*Advances in Economics, Business and Management Research, volume 128*
**International Scientific Conference "Far East Con" (ISCFEC 2020)**

# An Empirical Study on the Modeling of an Optimal Investment Portfolio Using Multivariate Model of Conditional Heteroscedasticity: Evidence from the Chinese Stock Exchanges

**A Shchankina[1], Zou Ping[2]**

[1]Master of Finance, School of Finance, Shanghai University of Finance and Economics, Shanghai, China
[2]Professor, School of Finance, Shanghai University of Finance and Economics, Shanghai, China

E-mail: shchankina.a@yandex.ru

**Abstract.** For any subject of economic relations, the main purpose of investments is to maximize income and minimize risks, as well as to save money from inflation. The best way to achieve this goal is a portfolio approach to investment. Nowadays, there are plenty of strategies and techniques for the construction and management of the investment portfolio, therefore, the problem of choosing the most effective investment policy is particularly relevant for any investor. Markowitz portfolio optimization model is very popular among investors around the world in recent decades, but also is criticized a lot by different scholars. To mitigate the problems of the Markowitz optimization model, we decided to apply multivariate time series forecast to an optimization process and examined, whether the use of multivariate time series models in a portfolio optimization process can improve portfolio performance and decrease overall volatility. Using a Markowitz optimization model and the BEKK GARCH model, portfolios for different levels of risk has been constructed for the Shanghai Stock Exchange and Hong Kong Stock Exchange. Backtesting results show, that in fact, portfolios constructed through a multivariate time series forecast decrease overall portfolio volatility. Also, we have found, that applied models outperformed the stock market indexes, which confirms the efficiency of the implementation of both models for portfolio construction techniques.

## 1. Introduction

### *1.1. Background of the study*

The following research is devoted to the construction of optimal investment portfolios using the classical Markowitz optimization model and the multidimensional model of conditional heteroscedasticity (GARCH).

Depending on goals and objectives the investor faces when constructing a particular portfolio, the portfolio manager must choose a certain percentage between different types of assets that make up the investor's portfolio. A portfolio constructed of different kinds of investments will, on average, yield higher returns and pose a lower risk than any individual investment found within the portfolio. For this reason, properly take into account the needs of the investor and create a portfolio of assets that combines reasonable risk and acceptable profitability, the main task of the portfolio manager.

Based on the literature review, we found, that today the Markowitz portfolio optimization model is the most popular model among investors. The model allows finding a combination of assets that provide a minimum risk for a required level of the expected return. And despite the fact that the model of Markowitz was proposed more than 60 years ago, it is still widely used by investors.

However, studies (King 1994; Koch 1991; Kaplanis 1988) have shown that mathematical expectations, variances, and covariances of asset yields, on which based the Markowitz optimization model, in some cases change over time and their selective evaluation does not take into account the possibility of time series autocorrelation of profitability, what causes estimation errors in a moment of return forecasting and makes the Markowitz model very sensitive to small changes in input data. So, given the challenges presented above, there was a need to model the expected returns, variances and covariances of returns with the purpose to predict their future value.

To solve problems, related to the Markowitz model, to decrease effects of time-varying risk, increase the efficiency of the portfolio and mitigate estimation errors we decided to use the generalized autoregressive conditional heteroscedasticity (GARCH) process, which widely used for modeling variances and covariances of time series of returns, and allows to make a fairly accurate prediction of these characteristics.

### *1.2. Theoretical and practical relevance of the topic*

The use of forecasts based on the GARCH models in the portfolio construction process has been previously studied in the work of Pojarliev and Polasek (2001). They found, that the volatility predictions gained through multivariate time series models can be successfully transformed into higher portfolio yields if the right combination of volatility modeling and portfolio strategy can be found. And despite the fact that the use of forecasts based on the GARCH models in portfolio construction process after Pojarliev and Polasek, also considered other scholars (Gupta and Donleavy, 2009; Miralles-Marcelo and Miralles-Quirós, 2013), all their researches based on index portfolios, which included only indexes of different countries, such as MSCI indexes, and none of them have not considered the stock portfolios, which are most widely managed by investors than index portfolios. So, the question about the influence of the forecasts, through multivariate GARCH models, on portfolios, consisting of the stocks, remains under-studied.

For this reason, the chosen topic represents high theoretical and practical value. One of the main purposes of the research is to establish whether the use of a multidimensional model of generalized autoregressive conditional heteroscedasticity (GARCH) can improve the accuracy of the portfolio volatility forecast and thereby reduce the risk of the optimal stock portfolio and increase its overall return.

### *1.3. Objectives of the study*

To achieve the main purpose of the research, we constructed two portfolios: the first portfolio based on a classical Markowitz optimization model, and second portfolio based on a data, predicted through a GARCH BEKK model. This allowed us to compare the performance of portfolios and decide whether the portfolio, based on predicted values through a GARCH BEKK model, performed better or not.

Also, we have checked our findings for two different stock exchanges. First, we ran two models for the stock portfolio constructed for the Shanghai Stock Exchange (SSE), then, we used the same models for a stock portfolio for the Hong Kong Stock Exchange (HKEX), which diversified by industry and country. Consequently, we got results for two different stock exchanges and compared, for which market our models work better: for a portfolio, consisted of the stocks of mainland Chinese companies, listed only on the SSE (single market portfolio), or for a portfolio, which contains stocks of the companies, diversified not only by industry, but also by country, which listed on the HKEX (diversified portfolio).

## 2. Theoretical Part

*2.1. Research methodology of the Markowitz portfolio optimization process*

In 1952 Harry Markowitz formulated the portfolio problem as a choice of the mean and variance of a portfolio of assets. He proved the fundamental theorem of mean-variance portfolio theory and quantified the difference between the risk of portfolio assets taken individually and the overall risk of the portfolio. Markowitz demonstrated that the portfolio risk came from the co-variances of the asset that made up the portfolio.

The Markowitz approach is often described as a mean-variance approach because it only takes mean return and variance of return into account to characterize the investor's portfolio. Further, we consider the mathematical basis of the classical Markowitz optimization model, used in an empirical part.

For the construction of an investment portfolio, the portfolio manager has to run an optimization model. In a classical Markowitz optimization model, there are two strategies among which investor has to make a choice:

1. Maximization of an investment portfolio return with limited risk;
2. Minimization of an investment portfolio risk at the minimum acceptable level of return.

Because the main focus of the research is volatility, in the empirical part of the paper we use the Markowitz model of minimum risk. The formula and restrictions, imposed on model, to find the optimal weights of financial instruments are presented below:

Mean-variance efficient portfolio of minimum risk:

$$\begin{cases} \sqrt{\sum_{i=1}^{n} w_i^2 \cdot \sigma_i^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} w_i \cdot w_j \cdot k_{ij} \cdot \sigma_i \cdot \sigma_j} \to min \\ \sum_{i=1}^{n} w_i \cdot r_i > r_p \\ \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0 \end{cases} \quad (1)$$

where,

$r_p$ – minimum required wealth;

$w_i$ – weight of asset i in the portfolio;

$\sigma_i$ – variance of asset i in the portfolio;

k – correlation between assets.

The formulation in Equation 1 gives the optimum portfolio (i.e. minimum risk) for the specified minimum required wealth, $r_p$. That is, no other portfolios can give a similar or higher return with a lower risk.

As we mentioned above, Harry Markowitz optimization model has some drawbacks, regarding variables estimation process, and in the next section, we consider the application of the GARCH process for variables forecasting in order to improve optimization model results.

*2.2. Multivariate time series forecasts for the portfolio optimization problem*

The GARCH model was developed independently by Bollerslev (1986) and Taylor (1986). Today, GARCH-type models primary used in forecasting volatility and conditional values of the assets.

Because univariate GARCH models model the conditional variance of each series entirely independently of all other series and don't allow to extend "volatility spillovers" between markets or assets, we use multivariate GARCH model which enable us to determine whether the volatility in one asset or market leads or lags the volatility in others.

The most popular multivariate GARCH models are the VECH and BEKK models (Engle and Kroner, 1995, p. 133). One disadvantage of VECH model is that this model does not guarantee that the matrix will be positive definite. However, taking into account that the components of the matrix are conditional variances and covariances, in our case such components cannot take negative values.

The BEKK model (Engle and Kroner, 1995, p. 133) solves such difficulty, ensuring that the H matrix is always positive definite. The positive definiteness of the covariance matrix is ensured owing to the quadratic nature of the terms on the equation's RHS. The BECK model also reduces the number of parameters for estimation, making it more convenient to use than the VECH model.

Parameters of the multivariate GARCH models can be estimated by maximizing the log-likelihood function (Brooks, 2014, p.470):

$$l\,(\theta) = -\frac{TN}{2}\,log2\pi - \frac{1}{2}\sum_{t=1}^{T}(log|H_t| + \epsilon`_t H_t^{-1}\epsilon_t) \quad (2)$$

where,

θ – all the unknown parameters to be estimated;

N – number of assets (number of series in the system);

T – number of observations;

$H_t$ - conditional covariance matrix;

$e_t$ – innovation vector.

Therefore, the second type of a portfolio in our research we develop based on the forecasted conditional variance matrix and forecasted vector of returns from a multivariate BEKK GARCH described above.

To estimate the parameters of the multidimensional GARCH model, as well as the sample numerical characteristics of the time series, we use the econometric package "EViews 10".

*2.3. Portfolio construction using Markowitz optimization model*

To prove an assumption, made in the first section of the paper, first, we constructed an investment portfolio for the Shanghai Stock Exchange and Hong Kong Stock Exchange based on the classical Markowitz optimization model.

Principles of stocks selection for the SSE portfolio are Liquidity; Industry performance; Diversification by industry; p/e (price/earnings) and p/b (price/book value) coefficients. Table 1 represents the stocks chose for optimization.

**Table 1.** List of the stocks for portfolio optimization for the SSE

| Ticker | Company | Industry |
|---|---|---|
| 600028 | China Petroleum & Chemical Corporation | Oil and Gas |
| 600030 | CITIC Securities Co Ltd | Financials |
| 600036 | China Merchants Bank Co Ltd | Banking |
| 600050 | China United Network Communications | Telecommunications |
| 600104 | SAIC Motor Co Ltd | Automotive |
| 600276 | Jiangsu Hengrui Medicine Co Ltd | Health Care |
| 600519 | Kweichow Moutai Co Ltd | Consumer Staples |
| 601088 | China Shenhua Energy Co Ltd | Energy |
| 601318 | Ping An Insurance Company of China Ltd | Insurance |
| 601668 | China State Construction Engineering Co | Construction |

For the in-sample estimation period, we use daily stock prices data for the period from 2013.01.04 until 2018.12.28 (1458 observations).

For the second portfolio construction experiment, we have chosen stocks from the Hong Kong Stock Exchange. In this case we were able to choose companies, operating in different countries and under different regulation, what made it possible, to construct more diversified portfolio with stocks, which are not exposed to similar market trends. Chosen stocks are presented in table 2.

**Table 2.** List of the stocks for portfolio optimization for the HKEX.

| Ticker | Company | Industry | Features |
|---|---|---|---|
| 0003.HK | Hong Kong & China Gas Co. | Public Utility | HK |
| 0005.HK | HSBC Holdings plc | Banking | UK |
| 0066.HK | MTR Corporation Ltd | Railroads | HK |
| 0388.HK | HKEx Ltd | Financial | HK |
| 0700.HK | Tencent Holdings Ltd | IT | H share |
| 0823.HK | Link Real Estate Investment | Property | HK |
| 0883.HK | China National Offshore Oil | Energy | Red Chip |
| 1128.HK | Wynn Macau, Limited | Resorts & | USA |
| 0019.HK | Swire Pacific | Conglomerate | UK |
| 2319.HK | China Mengniu Dairy Co. Ltd | Dairy | H share |

It can be seen, that our portfolio well diversified as it includes stocks of companies, registered in different countries, also, the portfolio contains stocks of companies operating in 10 different industries. All chosen stocks we can purchase on the Hong Kong Stock Exchange, so, all the stocks are listed in HK dollars.

For portfolio construction purposes, based on a Markowitz optimization model, in the empirical part, we use the following algorithm:

1. To obtain a number of historical data for an in-sample period;
2. To calculate log returns and covariance matrix of the chosen stocks;
3. To run an optimization model and find stock weights.

The structure of Markowitz optimal portfolio for the SSE shown in table 3.

**Table 3.** Markowitz optimal portfolios constructed for the SSE.

| Constraint | δ | Weights | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 600028 | 600030 | 600036 | 600050 | 600104 | 600276 | 600519 | 601088 | 601318 | 601668 |
| μp ≥ 0,02% | 1,62% | 34,99% | 9,00% | 7,16% | 8,17% | 8,07% | 3,59% | 4,73% | 11,13% | 6,24% | 6,91% |
| μp ≥ 0,03% | 1,57% | 22,40% | 9,49% | 8,55% | 9,06% | 9,02% | 7,08% | 7,31% | 10,58% | 8,08% | 8,42% |
| μp ≥ 0,04% | 1,55% | 9,31% | 10,03% | 10,08% | 10,05% | 10,05% | 10,19% | 10,14% | 9,97% | 10,10% | 10,08% |
| μp ≥ 0,05% | 1,55% | 0,00% | 6,96% | 11,48% | 10,88% | 10,94% | 14,03% | 12,92% | 9,12% | 12,03% | 11,63% |
| μp ≥ 0,06% | 1,51% | 0,00% | 0,00% | 10,97% | 10,58% | 10,74% | 19,35% | 16,27% | 5,67% | 13,77% | 12,67% |
| μp ≥ 0,07% | 1,51% | 0,00% | 0,00% | 8,25% | 9,20% | 9,49% | 25,21% | 19,58% | 0,23% | 15,03% | 13,01% |
| μp ≥ 0,08% | 1,55% | 0,00% | 0,00% | 0,00% | 2,32% | 8,23% | 33,70% | 24,59% | 0,00% | 17,22% | 13,94% |
| μp ≥ 0,09% | 1,60% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 44,17% | 26,49% | 0,00% | 16,82% | 12,52% |
| μp ≥ 0,10% | 1,73% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 61,81% | 22,94% | 0,00% | 10,41% | 4,84% |
| μp ≥ 0,11% | 1,95% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 80,68% | 17,95% | 0,00% | 1,37% | 0,00% |
| GMV | 1,430% | 19,60% | 0,00% | 22,05% | 4,14% | 7,15% | 22,07% | 20,82% | 2,75% | 0,00% | 1,41% |

As can be seen, for the Shanghai Stock Exchange we got 10 different portfolios with different requirements on a minimum return ($\mu_p$), so all portfolios have a different structure and different levels of risk. Also, we found a Global Minimum Variance (GMV) portfolio, with the lowest standard deviation, where there are no requirements on minimum return. From Table 3 we can see, that with an increase of the required level of $\mu_p$, the risk of the overall portfolio rises and stocks with lower yield are gradually excluded from the portfolio.

Same as for the SSE portfolio, for the HKEX portfolio we found optimal portfolio sets for requirements on risk from μp≥0,02% until μp≥0,11%. Also, we found a Global Minimum Variance (GMV) portfolio, with an expected level of risk of 0,76% (Table 4).

**Table 4.** Markowitz optimal portfolios constructed for the HKEX.

| Constraint | δ | Weights | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0003.HK | 0005.HK | 0019.HK | 0066.HK | 0388.HK | 0700.HK | 0823.HK | 0883.HK | 1128.HK | 2319.HK |
| μp ≥ 0,02% | 0,77% | 33,16% | 12,52% | 9,73% | 19,73% | 0,00% | 1,04% | 22,05% | 0,00% | 0,00% | 1,77% |
| μp ≥ 0,03% | 0,79% | 34,49% | 6,43% | 5,21% | 20,02% | 0,00% | 6,15% | 25,35% | 0,00% | 0,00% | 2,36% |
| μp ≥ 0,04% | 0,83% | 35,81% | 0,34% | 0,69% | 20,30% | 0,00% | 11,26% | 28,65% | 0,00% | 0,00% | 2,95% |
| μp ≥ 0,05% | 0,88% | 29,65% | 0,00% | 0,00% | 15,31% | 0,00% | 19,58% | 32,52% | 0,00% | 0,00% | 2,94% |
| μp ≥ 0,06% | 0,96% | 22,67% | 0,00% | 0,00% | 9,73% | 0,00% | 28,22% | 36,43% | 0,00% | 0,00% | 2,95% |
| μp ≥ 0,07% | 1,06% | 15,74% | 0,00% | 0,00% | 4,21% | 0,00% | 36,90% | 40,33% | 0,00% | 0,00% | 2,82% |
| μp ≥ 0,08% | 1,17% | 7,63% | 0,00% | 0,00% | 0,00% | 0,00% | 45,65% | 44,07% | 0,00% | 0,00% | 2,66% |
| μp ≥ 0,09% | 1,30% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 56,34% | 42,91% | 0,00% | 0,00% | 0,76% |
| μp ≥ 0,10% | 1,45% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 69,87% | 30,13% | 0,00% | 0,00% | 0,00% |
| μp ≥ 0,11% | 1,64% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 83,36% | 16,64% | 0,00% | 0,00% | 0,00% |
| GMV | 0,76% | 31,39% | 17,53% | 14,18% | 18,61% | 0,00% | 0,00% | 18,01% | 0,00% | 0,00% | 0,28% |

In this approach, the optimal portfolios constructed according to the classical Markowitz model, so weights of assets in the portfolio do not change until the end of the investment period and this step is the final for the investor. At the next section, we found an optimal portfolio for the same stocks but using multivariate time series forecast through a BEKK GARCH model.

*2.4. Optimal portfolio construction using multivariate time series forecasts*

To construct an optimal portfolio based on a multivariate time series forecast we used the BEKK GARCH model, described in the first part.

We use the econometric package "EViews 10" to estimate the parameters of the multidimensional GARCH model, as well as the sample numerical characteristics of the time series. So, the following algorithm implemented to solve the problem of construction of an optimal portfolio with a multidimensional GARCH model:

1. To obtain a number of historical data on profitability;
2. To conduct an analysis of time series data on stationarity and test for "ARCH-effects";
3. To estimate the parameters of the multidimensional GARCH model for the conditional covariance matrix of the income vector;
4. To model a forecast of the vector of conditional mathematical expectations of daily returns and conditional covariance matrix at the time of t+1;
5. To construct an optimal portfolio based on forecasted conditional estimates.

Because GARCH-type models are very sensitive to the number of data, they are more efficient for a time-series with a large number of observations. For this reason, for both stock exchanges we chose daily stock returns for the in-sample period from 2013.01.04 until 2018.12.28 (1458 observations in total).

First, we analyzed the descriptive statistics and checked all time-series on stationarity and the presence of ARCH effect through the Engle test in an Eviews. All time-series shows the presence of ARCH effect what allowed us to apply the GARCH model to chosen time-series data.

Then, using Akaike's information criterion (AIC), we evaluated diagonal BEKK GARCH models for different orders, to find the best fitted for the data series model. Table 5 shows the output of the analysis.

**Table 5.** AIC for different orders of the BEKK Model for SSE and HKEX.

| | BEKK (1,1) | BEKK (2,1) | BEKK (2,2) | BEKK (3,2) |
|---|---|---|---|---|
| AIC (SSE) | -54,01 | -52,9594 | -52,884 | -52,8085 |
| AIC (HKEX) | -59,058 | -59,033 | -58,592 | -58,579 |

According to AIC, we have to choose the model with the lowest value. Therefore, for the Shanghai Stock Exchange and for the Hong Kong Stock Exchange portfolios we selected a BEKK (1,1) model.

From the BEKK (1,1) model, we found estimates for a conditional vector of returns and estimates for a conditional covariance matrix, and we concluded, that all the values, estimated in the model are statistically significant, so we were able to use these parameters to predict the conditional values of daily returns and conditional covariance matrix for further portfolio optimization.

At the fourth step, mentioned in an algorithm above, through an Eviews, using BEKK (1,1) model we found new conditional mean returns and forecasted new conditional covariance matrix on a moment t+1.

Also, we observed, that for two examined stock exchanges, forecasted values in a conditional covariance matrix are different from the values in an unconditional covariance matrix, developed in a previous section.

In the fifth step, using the new forecasted conditional covariance matrix and new conditional vector of returns we run the same portfolio optimization model as for the classical Markowitz optimization model.

Optimal portfolios for the Shanghai Stock Exchange, constructed, using the BEKK (1,1) model, are displayed in Table 6. For comparison, in this table we also displayed weights of the stocks in optimal portfolios, calculated using the classical Markowitz model, which was already shown in Table 3.

**Table 6.** Optimal portfolios, constructed using the classic Markowitz model and optimization model with multivariate time series forecasts (SSE).

* - denotes portfolios, constructed using the BEKK model

| Constraint | δ | Weights | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 600028 | 600030 | 600036 | 600050 | 600104 | 600276 | 600519 | 601088 | 601318 | 601668 |
| μp ≥ 0,02% | 1,62% | 34,99% | 9,00% | 7,16% | 8,17% | 8,07% | 3,59% | 4,73% | 11,13% | 6,24% | 6,91% |
| *μp ≥ 0,02%* | *1,12%* | *9,00%* | *0,00%* | *0,00%* | *27,76%* | *21,42%* | *0,00%* | *9,85%* | *13,52%* | *13,31%* | *5,14%* |
| μp ≥ 0,03% | 1,57% | 22,40% | 9,49% | 8,55% | 9,06% | 9,02% | 7,08% | 7,31% | 10,58% | 8,08% | 8,42% |
| *μp ≥ 0,03%* | *1,15%* | *10,59%* | *0,43%* | *0,00%* | *31,67%* | *22,98%* | *0,00%* | *10,77%* | *0,22%* | *16,50%* | *6,85%* |
| μp ≥ 0,04% | 1,55% | 9,31% | 10,03% | 10,08% | 10,05% | 10,05% | 10,19% | 10,14% | 9,97% | 10,10% | 10,08% |
| *μp ≥ 0,04%* | *1,18%* | *8,13%* | *0,00%* | *0,00%* | *29,11%* | *23,01%* | *5,96%* | *16,49%* | *0,19%* | *8,56%* | *8,54%* |
| μp ≥ 0,05% | 1,55% | 0,00% | 6,96% | 11,48% | 10,88% | 10,94% | 14,03% | 12,92% | 9,12% | 12,03% | 11,63% |
| *μp ≥ 0,05%* | *1,24%* | *6,03%* | *0,00%* | *0,00%* | *26,77%* | *23,44%* | *12,51%* | *20,96%* | *0,00%* | *0,00%* | *10,29%* |
| μp ≥ 0,06% | 1,51% | 0,00% | 0,00% | 10,97% | 10,58% | 10,74% | 19,35% | 16,27% | 5,67% | 13,77% | 12,67% |
| *μp ≥ 0,06%* | *1,34%* | *2,27%* | *0,00%* | *0,00%* | *20,69%* | *22,20%* | *19,33%* | *24,91%* | *0,00%* | *0,00%* | *10,60%* |
| μp ≥ 0,07% | 1,51% | 0,00% | 0,00% | 8,25% | 9,20% | 9,49% | 25,21% | 19,58% | 0,23% | 15,03% | 13,01% |
| *μp ≥ 0,07%* | *1,47%* | *0,00%* | *0,00%* | *0,00%* | *13,59%* | *20,71%* | *26,44%* | *29,01%* | *0,00%* | *0,00%* | *10,24%* |
| μp ≥ 0,08% | 1,55% | 0,00% | 0,00% | 0,00% | 2,32% | 8,23% | 33,70% | 24,59% | 0,00% | 17,22% | 13,94% |
| *μp ≥ 0,08%* | *1,62%* | *0,00%* | *0,00%* | *0,00%* | *4,90%* | *18,86%* | *34,00%* | *33,33%* | *0,00%* | *0,00%* | *8,91%* |
| μp ≥ 0,09% | 1,60% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 44,17% | 26,49% | 0,00% | 16,82% | 12,52% |
| *μp ≥ 0,09%* | *1,80%* | *0,00%* | *0,00%* | *0,00%* | *0,00%* | *14,98%* | *41,86%* | *37,35%* | *0,00%* | *0,00%* | *5,82%* |
| μp ≥ 0,10% | 1,73% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 61,81% | 22,94% | 0,00% | 10,41% | 4,84% |
| *μp ≥ 0,10%* | *2,01%* | *0,00%* | *0,00%* | *0,00%* | *0,00%* | *8,46%* | *50,11%* | *40,98%* | *0,00%* | *0,00%* | *0,44%* |
| μp ≥ 0,11% | 1,95% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 80,68% | 17,95% | 0,00% | 1,37% | 0,00% |
| *μp ≥ 0,11%* | *2,24%* | *0,00%* | *0,00%* | *0,00%* | *0,00%* | *0,00%* | *60,60%* | *39,40%* | *0,00%* | *0,00%* | *0,00%* |
| GMV | 1,430% | 19,60% | 0,00% | 22,05% | 4,14% | 7,15% | 22,07% | 20,82% | 2,75% | 0,00% | 1,41% |
| *GMV* | *1,12%* | *9,12%* | *0,12%* | *0,00%* | *27,17%* | *21,31%* | *0,00%* | *8,25%* | *16,17%* | *13,29%* | *4,56%* |

As we can see from Table 6, all the weights in portfolios, constructed using the two different models, are different. Differences in weights and expected standard deviation in classical Markowitz portfolios and in Markowitz portfolios with use of the BEKK models are significantly different, especially in portfolios, with lower required μ. But we can notice, that in both models there is a similar trend when the required level of return increases: from all optimal portfolios with the required level of return μp ≥ 0,09%, are excluded stocks - 600028, 600030, 600036, 600050, 601088. It means, that these stocks have lower expected return and are more volatile, according to analyzed values. Also, can be observed, that the expected level of risk in the majority of portfolios is larger for portfolios, constructed using the classical Markowitz optimization model. It can be explained by an assumption, that conditional values, which we estimated through a BEKK model, allow us to decrease the volatility of an overall portfolio.

Table 7 represents new portfolio sets for BEKK optimized portfolios and classical Markowitz portfolios for the Hong Kong Stock Exchange.

**Table 7**. Optimal portfolios, constructed using the classic Markowitz model and optimization model with multivariate time series forecasts (HKEX).

* - denotes portfolios, constructed using the BEKK model

| Constraint | δ | Weights | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0003.HK | 0005.HK | 0019.HK | 0066.HK | 0388.HK | 0700.HK | 0823.HK | 0883.HK | 1128.HK | 2319.HK |
| μp ≥ 0,02% | 0,77% | 33,16% | 12,52% | 9,73% | 19,73% | 0,00% | 1,04% | 22,05% | 0,00% | 0,00% | 1,77% |
| *μp ≥ 0,02% | 0,70% | 41,89% | 13,86% | 9,98% | 18,36% | 2,46% | 0,00% | 13,45% | 0,00% | 0,00% | 0,00% |
| μp ≥ 0,03% | 0,79% | 34,49% | 6,43% | 5,21% | 20,02% | 0,00% | 6,15% | 25,35% | 0,00% | 0,00% | 2,36% |
| *μp ≥ 0,03% | 0,70% | 43,16% | 12,17% | 8,03% | 17,80% | 2,42% | 0,07% | 15,82% | 0,00% | 0,00% | 0,52% |
| μp ≥ 0,04% | 0,83% | 35,81% | 0,34% | 0,69% | 20,30% | 0,00% | 11,26% | 28,65% | 0,00% | 0,00% | 2,95% |
| *μp ≥ 0,04% | 0,72% | 44,93% | 7,35% | 5,54% | 17,58% | 0,13% | 4,92% | 18,33% | 0,00% | 0,00% | 1,23% |
| μp ≥ 0,05% | 0,88% | 29,65% | 0,00% | 0,00% | 15,31% | 0,00% | 19,58% | 32,52% | 0,00% | 0,00% | 2,94% |
| *μp ≥ 0,05% | 0,75% | 46,67% | 1,59% | 2,69% | 16,89% | 0,12% | 9,81% | 20,86% | 0,00% | 0,00% | 1,36% |
| μp ≥ 0,06% | 0,96% | 22,67% | 0,00% | 0,00% | 9,73% | 0,00% | 28,22% | 36,43% | 0,00% | 0,00% | 2,95% |
| *μp ≥ 0,06% | 0,79% | 44,94% | 0,00% | 0,00% | 13,81% | 0,08% | 15,30% | 24,03% | 0,00% | 0,00% | 1,84% |
| μp ≥ 0,07% | 1,06% | 15,74% | 0,00% | 0,00% | 4,21% | 0,00% | 36,90% | 40,33% | 0,00% | 0,00% | 2,82% |
| *μp ≥ 0,07% | 0,85% | 40,17% | 0,00% | 0,00% | 8,70% | 0,08% | 21,65% | 27,49% | 0,00% | 0,00% | 1,92% |
| μp ≥ 0,08% | 1,17% | 7,63% | 0,00% | 0,00% | 0,00% | 0,00% | 45,65% | 44,07% | 0,00% | 0,00% | 2,66% |
| *μp ≥ 0,08% | 0,92% | 35,51% | 0,00% | 0,00% | 3,87% | 0,00% | 28,12% | 31,02% | 0,00% | 0,00% | 1,48% |
| μp ≥ 0,09% | 1,30% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 56,34% | 42,91% | 0,00% | 0,00% | 0,76% |
| *μp ≥ 0,09% | 1,01% | 29,66% | 0,00% | 0,00% | 0,00% | 0,00% | 34,68% | 34,40% | 0,00% | 0,00% | 1,27% |
| μp ≥ 0,10% | 1,45% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 69,87% | 30,13% | 0,00% | 0,00% | 0,00% |
| *μp ≥ 0,10% | 1,11% | 20,18% | 0,00% | 0,00% | 0,00% | 0,00% | 41,77% | 37,43% | 0,00% | 0,00% | 0,62% |
| μp ≥ 0,11% | 1,64% | 0,00% | 0,00% | 0,00% | 0,00% | 0,00% | 83,36% | 16,64% | 0,00% | 0,00% | 0,00% |
| *μp ≥ 0,11% | 1,22% | 10,30% | 0,00% | 0,00% | 0,00% | 0,00% | 48,73% | 40,36% | 0,00% | 0,00% | 0,62% |
| GMV | 0,76% | 31,39% | 17,53% | 14,18% | 18,61% | 0,00% | 0,00% | 18,01% | 0,00% | 0,00% | 0,28% |
| *GMV | 0,70% | 41,89% | 13,86% | 9,98% | 18,36% | 2,46% | 0,00% | 13,45% | 0,00% | 0,00% | 0,00% |

As can be observed from the tables 6 and 7, on the in-sample period, expected values of risk are significantly lower for Hong Kong Stock Exchange portfolios, because the following portfolio is more diversified than portfolio for the Shanghai Stock Exchange.

As a result of calculations above, for the selected set of stocks, we applied two models of portfolio optimization: classical Markowitz optimization model, and optimization model with conditional fore-casted values, through a BEKK GARCH model. At the next step, we backtest all developed models on the out-of-sample period, to define, portfolios based on which model perform better.

*2.5. Portfolio Backtesting*

To demonstrate the viability of techniques that we have developed, we evaluate the performance of constructed portfolios through backtesting. To perform the backtesting we use historical prices of stocks included in final portfolios for the out-of-sample period from 02.01.2019 until 31.01.2019.

Analysis of real returns of constructed portfolios on the out-of-sample period, al-lows us to deter-mine, whether the forecast of numerical characteristics is accurate and enables to reduce the volatility of the overall portfolio. Actual portfolio return was calculated as fol- lows:

$$r_{t_p} = \sum_{i=1}^{10} w_{s_t} \times r_{s_t} \quad (3)$$

where,

$r_{tp}$ – portfolio return on the day t;

$w_{st}$ – weight of the stock in an optimal portfolio on the day t;

$r_{st}$ – actual stock return on the day t.

According to Markowitz optimization theory, unconditional forecasted values (covariance matrix and returns) obtained in tables 3 and 4, are stable. For this reason, optimal portfolios constructed at the beginning of the out-of-sample period and do not change until the end of the investment period. So, first, using the equation (3), we evaluated the performance of the portfolios, constructed through the classical Markowitz optimization model on the out-of-sample period for the SSE and HKEX.

At the next step, we calculated actual returns for portfolios constructed using BEKK GARCH model. Because in this section we developed portfolios using multivariate time series forecasts (BEKK model), estimated conditional covariance matrix and vector of daily stock returns are also forecasts, but only for the day, immediately following the end of the in-sample period (02.01.2019). So, fore-casted conditional covariance matrix and vector of returns for the day 03.01.2019 already will be different, since the information that appeared on 02.01.2019 also should be taken into account. Thus, for the entire out-of-sample period, it is necessary to obtain 21 more estimates of the conditional covariance matrix and the conditional mean of daily returns of considered stocks. To carry such analysis, parameters of the vector of daily returns and parameters of the BEKK (1,1) model should be re-evaluated every time, when new information appears. Therefore, the estimation of the parameters of the BEKK (1,1) model also realized on a sample of 1458 observations, however, we had to move our sample one day ahead 21 times, every time taking into account the new appeared values from the out-of-sample period. Moving "one day ahead" forecast also was carried through an econometric package EViews10.

The graphs below indicate the cumulative average performance of portfolios developed using two different models for a period from 02.01.2019 until 31.01.2019.

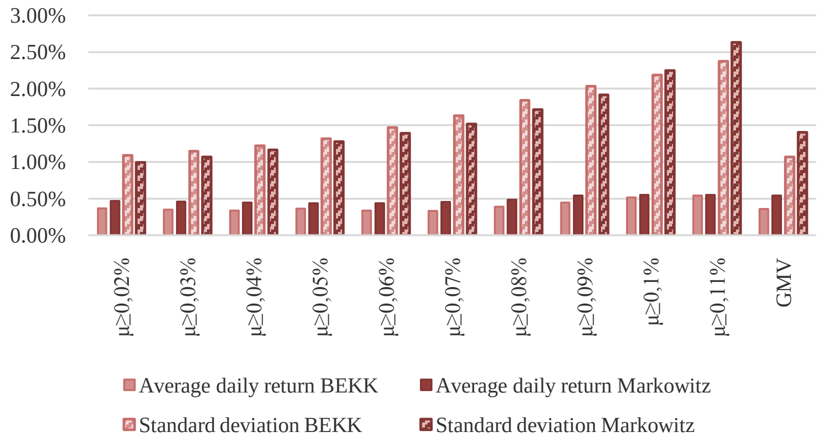


**Figure 1.** Mean and standard deviation of the optimal portfolios on the out-of-sample period for the Shanghai Stock Exchange.

According to an analysis of figure 1, it is obvious, that optimal Markowitz portfolios outperformed results of portfolios constructed using time series forecasts by overall values of return and risk. Only for portfolios, constructed with required $\mu \geq 0{,}10\%$ and $\mu \geq 0{,}11\%$, BEKK portfolio showed lower volatility.

For comparison of portfolios performance, we calculated the Sharpe Ratio. As a risk-free rate, we chose the rate of return of the China Government 1-year bond. Based on the analysis of the Sharpe Ratio we found, that the Sharpe Ratio of portfolios constructed using classical Markowitz optimization model is higher. It implies, that Markowitz portfolios performed better on the out-of-sample period, and showed a higher return with lower volatility.

Despite the fact, that most of the Markowitz portfolios showed higher returns, in comparison with BEKK portfolios, it is noticeable, that actual and forecasted risk data in portfolios, constructed with help of a BEKK model are almost the same and goes in the same direction, while in portfolios, based on the classical Markowitz model, these values are different almost for all portfolios with a different $\mu$. So, on the basis of our findings, we can conclude, that the final result of BEKK portfolios, on the out-of-sample period, is similar to forecasted values on the in-sample period towards to risk and average return values. By contrast, the in-sample and out-of-sample results, for classical Markowitz portfolios, are different. Therefore, portfolios constructed through multivariate time series forecasts, in practice, better follow the forecasted values, based on a historical return, what allow investor make more efficient decisions about capital allocation.

Figure 2 shows the overall performance of portfolios constructed for the Hong Kong Stock Exchange.

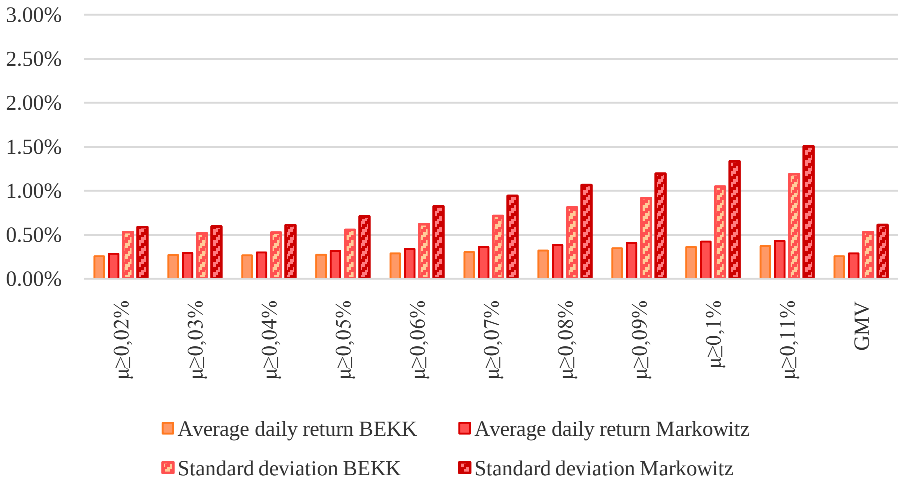


**Figure 2.** Mean and standard deviation of the optimal portfolios on the out-of-sample period for the Hong Kong Stock Exchange.

Based on the results for the out-of-sample period, it appears, that by return, Markowitz portfolios outperformed BEKK portfolios. On the other hand, in figure 2 clearly visible, that standard deviation in all BEKK portfolios significantly smaller, than in Markowitz portfolios. As an example, for the portfolio with μ≥0,11%, the standard deviation of the portfolio, constructed through the BEKK model, are smaller by 0,31 percentage points.

As a risk-free rate for Sharpe Ratio, we chose the 1-year rate of return of the Hong Kong government bond. We found, that the Sharpe Ratio of portfolios developed using BEKK GARCH model is lower than the ratio of Markowitz portfolios. Therefore, in the case of the Hong Kong Stock Exchange, BEKK portfolios more attractive for investors by its return and risk results.

Also, the standard deviation in Markowitz portfolios increases more extensively, when the required level of return increases. It implies, that portfolios, constructed with help of the multivariate time series forecasts, are less volatile, than portfolios, constructed through classical Markowitz approach.

In previous stages, based on two models of portfolio construction, we found optimal portfolios for the Shanghai Stock Exchange and Hong Kong Stock Exchange. At the conclusion part, we summed up all results, achieved through conducted research and gave recommendations according to the outcome of the study.

## 3. Conclusion

### *3.1. Main findings of the study*

The following research focused on the construction of optimal investment portfolios using the classical Markowitz optimization model and Generalized Autoregressive Model of Conditional Heteroscedasticity (GARCH).

The main objective of the study was to determine, whether the use of a multidimensional model of generalized autoregressive conditional heteroscedasticity (BEKK GARCH) can improve the accuracy of the portfolio volatility forecast and thereby reduce the risk of the optimal stock portfolio.

The final performance of Markowitz and BEKK portfolios for the Shanghai Stock Exchange shows, that optimal Markowitz portfolios outperformed results of the BEKK models by overall values of return and risk, and only for portfolios, constructed with required μ≥0,10% and μ≥0,11%, BEKK portfolio showed lower volatility. By contrast, performance results for the Hong Kong Stock Exchange shows, that all BEKK portfolios showed lower volatility than Markowitz portfolios. Also, by comparison of the final performance graphs for two stock exchanges we found, that in optimal BEKK portfolios, constructed for the Hong Kong Stock Exchange, the standard deviation is significantly smaller, than in Markowitz portfolios. And the standard deviation in Markowitz portfolios increases more significantly when the required level of return increases. It proves, that portfolios, constructed with help of the multivariate time series forecasts, are less volatile, than portfolios, constructed through classical Markowitz approach.

In portfolios for the Hong Kong Stock Exchange, real returns and risk results are better correlated with predicted values, in comparison with portfolios for the Shanghai Stock Exchange. Such differences indicate that results of the models in well-diversified portfolios are better follow predicted values. This finding justifies the construction of two different portfolios for the conducted research.

According to the main purpose of the research, on the basis of empirical analysis results, we confirmed, that in fact, optimal portfolios, developed using the multivariate BEKK GARCH model, have a lower standard deviation, than optimal portfolios, constructed using Markowitz optimization model. It confirms that multivariate time series forecasts can improve the accuracy of portfolio optimization models and reduce overall portfolio volatility. These results are almost identical for the Shanghai Stock Exchange, as well as for the Hong Kong Stock Exchange. But according to calculations outcomes, BEKK portfolios, constructed for the Hong Kong Stock Exchange, showed lower volatility for all portfolio sets with different requirements on a lower return. So, in terms of accuracy of prediction, more diversified portfolios perform better, than less diversified.

### *3.2. Practical recommendations and relevance of the research results*

As a conclusion, we confirmed, that according to the main research issue, time series forecasts conducted through the GARCH BEKK model actually decreasing portfolio volatility and allow more accurately predict future portfolio outcomes. Also, recalling the obtained results, we found, that portfolio developed using the GARCH model, better follows the structure of the predicted values, which is very important for investments decision making.

However, it is important to note, that the findings above also have limitations:

1. GARCH-type models are very sensitive to the size of the sample, therefore it is more appropriate to apply such models only for long time series.

2. In the optimal portfolios, constructed through multivariate time series forecasts, forecasted estimates depend on the size of the sample data (daily, weekly, monthly, etc.), hence an investor has to reevaluate model permanently and adjust portfolio according to new optimal weights. In other words, an active portfolio management strategy is a must.

3. Active portfolio strategy increases transaction costs, therefore it can be unsuitable for passive investors.

So, according to above mentioned features of the portfolio, developed using the GARCH model, it is possible to recommend such strategy mostly to active investors.

For passive investors, using “buy and hold strategy”, will be more appropriate to choose Markowitz portfolio optimization model, because, regarding our results, this model also shows high return and outperformed stock market indexes, as well as results of the portfolios, constructed using GARCH model, in terms of return. Also, because for passive investors portfolio volatility is not very essential, as opposed to active investors, more volatile portfolio will not affect an overall investor’s return, what makes it possible, for passive investors to use Markowitz optimization models.

The findings of the following research are providing a good basis for the present investors, who are looking for the methods of portfolio construction, which can improve the overall portfolio performance. Building upon the results of the research, real investors can apply studied models for their own investment strategies, or decide, how the particular models suit for their own needs on the examined markets.

Also, the results of the research will be useful in further study of the methodology of portfolio construction.

## References

[1] Bodie Z, Kane A, Marcus A 2011 Investments 9th ed. *McGraw-Hill Companies* (New York)

[2] Brooks C 2014 Introductory econometrics for finance 3rd ed. *Cambridge University Press* (United Kingdom)

[3] Graham B 2017 The intelligent investor 4th ed. *Alpina Publisher* (Moscow)

[4] Sharpe W, Alexander G, Bailey J 1998 Investments 6th ed. *Prentice Hall* (United States)

[5] Amenc N, Goltz F 2011 Practitioner portfolio construction and performance measurement: *Evidence from Europe Financial Analysts Journal* **67(3)** 39-50

[6] Andersen T, Bollerslev T 1998 Answering the skeptics: Yes, standard volatility models do provide accurate forecasts *International Economic Review* 885-905

[7] Black F, Litterman R 1990 Asset allocation: Combining investors views with market equilibrium Technical report, Goldman, Sachs, & Company

[8] Bollerslev T, Engle R, Wooldridge J 1988 Capital asset pricing model with time-varying covariances *The Journal of Political Economy* **96(1)** 116-131

[9] Engle R, Kroner K 1995 Multivariate simultaneous generalized ARCH *Econometric theory* **11(1)** 122-150

[10] Engle R, Colacito R 2006 Testing and valuing dynamic correlations for asset allocation *Journal of Business and Economic Statistics* **24** 238-253

[11] Fama F, French K 2004 The Capital Asset Pricing Model: Theory and Evidence *Journal of Economic Perspectives* **18(3)** 25-46

[12] Gupta R, Donleavy G 2009 Benefits of diversifying investments into emerging markets with time-varying correlations: An Australian perspective *Journal of Multinational Financial Management* **19(2)** 160-177

[13] Jorion P 2003 Portfolio optimization with tracking error constraints *Financial analyst journal* **59** 70-82

[14] Kaplanis E 1988 Stability and forecasting of the co-movement measures of international stock market returns *Journal of International Money and Finance* **7(1)** 63-75

[15] King M 1994 Volatility and links between national stock markets *Econometrics* **62(4)** 901-933

[16] Koch P 1991 Evolution in dynamic linkages across daily national stock indexes *Journal of International Money and Finance* **10(2)** 231-251

[17] Markowitz H 1952 Portfolio selection *The Journal of Finance* **7(1)** 77-91

[18] Markowitz H, Todd G, Sharpe W 1987 Mean-Variance Analysis in Portfolio Choice and Capital Markets (USA)

[19] Michaud R 1989 The Markowitz optimization enigma: Is “optimized” optimal *Financial Analysts Journal* **45(1)** 31-42

[20] Miralles-Marcelo J, Miralles-Quirós M 2013 Multivariate GARCH models and risk minimizing portfolios: The importance of medium and small firms *The Spanish Review of Financial Economics* **11(1)** 29-38

[21] Patton A, Sheppard K 2009 Evaluating Volatility Forecasts Handbook of Financial Time Series 801-838

[22] Pojarliev M, Polasek W 2001 Applying multivariate time series forecasts for active portfolio management *Financial Markets and Portfolio Management* **15(2)** 201-211

[23] Rockafellar R, Uryasev S 2000 Optimization of conditional value at risk *Journal of Risk* **2(3)** 21–41

[24] Sharpe W 1966 Mutual Fund Performance *The Journal of Business* **1** 119-138

[25] Tobin J 1958 Liquidity Preference as Behavior Toward Risk *Review of Economic Studies* **2** 65-86

[26] Ulucan A 2007 An analysis of mean variance portfolio selection with varying holding periods *Applied Economics* **39** 1399-1407