac{Y_0 + \displaystyle\sum_{k=1}^{n} \frac{\Delta_k e^{-\delta_k t}}{1 - e^{-\delta_k t}}}{\displaystyle\sum_{k=1}^{n} \frac{\Delta_k e^{-0.5\delta_k t}}{1 - e^{-\delta_k t}}} - e^{-0.5\delta_i t} \right]. \tag{9}$$

Substituting (9) in the target function, we obtain the maximum value of the accumulated capital at the time t on the set of investment processes

$$y_{\max} = \sum_{i=1}^{n} \frac{\Delta_i}{\left(1 - e^{-\delta_i t}\right)} \left[1 - e^{-0.5\delta_i t} \frac{\displaystyle\sum_{k=1}^{n} \frac{\Delta_k e^{-0.5\delta_k t}}{1 - e^{-\delta_k t}}}{Y_0 + \displaystyle\sum_{k=1}^{n} \frac{\Delta_k e^{-\delta_k t}}{1 - e^{-\delta_k t}}} \right]. \tag{10}$$

Figure 3 shows the change in the accumulated value of the aggregate investment processes over time the value of the initial capital of $Y_0 = 30$, intensities $\delta_1 = 0.2$, $\delta_2 = 0.1$, $\delta_3 = 0.05$, and limit capacities $\Delta_1 = 1000$, $\Delta_2 = 500$, $\Delta_3 = 500$.
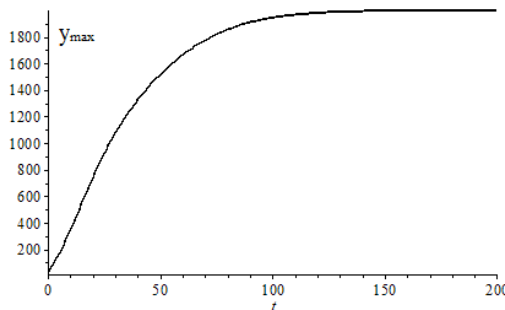

Figure 3. Dynamics of the maximum value of the accumulation function.

4. Conclusion

In the work were set and solved the problems of investment funds in the framework of investment projects with a limit capacity. Questions of participation of several investors in one project and participation of one investor in several projects are considered. The latter problem is reduced to the problem of mathematical programming and solved by the method of indefinite Lagrange multipliers. Graphs of the required economic parameters and functions are constructed. The influence of an investor with a larger accumulated amount on the savings of investors with a smaller amount is shown. The proposed mathematical models allow the investor to evaluate and control his position in the relevant investment process and, if possible, to influence it with the aim of obtaining maximum profit.

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