

# The Concept of the Historical Development of Geometric Knowledge in the Aspect of Synergetic Methodology

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**Abstract.** In this article the author explores the evolution of geometrical knowledge and gives a forecast of their further historical development. This is important from a practical point of view, because geometric knowledge and visually imaginative language of geometric modeling is used in many fields of professional activity. Such areas include the architecture, the design, the art, the science and etc. As we know, theoretical knowledge is the result of the scientific cognitive process. Practical professional activity may be the result of theoretical knowledge. Therefore, the concept of development of theoretical geometric knowledge will help to describe the panorama of changes and updates in the professional field. In particular, the study in this article can give some guidance for architects, designers, etc.

## 1. Introduction

The historical development is presented in the aspect of synergetic methodology. Synergetic studies the self-organizing system. Synergetic (from Others-Greek. συν is a prefix with the meaning of compatibility and ἔργον "activity") -an interdisciplinary direction of science, gives a description, explanation and prediction of any phenomenon of self-organization. A characteristic feature of the synergetic worldview is building the image (model) of a coherent picture of the world. The basis of this picture of peace is a multidisciplinary integration of different fields of knowledge. This knowledge is allocated after studying the various models of complex systems. The basic ideas of the synergetic methodology are subject to synergetic principles, including: openness, nonlinearity, instability, dynamic hierarchy (emergence), observability. All the principles are interrelated and are used in the applied value of our research.

The implementation of these principles, the author presented a scientific geometric knowledge as a complex system. This system is the basis of all of its component parts. The main parts of geometric knowledge are a scientific theory and a method of geometric modeling. It should be noted that geometric modeling is a visual-imaginative method of modeling. From the point of view of the synergetic principles, evolution of geometrical knowledge is represented as an unlimited sequence of processes of self-organization. This sequence of processes of self-organization is created by the combination of repetitive periods in the development of geometric knowledge [1,2].

## 2. The scenario of one of the periods in the development of geometric knowledge

The scenario of one of the periods in the development of geometric knowledge is represented in the following way.

- First, a relatively stable condition of the system of geometrical knowledge loses its stability. The instability occurs with the emergence of a new element in the system of geometrical knowledge or any external influence on the control parameter. This element triggers a dynamic process.
- Secondly, the point of bifurcation (bifurcation – lat. bifurcus - forked). The critical condition of the system leads to an instant exit from the unstable condition. The condition of instability of the system of geometrical knowledge involves the transition of quantitative changes into qualitative changes. From the point of view of quantitative changes in this period of time is a gradual process. From the point of view of qualitative change, this process appears to be jump. This jump brings the system to self-organize. In addition, the system can select one of the available trajectories of development. The trajectory of development is associated with the forces of the attractor, which forms the structure of geometric knowledge.
- Third, the end of the jump means the formation of a qualitatively new system and new structure of geometric knowledge. The evolving system of geometric knowledge is converted to new relatively stable system at the end of the process of self-organization.

It is known that the source of development of any process (phenomenon) is "unity and struggle of opposites". Alternative paths are considered to be opposites. Such alternative ways, in our view, are sustainable and unsustainable development. The stability of the theory consists in the development of different views, ideas, concepts, methods, theories (containing definitions, lemmas, theorems, proofs) that have arisen and arise for the purpose of in-depth knowledge of certain aspects of the material world. If the theory is in a stable state (equilibrium), it is not enough to know this fact alone. It is necessary to find out whether the equilibrium is stable, i.e. it is disturbed or not under random external influences, which in nature cannot be avoided [3,4].

The instability of the theory lies in the knowledge of the material world by obtaining new knowledge as a result of the synthesis of individual knowledge into a single integral knowledge. In the period of instability (even closed) the system becomes open, is a sensitive receiver of influences of other levels of being, receives information previously inaccessible to it. These States of instability and choice are called bifurcation points. They certainly arise in any situation of the birth of a new quality and characterize the boundary between the new and the old. Stability and instability are dialectical: stability grows out of instability, and stability sooner or later turns into instability. Stable and unstable stages of knowledge, replacing each other, determine the source of the theory.

Let us consider the evolution of geometric theory as a self-organizing system in more detail [5].

### **3. The first stage of development of geometric theory**

During this period, the geometric theory is just beginning to emerge, since the first geometric (or rather, graphic) models were visual images - drawings made on a solid medium (earth, stone, wood, etc.). The figure wore a silhouette-planar in nature, built without precise adherence to the dimensions of the object. However, such drawings allowed to create, store and transfer the accumulated information from one generation to another. These drawings reflected the culture, socio-economic development of society and practical activities of the time. This period is characterized by the steady development of geometric theory. It should be noted here that visual language has become the first language of human society.

Socio-economic development of society and practical human activity puts forward new tasks: the creation of geometric models with metric properties, the construction of projection images. The rules of execution of such images, i.e. rules of establishment of geometrical correspondence between points of the represented subject and points of its image on the plane, did not exist at that time. There is a new medium - paper. In this period of unstable development of geometrical knowledge, i.e. inconsistencies existing views of knowledge of reality to new ideas, there is a science called "geometry". Geometry emerged through the accumulation of geometric knowledge, elucidation of relationships between various geometric facts; development of methods of evidence, the formation of

concepts, and, finally, there were the first attempts to establish the rules of construction of images on the plane.

Based on the above, we can conclude that the process of sustainable development of the theory, which occurs through the accumulation of different geometric facts and acts in time as a gradual quantitative change, leads the system to an unstable state, i.e. there is a lot of knowledge. The output of the system from the unstable state is characterized by qualitative changes as a leap, i.e. there is a new holistic knowledge. The emergence of a new system is associated with the loss of stability and the transition of the original system to a new stable state. In this case, the system structure changes. Thus, geometry has become an independent science with its systematic presentation. As for the construction of images – models, then, firstly, the model has become metric, ie built exactly in size; secondly, the model remained two-dimensional, although in this period there is a concept of reversibility of the model and the original, which allows you to use these models in the manufacture of various objects (architectural, technical, etc.).

The completion of the scientific jump is characterized by the transition of the system through the bifurcation point, in which the evolutionary path of the system is branched. At the first stage, two ways of development were formed and distinguished, which include several independent directions: 1) the development of theories: geometry, mathematics, astronomy, mechanics, optics, geography, etc.; 2) the development of the theory of images, including: calligraphy, which later gave the alphabet and writing; artistic modeling (painting, sculpture, design, advertising, etc.); geometric modeling (construction of one-to-one maps, i.e. reversible).

In our analysis we will focus only on the development of the theory of geometric modeling. Thus, the emergence of geometry completes the first period of development of the theory of geometric modeling, which is the first turn in the spiral of development of the dialectical model of this field of knowledge. During this period of development, the theory of geometric modeling can be defined as the theory of *image methods and their practical application* [2].

#### **4. The second stage of development of the theory of geometric modeling**

The first period ended with the emergence of geometry and began the second stage of development of the theory of geometric modeling, which was based on the development of various projection methods of image construction. These methods were scattered, but United by a common theory and some of them were only ways to solve individual practical problems. This was a period of steady development of geometric theory, characterized by the presence of a large variety of methods and representing the quantitative accumulation of knowledge in this subject area. A major role in this period was played by the work of R. Descartes "Geometry", in which he developed the method of coordinates. The method of coordinates is allowed to link geometry with algebra, then developing and emerging analysis. Application of methods of these Sciences in geometry has generated analytical geometry, and then differential. The theory of images also developed. At this time, there are the first concepts of projective geometry, the theory of axonometric projections, perspective. Geometry has moved to a qualitatively new level compared to the geometry of the ancients.

However, the development of human society, the expansion of trade relations, the development of industry demanded from the geometric theory of the development of new methods of imaging rather complex three-dimensional objects. Such intensive information influence on the theory of geometric modeling, characterized by socio-economic development of the society of that time, led to instability of the theory of geometric modeling.

Exit from the state of instability of the system (theory) involves the transition of quantitative changes in quality. From the side of quantitative changes, this period appears in time as something gradual, and from the side of qualitative changes it appears as a leap. At this time, dramatically increases the role of personality (and other accidents), not too important in the phase of evolutionary period. Such a qualitative leap was made by the French mathematician and public figure G. Monge, who combined the existing methods at that time in his work "Geometry descriptive" (1799), thereby initiating the existence of descriptive geometry as a science. Descriptive geometry is an *engineering*

*discipline that represents a two-dimensional geometric apparatus and a set of algorithms for studying the properties of geometric objects [2].*

Geometric models (drawings) created by the methods of descriptive geometry differed from the models of the previous period of development of geometric theory. These models had reversibility, i.e. allow to restore (reconstruct) the three-dimensional model of the object, as well as to investigate the geometric characteristics of the simulated object and make it.

Thus, the completion of the qualitative leap (bifurcation point) is characterized by the branching of the theory into pure geometry theory (differential, analytical, projective) and into the theory of image methods and their applications in various fields of knowledge.

### **5. The third stage of development of the theory of geometric modeling**

The third stage of development of the theory of geometric modeling came, which completed the second round of the dialectical spiral. The emergence of various geometries, the systematization of which was given by F. Klein in his Erlangen program, contributed to the development of the concepts of Euclidean space, projective, affine, and others. The development of the concept of different spaces led to the concept of multidimensional space, in which three variables - the spatial coordinates of a material point - are added more variables and are considered as Cartesian coordinates of a point of multidimensional space.

In the middle of the XIX century was born topology, which introduced a new concept of the dimension of space. The concept of dimension is related to the number of variables (parameters), which is necessary to set the point of the object in a particular space. In the 70s of the XIX century at the junction of analysis and geometry there was a General theory of point sets, which later formed a special discipline called set theory. Thus, it is possible to set geometric sets in parametric form. The development of new geometric theories and the development of already existing areas (elementary geometry, analytical, differential, etc.) greatly influenced the development of the theory of geometric modeling: there are many new structural models. These models are based on the concept of dimension of spaces, i.e. parameterization of geometric sets.

Thus, we can say that the considered stage of this period was first stable in the development of the theory of geometric modeling, then the variety of methods of descriptive geometry led to an unstable state of the system. Therefore, there is a need to transform descriptive geometry into a generalized theory of geometric modeling, which is now understood as the *theory of modeling methods of spaces and varieties of different number of dimensions and different structures [2].*

In addition, at the end of the XX century the time of computer technology and intensive integration of science, production and education, the widespread introduction of electronic computer technology in various fields of activity required new research in the theory of geometric modeling. The ability to specify geometric sets in parametric form, taking into account their dimension, called at that time descriptive geometry, allowed to build three-dimensional geometric objects (models). Since this model has become three-dimensional, and its dimension coincided with the dimension of the object modeling (original). It has become more visual, informative, allowing to obtain and explore not only the geometric parameters of the object, but also other, for example, mechanical. The computer screen is no longer seen as a plane, but as a two-dimensional space separating two three-dimensional spaces: the physical and electronic space of the computer. This idea is based on the intersection dimension theorem for four-dimensional space: two three-dimensional spaces intersect in four-dimensional space along a two-dimensional space – plane. Here we can conclude that if the dimension of the space of the computer screen would be three-dimensional (and not two-dimensional as it is now), we would be able to build a visual four-dimensional model.

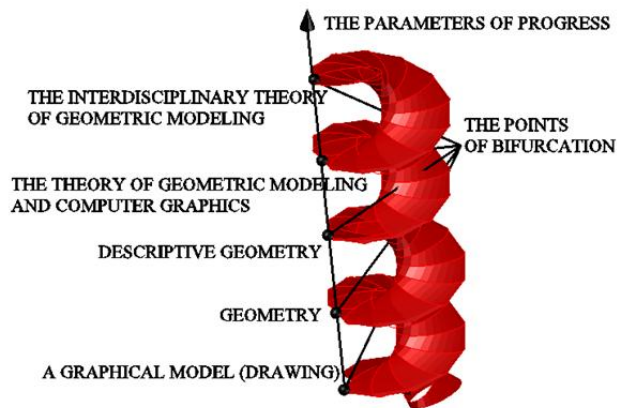
Thus, there was a computer graphics, considered as a part of the theory of geometric modeling, as computer graphics by definition of the International organization for standardization – a set of methods and tools for converting data into a graphical representation using a computer. In other words, a computer is only a tool (an "electronic pencil") for creating models. So, the third round

(jump) of the spiral of development was completed and the fourth period in the development of the theory of geometric modeling came, which continues to the present time.

## 6. Conclusion

The results obtained in the historical and logical study of the theory of geometric modeling, allow us to conclude that the dialectical model of the theory of geometric modeling is a dialectical spiral. In accordance with the concepts of modern natural science and philosophy is considered (or is proven) that the development is carried out in a dialectical spiral, if according to the basic laws of dialectics identified and shown one full turn and at least part of the second. We have revealed three turns of the evolutionary development of geometric theory.

Thus, the research reveals the sources of development of the theory of geometric modeling, the direction and result of the process of knowledge, the General mechanism of its development, as well as the principles of construction of educational information based on the principles of synergy. The principal feature of synergetic is the construction of mathematical (geometric) models of complex developing systems. This makes it possible to make a breakthrough in the creation of models that have hitherto belonged to the humanitarian field of knowledge. Modeling allows not only to quantify existing theories, but also to create conceptual models of new theories— ‘Figure 1’.



**Figure 1.** The model of evolution of geometric knowledge.

## References

- [1] Budanov V G 2007 Methodology of Synergetics in Post-Non-Classical Science and Education (Moscow: LKI Publishing house) p 240
- [2] Shangina E I 2010 Methodological Basis for the Formation of the Structure and Content of Geometric and Graphic Education in a Technical University in Terms of Integration with General Engineering and Special Disciplines (Moscow: diss. ... dock. ped. science 13.00.08) p 365
- [3] Chernavskii D S 2001 Synergetics and Information (Moscow: Publishing house “Nauka”) p 105
- [4] Haken G 1985 Synergetics. Hierarchies of Instabilities in Self-Organizing Systems and Devices (Moscow: Publishing house “Mir”) p 424.
- [5] Haken G 2003 Secrets of nature Synergy: *Science of Interaction* (Moscow-Izhevsk) p 320