Optimization of Enterprise Activity

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Abstract. This paper dwells upon the use of optimization measures proposed by the theory of the securities portfolio (SP). The uncertainty of what consequences production planning may have makes imperative to take sound decisions by means of technologies tested in well-developed financial markets. This study adapts the principles behind EGP index models to managing a 'multiprofile' enterprise. The principles rely not only on the profitability of goods and its fluctuations, but also on the production costs. Enterprise activity optimization boils down to managing the product line. In the second part of the paper, the built models are tested by applying fuzzy sets. The profitability-to-risk ratio is a fuzzy function. When the function reaches its extreme, the enterprise is run optimally. The paper presents empirical evidence of how this model works. It shows how the proportions of goods in a company's line can be optimized. The theoretical data is further compared against empirical data.

1. Introduction

An industrial enterprise should, among other things, boost its sales by optimizing them. Optimizing the product line is of particular interest. Given that such enterprises offer a wide range of products, systematizing and optimizing the stocks might be challenging. Storage conditions, delivery methods, etc. impose multiple restrictions. Despite this, each item in the product catalog can be associated with specific profitability and costs. This is a fundamental provision that helps formulate a whole new approach to optimization.

This study seeks to optimize the product line by finding the most reasonable breakdown of the product range. The first part simulates a 'multiprofile' enterprise. It is proposed to make up the product line by applying the EGP model under the optimal SP theory [1–3]; the model is essentially about finding the maximum profitability-to-risk ratio when optimizing the product line. Note that maximizing the profitability and minimizing the risks is impossible without such studies [4].

Unfortunately, lack of data on the actual value of goods means this data cannot be stochastic, and the above methods do not apply. This is why the second part of the paper dwells upon how to address such problems by fuzzy sets. The mathematical foundations of the fuzzy set theory were mainly laid in the second half of the 20th century [5-7].

Interval analysis is an efficient tool for testing mathematical models in the light of uncertainty. While first created for the needs of computational mathematics, it has been made active use of in the control theory [8-11], operation research [12,13], and game theory [14,15].

Fuzzy sets have been in use in economic since the late 1970s. Notable is the monograph [16] that presents a variety of applications for the fuzzy set theory. Most notable are triangular fuzzy numbers. Some papers apply this methodology to investment problems [16-20]. Triangular numbers can be used to quantify qualitative expert assessments.

This paper covers a situation where the value of goods is rendered as triangular fuzzy numbers. Optimizing an enterprise effectively boils down to finding the extreme of some triangular fuzzy function. The paper concludes with an empirical evidence of how this model works. It shows how the proportions of goods in a company's line can be optimized. The theoretical data is further compared against empirical data.

2. Enterprise optimization model

Consider a manufacturing enterprise. Assume that the enterprise itself does not influence the pricing, is confident that the entire product stock can be sold, and seeks to optimize its product line. The product line is defined herein as the ratio of item-specific product quantities on the catalog.

Let us adapt the well-known EGP method to the situation under consideration [4]. Consider products as shares, i.e. divide the entire output into shares. This helps optimize the production planning for maximum profitability.

Find the profitability S_{it} of the ith item, $i=1,...,n$ at time t, $t=1,...,T$, T is the number of observations in the sample; apply the formula:

$$S_{it} = R_{it} - f_i(k_i)Q_t.$$

where $R_{it} = \left(D_{i,t+\Delta t} - D_{it}\right)/D_{it}, Q_t = \left(C_{t+\Delta t} - C_t\right)/C_t, D_{it}$ is the unit price of the ith item at time t, C_t are production costs at time t, k_i is the share of the ith item in the total product, $f_i(k_i)$ is a coefficient that describes the share of costs of making the i th item. Assume that $f_i(k_i)$ depends on k_i, $i=\overline{1,n}$,,, while R_{it} and Q_t are independent random variables. This means that at time t, the profitability equals

$$P_t = \sum_{i=1}^{n} k_i S_{it} = \sum_{i=1}^{n}\left(k_i R_{it} - k_i f_i(k_i)Q_t\right).$$

Mathematical expectation of profitability $\bar{P} = E(P_t)$ and standard profitability deviation $\delta_P^2 = E\left(\left(P_t - \bar{P}\right)^2\right)$ can be calculated by the standard method. This means the securities portfolio theory methods can be applied to optimize an enterprise, see [5-6]. In particular, it becomes clear that the best profitability-to-risk ratio can be found by maximizing the function $\theta = \left(\bar{P} - R_0\right)/\delta_P$ [4], where R_0 corresponds to such $\bar{R}_i$, for which the variance is zero.

Optimizing this function boils down to finding such coefficients k_i that correspond to the maximum of the function

$$q = A^{-\frac{1}{2}}\left(\sum_{i=1}^{n} k_i\left(\bar{R}_i - R_0 - f_i(k_i)\bar{Q}\right)\right), \quad A = \sum_{i=1}^{n} k_i^2\delta_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} k_i k_j \delta_{ij} + \delta^2\sum_{i=1}^{n} k_i^2 f_i^2(k_i).$$

Assume that the functions $f_i(k), i=1,...,n$ are differentiable. Find the partial derivatives of the function θ by the desired parameters, then equate them to zero. This generates a system of n equations to find the coefficients k_i.

$$\frac{\partial\theta}{\partial k_s} = \left[\sum_{i=1}^{n} k_i\left(\bar{R}_i - R_0 - f_i(k_i)\bar{Q}\right)\right]\cdot\left[-\sum_{\substack{j=1\\ j\neq s}}^{n} k_j\delta_{sj} - k_s\delta_s^2 - \frac{\delta^2}{2}\left(k_s^2 f_s^2(k_s)\right)'_{k_s}\right]\cdot A^{-\frac{3}{2}} +$$

$$+A^{-\frac{1}{2}}\left(\bar{R}_s - R_0 - \left(k_s f_s\left(k_s\right)\right)'_{k_s} \bar{Q}\right), \ s=1,\dots,n. \qquad (1)$$

By finding the special form of the functions $f_i\left(k\right)$, one can study the system (1) and find the optimal solution $k=\left(k_1,\dots,k_n\right)$. In particular, assuming that $f_i\left(k\right)=a_i, i=\overline{1,n}$, $k=\left(k_1,\dots,k_n\right)$ can be found by a linear system

$$\sum_{\substack{j=1\\ j\neq s}}^{n} z_j\delta_{sj} + z_s\delta_s^2 + \delta^2 z_s a_s^2 = \bar{R}_s - R_0 - a_s\bar{Q}, s=1,\dots,n,$$

where z_s and k_s are linked by the ratio $z_s=\lambda k_s$, $\lambda=\sum_{i=1}^{n} z_s$.

3. Fuzzy sets for enterprise optimization

Processes in markets for goods are often volatile and heterogeneous. Uncertainty is integral to market pricing. This leads to a logical conclusion that the correlation between various good-specific profitabilities cannot be described statistically. Triangular fuzzy numbers are a good descriptor that takes into account the temporal fuzziness of the source data. To that end, modify the known securities portfolio structure optimization model [4] so as to apply the fuzzy set theory.

Let three parameters of the i th item be defined for the time interval $[t,t+\Delta t]$: the value as of opening the bidding (S_{it}^{o}), maximum value ($S_{it}^{\max}$), and minimum value ($S_{it}^{\min}$), $i=1,\dots,n$, $t=1,\dots,T$. Let the profitability at time t be a triangular fuzzy number, $\tilde{R}_{it}=\left\langle R_{it}^{\min}, R_{it}, R_{it}^{\max}\right\rangle$, where

$$R_{it}^{\max}=\frac{S_{it}^{\max}-S_{it}^{o}}{S_{it}^{o}}, \quad R_{it}^{\min}=\frac{S_{it}^{\min}-S_{it}^{o}}{S_{it}^{o}}, \qquad R_{it}=\frac{R_{it}^{\max}-R_{it}^{\min}}{2}.$$

Given a history of value of these goods over some time, substitute the profitability for each specific time to find a fuzzy random profitability value. Mathematical expectation and variance are defined for fuzzy random values similarly to ordinary random values [16]. This helps find the profitability $m_p=\left\langle m_p^{\min}, m_p, m_p^{\max}\right\rangle$ and the risk $d_p=\left\langle d_p^{\min}, d_p, d_p^{\max}\right\rangle$.

To optimize the product line, maximize the function

$$\tilde{\theta}=\frac{\tilde{m}_p}{\tilde{\delta}_p}=\left\langle \frac{m_p^{\min}}{\delta_p^{\max}}, \frac{m_p}{\delta_p}, \frac{m_p^{\max}}{\delta_p^{\min}}\right\rangle=\left\langle \theta_{\min}, \theta, \theta_{\max}\right\rangle.$$

Move to a levels and solve the systems

$$\sum_{\substack{j=1\\ j\neq s}}^{n} z^q{}_j\delta_{sj}^q\left(\alpha\right)+z^q{}_s\delta_s^{q2}\left(\alpha\right)+\delta^{q2}\left(\alpha\right)z^q{}_s a^{q2}{}_s\left(\alpha\right)=\bar{R}_s^q\left(\alpha\right)-R^q{}_0\left(\alpha\right)-a^q{}_s\left(\alpha\right)\bar{Q}^q\left(\alpha\right), s=1,\dots,n, \qquad (2)$$

to find the corresponding values of $q^q\left(a\right)$. Here, the index q should be omitted once, then assume $p=\min$ and $p=\max$.

This approach effectively generates a membership function $m_{\tilde{q}}$ for the maximum value of the function $\tilde{q}$. Thus, the solution comprises such values of the inputs that correspond to the maximum of this membership function.

4. Implementation

Let us apply this model to empirical evidence so as to show how a mechanical engineering enterprise could be optimized. The Company produces more than 250 items. Production data is sourced from [21]. Divide the entire product line into 9 Product Families, items within each family having similar specifications. Families are numbered. Calculations below represent an attempt to optimize the Company's activity for April 2018. Assume that T = 28. Prices for products in each family are fuzzy numbers, as they depend on multiple parameters and terms of transactions.

Construct systems type (2) for all $a = 0{,}85 + 0{,}01(j - 1)$, $j = \overline{1{,}15}$; solve them. This returns a membership function for the function $\tilde{\theta}$. See Figure 1.

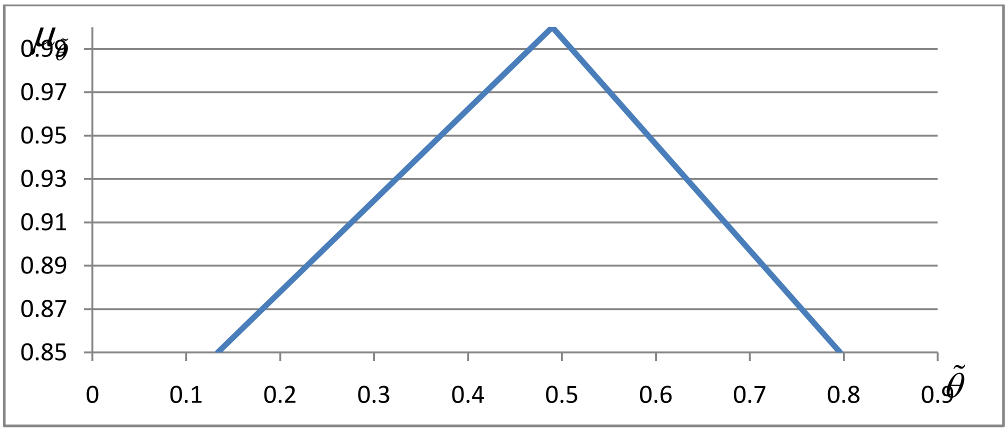

Figure 1. Membership function for the production performance criterion.

The maximum of the membership function corresponds to the best value of $\theta^* = 0{,}48963$. This means we can clearly define such product line that will match this θ^*.

Table 1 below shows the optimal breakdown of the pump lineup. It also presents the actual planned production figures.

Table 1. Shares of items.

Family	Actual share	Optimal share
1	0.036	0.103
2	0.016	0.108
3	0.004	0.075
4	0.004	0.027
5	0.002	0.075
6	0.059	0.068
7	0.022	0.079
8	0.797	0.295
9	0.059	0.208

The data clearly shows the enterprise is boosting its Product Family 8. The model shows the most optimal solution is to develop all product families with a focus on 8 and 9. Production planning will thus help maximize profitability, while Families 8 and 9 will remain a priority.

5. Conclusions

Thus, this research effort has effectively produced an enterprise optimization model. The presented method takes into account fuzzy data to maximize the company's profits. Evidence presented prove the model feasible. Besides, the method is clearly adaptable to even more restrictions or to greater timeframes.

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