

# Solution of the Decentralized Task of Evaluating and Improving Product Quality

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**Abstract.** Management of products quality needs from a producer to decide following tasks: technologies of production, economical issues, management of purchases. All these tasks are aimed on the obtaining of a rational quality level of products. For this various methods and models for monitoring a quality of product are developed. The goal of this paper is to develop a method for assessing and improving product quality, based on a multi-level optimization. The problem of quality evaluating is considered, based on decentralization, quality objective functions, developed methodology for assessing and improving product quality and proposed ways to improve the developed methodology.

## 1. Introduction

One of the key factors for ensuring of competitiveness of instrumentation organizations is a rational decision-making in regard to the quality of products. The rationality means that before deployment of decision-making in regard to the quality we must analyze both qualitative and quantitative information, and define best of possible alternatives.

The decision-making task becomes difficult, when a lot of factors should be taken into consideration. The main factors include those, which effect on the effectiveness of the quality management system, and the example of these is described in [1].

While solving a complex problem, which referred with uncertainty and difficulty of formalizing of a decision-making system, methods based on the expert knowledge are used. The decision of a problem, in this case, could be defined as a set-theoretic model:

$$\langle Q, f_1(A), f_2(A), \dots, f_n(A), R, K_1, K_2, \dots, K_m, N \rangle,$$

where  $f_i(A)$  is the preference relationship function of one alternative over others;  $R$  is a binary relations on set;  $A$ ,  $K_j$  are comparisons criteria of alternatives;  $N$  is logics normalization of criteria;  $Q$  is the objective function for finding the numerical value of the solution on multi- criteria on the multicriteria set.

The task becomes more complicated, if several structural units are involved in the decision-making process. If methods from classical qualimetry are used (or methods described above), the time for decision-making increases, and objectivity of a product quality assessment is going down.

Among the most popular methods for solving decentralized decision-making tasks, methods from the theory of multi-optimization are known. These methods were applied for managing of activities of the railway transport hub [2], for optimal designing of load lifting mechanisms [3]. Other works, using the theory of multi-optimization, are [4], [5].

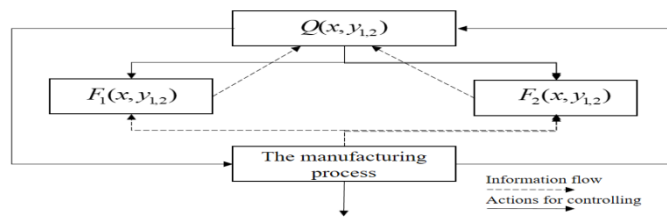
**2. Formulation of the problem of evaluating of the quality level of instrumentation products**

Objective functions for the decentralization task, based on requirements to a quality monitoring model, is defined in the following way:  $Q(x, y_{1,2})$  is the performance monitoring model,  $F_1(x, y_{1,2})$  is a function of quality costs,  $F_2(x, y_{1,2})$  is a function of purchasing management. The function of quality is such, that  $Q: X \times Y_1 \times Y_2 \rightarrow R$ , where  $y$  is the quality criteria,  $y_i \in Y_i \subset R^{m_i}$ , where  $F_i: X \times Y_i \rightarrow R$ .

In the feasible region, for solving the above problem, we need to find a minimum point:

$$S = \{(x, y_1, y_2) \in X \times Y_1 \times Y_2 : Ax + \sum_{i=1}^k B_i y_i \geq b, A_i x + C_i y \geq b_i, i = 1, \dots, k\}.$$

The structure of the task is shown in figure 1.



**Figure 1.** The structure of the task.

The task could be described by the following analytical model:

$$\begin{aligned}
 & \min_{x \in X} Q(x, y_1 = y, y_2 = z, y_3 = w) = cx - d_1 y + d_2 z \\
 & Ax + B_1 y_1 + B_2 y_2 \leq b_1 \\
 & \min_{y_i \in Y} F_1(x, y) = cx + d_1 y \\
 & Ax + B_1 y \leq b' \\
 & \min_{z_i \in Z} F_2(x, z) = cx + d_2 z \\
 & Ax + B_2 z \leq b''
 \end{aligned} \tag{2}$$

The top level is intended for formulation of additional criteria for bottom levels. These criteria need for managing of bottom levels. According with [6], “The leader goes first and chooses in an attempt to optimize (maximize or minimize) his own objective function  $F(x, y(x))$ , subject to additional constraints”.

The state of work for the top level is defined on following output properties:

- the level of yield of products:  $x_1$ ;
- the degree of effectiveness of the developed preventive activities:  $x_2(y_2 + z_2)$ ;
- the degree of effectiveness for new introduced technologies and techniques:  $x_3(y_3 + z_3)$ .

Variables  $x_2, x_3$  will be defined the using of three variables of the bottom level  $y, z$  (more detailed information about bottom variables is below).

The field of the definition of function  $Q(x)$  we'll set through the polyhedron  $X^5$ , with the functional blocks of the device, shown in figure 2.

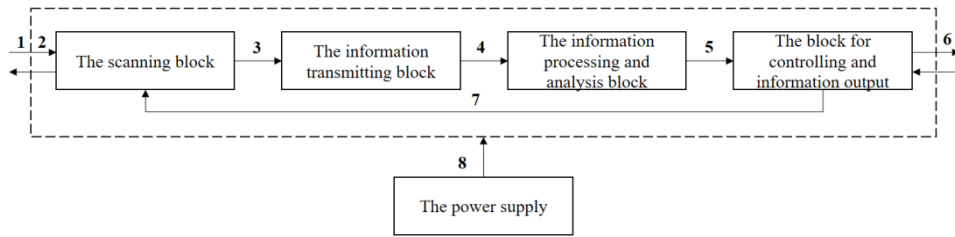


Figure 2. The device.

The feasible region of  $Q(x)$  is defined by the following matrix:

$$\begin{pmatrix} a_{11}x_1 & a_{12}x_2(y_2 + z_2) & a_{13}x_3(y_3 + z_3) \\ a_{21}x_1 & a_{22}x_2(y_2 + z_2) & a_{23}x_3(y_3 + z_3) \\ a_{31}x_1 & a_{32}x_2(y_2 + z_2) & a_{33}x_3(y_3 + z_3) \\ a_{41}x_1 & a_{42}x_2(y_2 + z_2) & a_{43}x_3(y_3 + z_3) \\ a_{51}x_1 & a_{52}x_2(y_2 + z_2) & a_{53}x_3(y_3 + z_3) \end{pmatrix}$$

Criteria  $c_1x_1$  are calculated through an indicator of defect per unit in the following way:

$$a_{i1}x_1 = 1 \times \left(1 - \frac{d}{m}\right), \tag{3}$$

where  $d$  is the number of founded defects,  $m$  is number of inspected units.

Criteria  $c_2x_2(y_2, z_2)$  are calculated as:

$$a_{i2}x_2(y_2, z_2) = x_2(y_2, z_2)^{1-c_2}, \tag{4}$$

where  $x_2 = \frac{P}{B}$  is the relation of the working corrective and preventive measures to the implemented

ones (in the area of purchasing management and cost management),  $c_2 = \frac{C(P)}{C(B)}$  is the relation of expenses of working activities to the implemented cost (in the area of purchasing management and cost management).

Criteria  $c_3x_3(y_3, z_3)$  are calculated in the same way, as the criteria  $c_2x_2(y_2, z_2)$ , where the numerator is the ratio of working technologies, and the denominator is the number of embedded.

### 3. Formalization objective functions for bottom levels

The most popular methods for classifying of quality costs can be found in [7]. For evaluating costs of quality  $F_1(y)$  on a manufacturing stage we introduce the following criteria:

- costs of quality assessment -  $y_1(x_1)$ ;
- costs of preventing of defects -  $y_2(x_1)$ ;
- costs for eliminating of internal defects -  $y_3$ .

In order to determine a feasible region for function  $F_1(y)$ . In articles [8], [9], a lot of causes are listed for classification costs of quality. Considering the assessment of the quality level at the manufacturing stage, we will determine the following causes:

- costs for a control of technology stage -  $b_1'$ ;
- costs for inspection of products -  $b_2'$ ;
- costs for analyzing of defects -  $b_3'$ ;



$$b''_{i_2} z_2 = \left( \frac{\sum_{i=1}^n z_2(t)_i}{n} \right) \cdot \frac{1}{z_2(t)_u} \cdot 1^{1-d_2}, \tag{6}$$

where  $z_2(t)_i$  is time points for delivering components,  $n$  is the number of reference points,  $z_2(t)_u$  is the time to deliver components under the contract,  $d_2$  is the ratio of the amount of the cost of components and the cost of applying them to their destination to the cost of the flaw detector.

Criterion  $d_3 z_3$  will found by the formula:

$$b''_{i_3} z_3 = z_3, \tag{7}$$

where  $z_3$  is the relation of closed questions to the total number of questions.

#### 4. Calculating the quality level of the product and defending ways for improving product quality

The optimization of a double-level model with two followers could be done with using of Kuhn-Tucker conditions and the simplex method. Let's set dual variables  $u_i \in R^q$ ,  $v_i \in R^m$  ( $i=1, \dots, k$ ), which are related to functions  $Q(x, y, z)$ ,  $F_1(y)$ ,  $F_2(z)$ . Constraints for systems of linear equations are determined in the following way. If partial values of the  $i$ -th equation are differed from 1, we need to put at the right-hand side the average value of these quantities with the sign  $\geq$ . If all of partial values of the  $i$ -th equation are equal to 1, then at the right side we put number 1 with the sign  $\leq$ . If the equation has one variable, then the constraint is put with the sign  $\leq$ .

The analytical model of optimization task (2) looks in following way:

$$\begin{aligned} \min Q(x, y, z) &= Cx + d_1 y + d_2 z \\ Ax + B'y + B''z &\leq b, Ax + C_i y_i \leq b_j \\ u_i B' - s_i &= -e_i \\ u_i (b_i - Ax - B'y) + s_i y &= 0 \\ v_i B'' - s_i &= -e_i \\ v_i (b_i - Ax - B''z) + s_i z &= 0 \\ (x, y, z) &\geq 0, u_i \geq 0, v_i \geq 0. \end{aligned} \tag{8}$$

The example of task is listed in the table 1.

**Table 1.** The example.

$Q$	$x_1$	$x_2(y_2)$	$z_2$	$y_3$	$z_3$	$b$
E1	0,8	0,3	0,1	0,8	0,8	$\geq 0,68$
E2	0,3	0,1	0,6	0,9	1	$\geq 0,63$
E3	0,7	0,9	1	0,7	1	$\geq 0,84$
E4	0,1	0,9	1	1	0,5	$\geq 0,68$
E5	1	1	1	1	1	$\leq 1,00$
$F_1$	$x_1$	$x_2(y_2)$	$z_2$	$y_3$	$z_3$	$b'$
E1	0,1		-		-	$\leq 0,1$
E2		0,5	-		-	$\leq 0,5$
E3		1	-		-	$\leq 1,0$
E4			-	1	-	$\leq 1,0$
E5			-	0,9	-	$\leq 0,9$
E6	0,9	0,7	-		-	$\geq 0,8$
$F_2$	$x_1$	$x_2(y_2)$	$z_2$	$y_3$	$z_3$	$b''$
E1	0,3	-	0,5	-	0,8	$\geq 0,53$
E2	1	-	1	-	1	$\leq 1$

Taking into account the example table 1, the task (8) will look like:

$$\begin{aligned}
 \min Q(x, y, z) &= 1 + x_1 - y_2 - z_2 - y_3 + z_3 \\
 0,8x_1 + 0,3y_2 + 0,1z_2 + 0,8y_3 + 0,8z_3 &\geq 0,68 \\
 0,3x_1 + 0,1y_2 + 0,6z_2 + 0,9y_3 + z_3 &\geq 0,63 \\
 0,7x_1 + 0,9y_2 + z_2 + 0,7y_3 + z_3 &\geq 0,84 \\
 0,1x_1 + 0,9y_2 + z_2 + y_3 + 0,5z_3 &\geq 0,68 \\
 x_1 + y_2 + z_2 + y_3 + z_3 &\leq 1, 0,1x_1 \leq 0,1 \\
 0,5x_2 &\leq 0,5, x_2 \leq 1, y_3 \leq 1 \\
 0,9y_3 &\leq 0,9, 0,9x_1 + 0,7x_2 &\geq 0,8 \\
 0,3x_1 + 0,5z_2 + 0,8z_3 &\geq 0,53 \\
 x_1 + z_2 + z_3 &\leq 1, u_1 + 0,9u_2 - u_3 = -1 \\
 u_1(1 - y_3) = 0, u_2(0,9 - 0,9y_3) = 0, u_3(y_3) = 0 \\
 -0,5v_1 + v_2 - v_3 = -1, -0,8v_1 + v_2 - v_4 = -1 \\
 v_1(0,3z_1 + 0,5z_2 + 0,8z_3 - 0,53) = 0 \\
 v_2(1 - z_1 - z_2 - 0z_3) = 0 \\
 v_3(z_1) = 0, v_4(z_2) = 0, (x, y, z) \geq 0, u_i \geq 0, v_i \geq 0.
 \end{aligned}$$

Optimization model with using of Kuhn-Tucker theorem will be as following:

$$\begin{aligned}
 \min Q(x, y, z) &= 1 + x_1 - y_2 - z_2 - y_3 + z_3 \\
 0,8x_1 + 0,3y_2 + 0,1z_2 + 0,8y_3 + 0,8z_3 &\geq 0,22 \\
 0,3x_1 + 0,1y_2 + 0,6z_2 + 0,9y_3 + z_3 &\geq 0,08 \\
 0,7x_1 + 0,9y_2 + z_2 + 0,7y_3 + z_3 &\geq 0,32 \\
 0,1x_1 + 0,9y_2 + z_2 + y_3 + 0,5z_3 &\geq 0,02 \\
 x_1 + y_2 + z_2 + y_3 + z_3 &\leq 1 \\
 0,1x_1 \leq 0,1, 0,5x_2 \leq 0,5, x_2 \leq 1, y_3 = 0 \\
 0,9x_1 + 0,7x_2 &\geq 0,8 \\
 0,3x_1 + 0,5z_2 + 0,8z_3 &\geq 0,1 \\
 x_1 + z_2 + z_3 &\leq 1, z_2 = 0, z_3 = 0 \\
 u_1 + 0,9u_2 - u_3 &= -1 \\
 u_1(1 - y_3) = 0, u_2(0,9 - 0,9y_3) = 0, u_3(y_3) = 0 \\
 -0,5v_1 + v_2 - v_3 = -1, -0,8v_1 + v_2 - v_4 = -1 \\
 v_1(0,3z_1 + 0,5z_2 + 0,8z_3 - 0,53) = 0 \\
 v_2(1 - z_1 - z_2 - 0z_3) = 0 \\
 v_3(z_1) = 0, v_4(z_2) = 0, (x, y, z) \geq 0, u_i \geq 0, v_i \geq 0.
 \end{aligned}$$

More detail information about bi-level programming one can see in [14]. Based on the results of implementation of simplex method, the quality level of flaw detector at the production stage was determined as  $Q(x, y, z) = 0,67$ . Values of variables are introduced in the Table 2.

**Table 2.** Values of variables.

N		1	2	3
1	x	0,33333333	0,7142857	0
2	y	0	0,66666667	0
3	z	0,00	0,00	0,00

Thus, for enhancement of the product quality it is needed:

- To replace suppliers, or to update procedures that regulate relations with suppliers;
- To reduce expenses for preventive measures (or update them), and update procedures for assessment of product quality in operational inspection.

## 5. Conclusions

Presented approaches for assessing of the level of quality possess advantages in comparing with standard methods of qualimetry. These advantages are achieved by including of a lot of factors into the model, which affect on the quality of products. The fact that the model has many factors makes the method more complicated for calculating the quality of product. For this it is recommended to use at intermediate stage with multidimensional cluster analysis methods or fuzzy logic methods.

Recommended to change the linear type of model optimization for nonlinear, in order to increase the feasible region. To assess the accuracy of decision-making, it is necessary to involve a probability-theoretic approach in the proposed methodology.

The developed methodology based on multi-level optimization allows to take into account many factors, and flexibly manage the quality of the product. Quality management is carried out by making changes to certain indicators of product quality, both from the side of economic losses and from the side of production technology.

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