Development of Zoning Method for Solving Economic Problems of Optimal Resource Allocation to Objects of Various Importance in Context of Incomplete Information

A V Terentyev[1], S S Yevtukov[2], E A Karelina[3]

[1]Moscow Automobile and Road State Technical University (MADI), Leningradsky ave. 64, Moscow, 125319, Russia
[2]Saint-Peterburg State University of Architecture and Civil Engineering, 2nd Krasnoarmeyskaya 4, St. Petersburg, 190005, Russia
[3]TMoscow State University of Technology "STANKIN", 1 Vadkovsky lane, Moscow, 127055, Russia

E-mail: dm@madi.ru

Abstract. The article is devoted to the development of the zoning method, which allows, in the presence of minimal information about states of the research environment, to build algorithms that provide a Pareto-optimal solution within the framework of the theory of a game with "nature". The idea of the method is to partition the set of possible states of nature into subsets of dominance of individual actions. However, in a number of cases, the use of this method does not allow to obtain the desired optimal solution, although it significantly simplifies its search. This circumstance determined the need for developing the method. The obtained results were defined as a zoning method based on the principle of observing a hierarchical ratio of probabilities of possible states of the external environment of the study. The area of application of the zoning method developed is wide enough according to the principle of observing the hierarchical ratio of probabilities of possible states of the external research environment, but it is most effective to use it to solve economic problems of optimal allocation of resources to objects of various importance in conditions of incomplete information.

1. Introduction

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Uncertainty, determined by the incompleteness of information about the processes studied, is not only a condition that has to be taken into account when solving various economic and production problems, but also a factor inherent in any research. The uncertainty of situations that one encounters when conducting research has very different nature and significance. It may be:

1) uncertainty caused by the lack of information and its reliability due to technical, economic, social and other reasons;

2) uncertainty caused by the behavior of the external environment iSf the study;

3) uncertainty generated by a large number of objects or elements included in a situation;

4) "fundamental" uncertainty caused by the lack of material on the topic of research.

The presence of a number of situations with a varying degree of uncertainty in a research requires a certain mathematical apparatus to describe it. The task of removing uncertainty becomes more complicated if it is necessary to evaluate the effectiveness of management actions in a system according to several efficiency criteria [1,2].

2. Problem statement

Multi-criteriality is a direct consequence of the incompleteness of information about a situation, determined by a combination of factors that need to be considered when forming and finding effective solutions in the complex interaction of the system under study with the external environment [3,4,5]. It is precisely because of the lack of sufficiently complete and reliable information necessary for making effective decisions that it is impossible to unambiguously determine the nature of the operations being conducted, which necessitates the development of an analytical apparatus optimizing the decision taken on several performance indicators taking into account possible changes in the environmental conditions of a system. An analysis of decision-making methods under conditions of uncertainty and methods for solving multi-criteria tasks [6, 7, 8, 9] allows us to formulate a comprehensive method for removing uncertainties in solving such tasks, which allows us to increase the reliability of decisions made in information situations typical of optimal resource allocation to objects of different importance in the context of incomplete information.

3. Research questions

Let us form a multi-criteria task of optimizing the generation and (or) choice of effective solutions. It is characterized by three main terms: variety of possible solutions; variety of informational states; efficiency of any decision for each information state.

Let us accept the following notation:

m – number of possible options for an action;

n – number of relevant criteria;

a_{ij}– efficiency of the i-th action for the j-th criterion, $i= \overline{1, m}, \ j = \overline{1, n}$.

Then the matrix of efficiencies of various actions looks like:

$$\|a_{ij}\| = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & & & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}.$$

The zoning method consists in dividing the set of possible states of "nature of factors" or "nature" into subsets of dominance of individual actions. In a number of cases, the use of this method does not allow to obtain the desired optimal solution, although it significantly simplifies its search. This circumstance determined the direction of development of this method in a number of ways. Let us formulate the main principles of the zoning method according to the principle of hierarchical ratio of probabilities of possible states of the external environment [10,11].

1. Zoning is an inverse parametric linear programming problem, therefore, to obtain the desired solution, it is important to choose the zoning method.

2. The matrix game with "nature" is reduced (subject to continuity of the function of performance indicators from changing the nature state vector) to a linear vector optimization problem and vice versa. Consequently, to solve games with nature, one can use vector optimization methods, and multi-criteria problems can be solved using the apparatus of game theory with "nature" in many cases.

3. In transition from a multi-criteria problem to problems of games with "nature", the probabilities of the states of nature p_j are adequate to the coefficients of relative importance of the criteria c_j, i.e. $p_j \equiv c_j$.

4. The method is aimed at researching the phenomena of external environment of a study, which are caused by individual information situations.

5. Zoning should be carried out not according to the principle of dominance of individual actions, but according to the principle of maintaining a given hierarchical ratio of possible states of "nature".

4. Research methods

Let the distribution of coefficients of relative importance (CRI) for certain efficiency criteria be subject to restrictions

$$0 \leq c_j \leq 1, \quad j = \overline{1,n}, \quad \sum_{j=1}^{n} c_j = 1, \tag{1}$$

that is, determined by a set of $(n-1)$ independent values.

The distribution field will correspond to the set of distributions in the form of a rectangular unit hypertetrahedron in the $(n-1)$ dimension space $\sum_{j=1}^{n-1} c_j$. In the Cartesian coordinate system $c_1, c_2 \dots, c_{n-1}$ such a hypertetrahedron is the result of the intersection of a positive hyperoctant with a hyperplane, which cuts off a unit on each of the coordinate axes. Let us consider a situation where there are grounds for location of these coefficients in a sequence, in which the following condition is fulfilled

$$c_1 \geq c_2 \geq \cdots \geq c_i \geq \cdots \geq c_{n-1} \geq c_n \tag{2}$$

The total number of sequences of this type for the distributions of system (1) is determined by the number of permutations $n!$ When $n = 3$, the field of distribution of coefficients of relative importance degenerates into a right-angled triangle with single legs (Figure 1). The number of subsets, each of which has its own ratio between coefficients of relative importance of the indicators, is $P_3 = 3! = 6$.
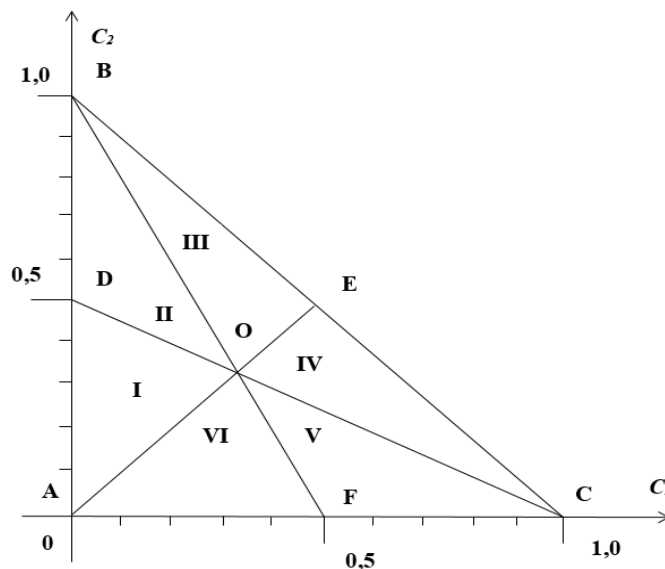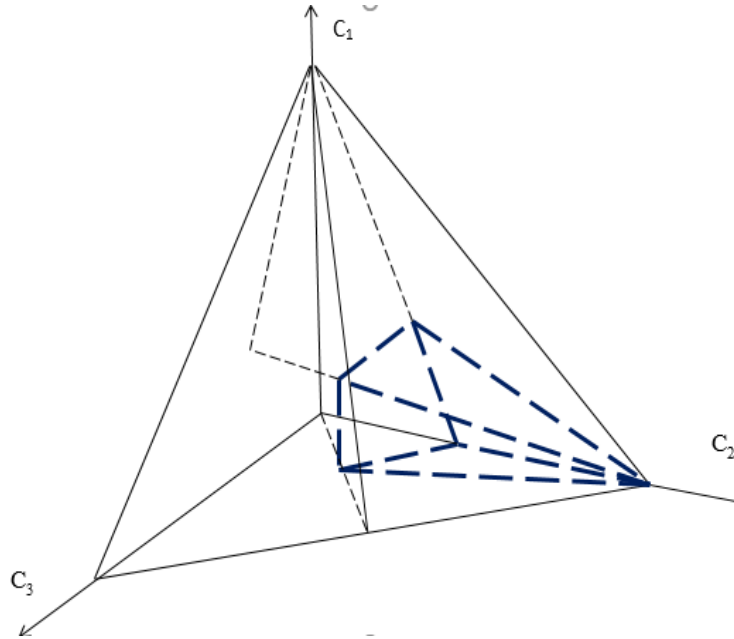

Figure 1. Coefficient distribution field C_i, $P_3 = 3! = 6$.

Let us look at Figure 1. The axes of the coordinates show the values of the coefficients c_1 and c_2, and the right triangle ABC reflects the distribution of the coefficients described by the system

$$0 \leq c_i \leq 1; \quad i = 1,2,3; \quad c_1 + c_2 + c_3 = 1, \tag{3}$$

That is, all possible solutions of system (1) are located inside the triangle ABC and on its sides AB, BC and CA. In this triangle there are medians AE, BF and CD, which break the triangle ABC into $3! = 6$ triangles (AOD, DOB, BOE, EOC, COF and FOA). When $n = 4$, (Figure 2) the number of subsets, each of which has its own ratio between coefficients of relative importance of the indicators, is $P_4 = 4! = 24$.

Figure 2. Coefficient distribution field C_i, $P_4 = 4! = 24$

Let us return to Figure 1. It is easy to show that the triangles obtained in the figure are equal to each other, and the area of each of them is equal to 1/12. At the same time, the area of the large triangle is 1/2. Point O is the center of the triangle and divides each median in the ratio 1:2. Table 1 presents the equations of the sides and medians of the ABC triangle, and in Table 2, each of the six subsets is matched with its own distribution of CRI indicators.

Table 1. Equations of sides and medians of triangles.

Triangle segments	Equations of segments
Side AB	$c_2 + c_3 = 1$; $c_1 = 0$
Side AC	$c_1 + c_3 = 1$; $c_2 = 0$
Side BC	$c_1 + c_2 = 1$; $c_3 = 0$
Median AE	$c_1 = c_2$; $c_1 + c_2 + c_3 = 1$
Median BF	$c_1 = c_3$; $c_1 + c_2 + c_3 = 1$
Median CD	$c_2 = c_3$; $c_1 + c_2 + c_3 = 1$

Table 2. Geometric field of the distribution of CRI.

Subset	Triangle	Coefficient ratio
I	AOD	$c_1 < c_2 < c_3$
II	DOB	$c_1 < c_3 < c_2$
III	BOE	$c_3 < c_1 < c_2$
IV	EOC	$c_3 < c_2 < c_1$
V	COF	$c_2 < c_3 < c_1$
VI	FOA	$c_2 < c_1 < c_3$

Let us analyze the contents of these tables. The segment AB is described by the equation $c_2 + c_3 = 1$, with $c_1 = 0$. In the same format, the remaining sides of the large triangle are described. Median AE corresponds to the equation $c_1 = c_2$, while $c_1 + c_2 + c_3 = 1$. Similarly, the remaining medians are described. From Table 2 it can be seen that each of the six possible subsets is assigned its own distribution of coefficients of importance. For example, all possible solutions of a system of equations and inequalities

$$0 \leq c_i \leq 1 \,; i = 1,2,3 \,; c_1 + c_2 + c_3 = 1 \,; c_3 \leq c_2 \leq c_1 \qquad (4)$$

are in subset IV, i.e. in the area of the triangle EOC. Point (O) has coordinates $c_1 = c_2 = c_3 = 1/3$.

The approach described above allows us to formulate an algorithm for choosing the optimal variant of the desired solution (D_i), which is implemented using the zoning method according to the principle of hierarchical ratio of probabilities of possible external environment states:

1. The relative importance of indicators C_j is ordered in the form of a sequence (2)
2. For each compared variant i a linear programming problem is solved:

$$\begin{cases} D_i = \sum_{j=1}^{n} a_{ij} c_j \to max, \\ \sum_{j=1}^{n} c_j = 1, \ 0 \leq c_j \leq 1, c_j \geq c_{j+1}, j = \overline{1, n-1}. \end{cases} \qquad (5)$$

Formulation of this task makes it possible to obtain a number of analytical solutions, in particular:

$$c_j = \begin{cases} \frac{1}{k}, \text{if } j \leq k, \\ 0, \text{if } j > k, \end{cases} \qquad (6)$$

where the index k is determined from the condition $a_{kj} = \max_j a_{ij}$.

The principal difference of the developed method from the existing ones is the absence of a subjectively formalized connection between the obtained values of coefficients of relative importance according to individual criteria and for individual solution options [12].

Let us apply the developed method to solve the example

$$. \ A = \begin{pmatrix} 0,20 & 0,24 & 0,22 \\ 0,76 & 0,18 & 0,34 \\ 0,18 & 0,80 & 0,26 \\ 0,84 & 0,02 & 0,46 \end{pmatrix}. \qquad (7)$$

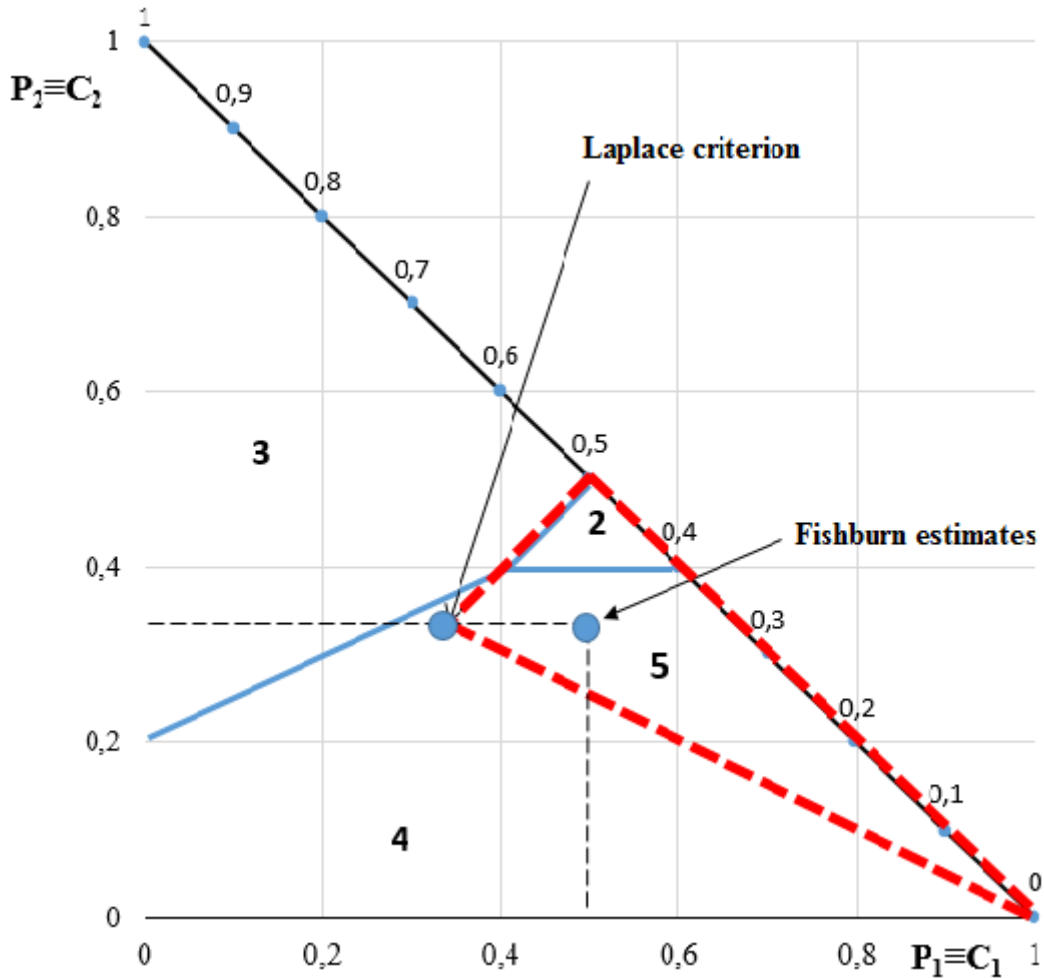Let us show a graphical solution of the problem (7) in Figure 3.

Figure 3. Graphical solution of problem (7).

Let us analyze Figure 3. The graphical solution of the end-to-end example allows you to compare the results of solutions obtained by various methods:

1) 2, 3 and 4 are areas of probability of having effective solutions when applying the zoning method based on the principle of dominance of individual actions;

2) 5 is the area of probabilities of the presence of effective solutions when applying the zoning method according to the principle of a given hierarchical ratio (IV) of possible states of nature (for n=3);

3) The point of "Fishburn estimates" is the point corresponding to an effective solution obtained from the Fishburn weights system;

4) The "Laplace criterion" point is the point corresponding to an effective solution obtained by applying the single-component Laplace criterion.

The result obtained (the method developed) coincides with solutions acquired in case of applying the Fishburn estimation method and also when the Laplace criterion is applied. In this case, the value of an effective solution in quantitative estimates is maximum in case of applying the zoning method according to the principle of a hierarchical ratio of probabilities of possible external environment states. Table 3 compares the results of applying various methods of removing uncertainty [13,14,15].

Table 3. Comparison of results of applying various methods for removing uncertainty.

Decision making method	Variant of solution	Efficiency of solution
Wald criterion	1	0,200
Savage criterion	2	0,620
Hurwitz criterion for an optimistic estimate of $\lambda = 0,8$ and pessimistic estimate ($\lambda = 0,3$)	3	0,676 (0,366)
Laplace criterion	4	0,440
Fishburn estimates	4	0,503
Modification of the zoning method	4	0,840

5. Findings

To solve the problem stated, a mathematical apparatus has been used, based on the theories of probability, linear algebra and mathematical programming. Obtaining an optimal variant of the desired solution is achieved by solving a linear programming problem for each variant being compared. The developed method will have the greatest efficiency when solving problems with a relatively small number of possible states of the research environment, but there are no significant limitations on the number of possible states of the external environment. The developed method solves an important scientific problem related to finding solutions to vector optimization problems, namely, problems solved by several efficiency criteria. This approach to the zoning method, namely the choice itself of the zoning method allows us to bring maximum removal of uncertainty and clarification of information about the probabilities of the states of factor nature to a new qualitative level, that is, it represents an objective apparatus for making a subjective decision.

The principal advantages of the developed method are:

1. Absence of a formalized relationship between the CRI obtained for individual criteria and different solutions.

2. To obtain a solution to the problem of making a decision on several efficiency criteria, it is sufficient to establish the order of priority between them.

3. The resulting solution is the maximum possible in quantitative estimates, taking into account the initial values of efficiency indicators for the criteria under consideration.

4. The apparatus for obtaining the weight of a criterion is formalized, that is, it is objective in terms of obtaining the weight of an individual criterion for each individual solution.

Thus, according to the principle of a hierarchical ratio of probabilities of possible external environment states, the zoning method allows us to build an algorithm that provides an optimal solution to the problem if there is minimal information about the states of "nature" or environmental factors of the study. The scope of the developed zoning method is quite wide. Let us list only some of the possible directions:

1. games with nature;

2. vector optimization (multi-criteria tasks);

3. any tasks in which a solution is sought on the basis of expert assessments;

4. tasks based on the idea of ranking;

5. tasks of optimal allocation of resources to objects of various importance;

6. problems of optimal distribution of homogeneous or heterogeneous resources in conditions of incomplete information;

7. parametric programming problems;

8. tasks of determining the optimal structure of required resources (material, financial, etc.).

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