Human Resource Management in a Project Co-Investment Model Under the Conditions of the Multiple Interests of the Participating Agents

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Abstract. Many practical tasks of economic activity and a number of important issues of economic theory are associated with tasks of determining the optimal solution. This article proposes a game-theoretic version of the transport problem with changing conditions, that is, a problem in which the parameters of the system depend on time. For this model, the problem of finding the compromise values of the income functions of all participants and the corresponding sequence of control sets that implement the compromise trajectory is solved. The dynamic programming approach in this case is that the problem being solved "plunges" into a wider class of problems described by a number of parameters, and after that, using the principle of optimality, the main recurrent relation is determined based on the definition of a compromise set. When a compromise is found at each stage, an optimal trajectory emerges on our tree, the movement along which gives a compromise in the entire multi-step problem.

1. A formal problem statement

Consider a deterministic model, that is, a model in which the system goes from one state to another when a control parameter is selected with a probability equal to 1. Several employers are organizing the process of providing vacancies to a certain number of labor resources. At the initial moment of time, for employers, the types of labor resources and their quantities, the costs of their maintenance are defined, and for labor resources – the prices and utility functions of vacancies. The general view of the network, the capacity functions at its edges and the cost of moving a unit of labor resources along the edges are also considered defined. Thus, we have a system $\Omega = \{ N, M, \sigma_i, \beta_i, \alpha_j^s, K_{(x,y)}, d_{(x,y)}, \Theta_s(\delta_j^s) \big| i = \overline{1,N}; j = \overline{1,M}; s = \overline{1,N}; (x,y) \in R \}$, where N – is a number of employers, M - number of vacancies, σ_i - amount of labor i type of production, δ_j^s - labor needs s in j at the point of production, β_i - the cost of maintaining a unit of labor resources in the i production point, α_j^s - unit cost of labor s at the j consumption point, $K_{(x,y)}$ - edge bandwidth *(x,y)*, d_{xy} - cost of moving a unit of labor resources along the edge *(x,y)*. For the player $r = \overline{1, N+M}$ at each moment of time $t = \overline{0, T-1}$ is determined by the set of controls

$$U_r^t = \{u_{l_r}^t\}, \qquad (1)$$

where the power of the set $|U_r^t| = p_r^t$, $l_r = 1,2,...p_r^t$ is the number of elements of the set U_r^t, r - is the player number. It is necessary to determine the trade-off values of the player's income functions and the sequence of control sets that implements the trade-off trajectory. At the 1st step, each player independently chooses his own control. It turns out a set of controls

$$u_{q_1}^1 = (u_1^1,.....u_{N+M}^1),\ u_{q_1}^1 \in U^1 = \prod_{r=1}^{N+M} U_r^1, \qquad (2)$$

where $U^1 = \{u_1^1,...,u_{z_1}^1\}$ - many possible sets of player controls, z_1 - power set U^1, q_1 - number of sets of controls on the 1st step, i.e. $q_1 \in \overline{1, z_1}$, according to which the model parameters change. By the beginning of the next step, depending on the departments selected at the previous step, the employers' capacities, the cost of maintaining a unit of labor resources, the price of labor resources at consumption points, the carrying capacity of ribs and the cost of moving a unit of labor resources change. Consequently, the entire system goes into a new state in which players again solve the problem of choosing control parameters. Thus, a dynamic problem arises, which is solved by the dynamic programming method.

2. Mathematical formulation of the problem

Let the set of employers be *I*. Enumerate them by index $i = \overline{1, N}$, *I={i}*. There is a set of manpower resources *J*. Enumerate them by index $j = \overline{1, M}$, *J={j}*. The process of economic relations between employers and human resources takes place over T periods of time $t = \overline{0, T}$. The amount of labor resources σ_i^0 and the cost of maintaining a unit of labor resources β_i^0 are also initially known is. The throughput $K_{(x,y)}^0$ and the cost of moving a unit of labor $d_{(x,y)}^0$ are considered to be given in the first stage for each edge of the network. Thus, the initial state of the system Ω can be designated as

$$\Omega^0 = \{(N, M, \sigma_i^0, \beta_i^0, {}^0\alpha_j^s, K_{(x,y)}^0, d_{(x,y)}^0, \Theta_s(\delta_j^s)) \big| i = \overline{1, N};\ j = \overline{1, M}; s = \overline{1, N}; (x, y) \in R\} \qquad (3)$$

Further, in the description $\Omega^t, t = \overline{1, T}$ of the system Ω states the number of vacancies M, the number of vacancies N and the utility functions $\Theta_s(\delta_j^s)$ will not be included, since they do not depend on the choice of the set of controls. The income functions of players are set as follows. For employers:

$$H_{A_i}^t(u_{q_1}^t) = \sum_{j=1}^{M} ({}^t\delta_j^s\, {}^t\alpha_j^s) - \sigma_i^t \beta_i^t - \gamma^t(p, \delta), \qquad (4)$$

where q_t - control dialing number at time *t*, $\sum_{j=1}^{M} ({}^t\delta_j^s\, {}^t\alpha_j^s)$ - the set of controls number at the time *t*, $\sigma_i^t \beta_i^t$ - labor costs at time *t*, $\gamma^t(p, \delta)$ - the cost of transporting goods at a time *t*, depending on the path of movement *p* and on the amount of labor resources δ transferred.

For labor resources:

$$H_{B_j}^t(u_{q_1}^t) = \sum_{s=1}^{N} \Theta_s({}^t\delta_j^s) - \sum_{s=1}^{N} {}^t\delta_j^s\, {}^t\alpha_j^s, \qquad (5)$$

where $\Theta_s\, {}^t\delta_j^s$ - the utility of the number of *s* labor resource *s*-x for the *j* consumer at the time *t* and is set by means of the function, ${}^t\delta_j^s\, {}^t\alpha_j^s$ - the cost of purchasing δ_j^s units *s* labor resources by the *j* consumption point at the price α_j^s at the time *t*.

Along with the income functions for each player at each moment of time $t = \overline{0, T-1}$ a set of controls is defined $U_r^t = \{u_{l_r}^t\}$, where $r = \overline{1, N+M}$ - is the player number, $l_r = 1,2,..,p_r^t$ - is the number of the element of the set U_r^t, p_r^t - is the power of the set U_r^t. In this problem, the sets of controls have the following form. For employers, these are all kinds of routes for moving labor resources to all places of work, i.e.

$W_i^t = \{(A_i \to B_1, ...A_i \to B_M)\}$, where $i = \overline{1, N}$, $t = \overline{0, T}$. (6)

Each element of the set W_i^t must satisfy the edge bandwidth limitations. For labor resources, the set of departments V_j^t in each period of time will consist of different orders of each type of labor resources, i.e.

$V_j^t = \{(\delta_j^1, ...\delta_j^s)\}$, where $j = \overline{1, M}$. (7)

Each element of the set V_j^t must satisfy the conditions:

$\sigma_i^t = {}^t\delta_1^s + ...+ {}^t\delta_M^s$, где $s = \overline{1, N}$, $i = \overline{1, N}$, $t = \overline{0, T}$.

(8)

Every player $r = \overline{1, N+M}$ chooses an element from his set of controls U_r^t. If controls are chosen by players and a common set of controls is obtained:

$u_{q_t}^t = \{(A_i \to B_1, ...A_i \to B_M), ..., (\delta_j^1, ...\delta_j^s)\}$, (9)

where $i = \overline{1, N}$, $j = \overline{1, M}$, $t = \overline{1, T}$, then the system Ω goes from the state $\Gamma_{q_{t-1}}^{t-1}$ to the next state $\Gamma_{q_t}^t$. Then the players get their winnings, determined by the formulas (7), (8) and depending on the period and the chosen control. Thus, sorting through all possible controls of this kind and obtaining in accordance with them the values of the function of income of the players at each stage, we find a compromise income of the players and the corresponding set of controls. To find the compromise values of the income functions of the players and the corresponding sequence of control sets for the system Ω, that is in the state Ω^0, in T steps, it is necessary to use the principle of dynamic programming. We denote the total income of the system Ω, which is in the state Ω^0 in T steps in

$R^T(\Omega^0, \Omega_{q_1}^1, ...\Omega_{q_T}^T) = H^T(u_{q_1}^1, u_{q_2}^2, ..., u_{q_T}^T)$, (10)

where Ω^0 - the initial state of the system Ω, $\Omega_{q_t}^t$ - the status of the system Ω by the number $q_t \in \overline{1, z_t}$ in the moment of time *t*, $u_{q_t}^t$ - a set of controls under the number $q_t \in \overline{1, z_t}$ at time *t*, according to which the system Ω goes from the state $\Omega_{q_{t-1}}^{t-1}$ to the state $\Omega_{q_t}^t$. Total revenue depends on the states in which the system was located at each time point $t = \overline{0, T}$. Then we construct the Bellman function, the values of which reflect the system's revenue Ω^0 in T steps when implementing a compromise sequence of control sets.

$$C^T(\Omega^0) = \underset{u_{q_1}^1, u_{q_2}^2, ..., u_{q_T}^T}{comp} R^T(\Omega^0, \Omega_{q_1}^1, ...\Omega_{q_T}^T), \quad (11)$$

where Ω^0 - the initial state of the system Ω, $\Omega_{q_t}^t$ - the state of the system Ω under the number $q_t \in \overline{1, z_t}$ at the time *t*, $u_{q_t}^t$ - number of controls $q_t \in \overline{1, z_t}$ at the moment of time *t*, whereby the system Ω goes from the state $\Omega_{q_{t-1}}^{t-1}$ to the state $\Omega_{q_t}^t$, $t = \overline{0, T}$, *comp* - is an operator of computation of a compromise point. Consider the space of all models with a fixed set of controls and variable income functions. In each such model, the only compromise point is chosen. Thus, we obtain the operator *comp* on the space of all models. Now for the Bellman function, we construct the recurrence relation.

$$C^T(\Omega^0) = \underset{u_{q_1}^1}{comp}[H^1(u_{q_1}^1) + C^{T-1}(\Omega_{q_1}^1)], \quad (12)$$

where $C^T(\Omega^0)$ - compromise revenue for the system Ω, that is in the state Ω^0 *at t = 0* in T steps, *comp* is the operator of the computation of a compromise point, $H^1(u^1_{q_1})$ is the player's income at the first time point when the total number of controls q_1 is implemented, $C^{T-1}(\Omega^1_{q_1})$ is the compromise income for the system that is in the state $\Gamma^1_{q_1}$ in the next *(T-1)* step. Another technique for describing this game is that it indicates which moves players can make and what the size of the payments can be at each step.

3. Algorithm of the decision

The solution algorithm consists of (*T + 1*) steps.

Step 1. Consider all possible states of the system Ω at *t=T-1* - Ω^{T-1}. Using the recurrence relation $C^T(\Omega^0) = \underset{u^1_{q_1}}{comp}[H^1(u^1_{q_1}) + C^{T-1}(\Omega^1_{q_1})]$ we find a compromise income for all possible system states Ω^{T-1} for the last step as follows:

$$C^1(\Omega^{T-1}_{q_{t-1}}) = \underset{u^T_{q_T}}{comp}\, H^T(u^T_{q_T}).$$

Step 2. Consider all possible states of the system Ω at *t=T-2* - Ω^{T-2}. For them we will find a compromise income for the last two steps according to the formula:

$$C^2(\Omega^{T-2}_{q_{T-2}}) = \underset{u^{T-1}_{q_{T-1}}}{comp}[H^{T-1}(u^{T-1}_{q_{T-1}}) + C^1(\Omega^{T-1}_{q_{T-1}})]$$

Step T. Consider the initial state Ω^0 of the system Ω. For it we find a compromise income for *T* steps

$$C^T(\Omega^0) = \underset{u^1_{q_1}}{comp}[H^1(u^1_{q_1}) + C^{T-1}(\Omega^1_{q_1})].$$

Step (T+1). Substituting the obtained relations one into another, we obtain a compromise income of players for the entire *T* step process and a sequence of sets of controls that implement a compromise trajectory on the game tree.

4. Conclusions

The paper proposes a game-theoretic version of the transport problem of integer programming. A dynamic model is built for this option. For a dynamic model, an algorithm is presented for finding the compromise income of players and the corresponding sequence of sets of controls that implement a compromise trajectory on the game tree.

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References

[1] Kriulina E, Tarasenko N, Miroshnitchenko N, Zaitseva I, Dedyukhina I 2016 Environmental Management in Agriculture: Problems and Solutions, Research Journal of Pharmaceutical *Biological and Chemical Sciences* **7(3)** 1908-1912

[2] Zaitseva I, Popova M 2013 Technique to study the employment potential of the region: economic-mathematical aspect *World Applied Sciences Journal* **22(1)** 22-25

[3] Zaitseva I, Kriulina E, Ermakova A, Shevchenko E, Vorokhobina Y 2016 Application of Factor Analysis to Study the Labour Capacity of Stavropol Krai *Research Journal of Pharmaceutical, Biological and Chemical Sciences* **7(4)** 2183-2186

[4] Zaitseva I, Popova M, Ermakova A, Bogdanova S, Rezenkov D 2016 Determination Prospects Of Development Labor Potential In Agriculture Stavropol Territory Based On Assessment His Condition *Research Journal of Pharmaceutical, Biological and Chemical Sciences* **7(3)** 2592-2595

[5] Zaitseva I, Malafeyev O, Strekopytov S, Ermakova A, Shlaev D 2018 Game-theoretical model of labour force training *Journal of Theoretical and Applied Information Technology*

[6] Zaitseva I, Malafeyev O, Strekopytov S, Bondarenko G, Lovyannikov D 2018 Mathematic model of regional economy development by the final result of labor resources *AIP Conference Proceedings, American Institute of Physics*

[7] Zaitseva I V, Ermakova A N, Shlaev D V, Shevchenko E A, Lugovskoy S I 2017 Workforce planning redistribution of the region's results *Research Journal of Pharmaceutical, Biological and Chemical Sciences* **8(1)** 1862-1866

[8] Kostyukov K, Zaitseva I, Bondarenko G, Svechinskaya T, Nechayeva S 2016 Workforce Planning as An Element of Control System *Research Journal of Pharmaceutical, Biological and Chemical Sciences* **7(6)** 2315–2319

[9] Malafeyev O A, Redinskikh N D, Nemnyugin S A, Kolesin I D, Zaitseva I V 2018 The optimization problem of preventive equipment repair planning *AIP Conference Proceedings 1978* 100013 https://doi.org/10.1063/1.5043757

[10] Malafeev O, Nemnyugin S 1996 Generalized dynamic model of a system moving in an external field with stochastic components *Theoretical and Mathematical Physics* **107(3)** 770

[11] Malafeev O 1995 On the existence of nash equilibria in a noncoope rative n-person game with measures as coefficients *Communications in Applied Mathematics and Computational Science* **5(4)** 689-701

[12] Malafeyev O, Awasthi A, Zaitseva I, Rezenkov D, Bogdanova S 2018 A dynamic model of functioning of a bank *AIP Conference Proceedings, American Institute of Physics*

[13] Malafeyev O, Rylow D, Zaitseva I, Popova M, Novozhilova L 2017 Game-theoretic model of dispersed material drying process *AIP Conference Proceedings, American Institute of Physics*

[14] Malafeyev O, Saifullina D, Ivaniukovich G, Marakhov V, Zaytseva I 2017 The model of multi-agent interaction in a transportation problem with a corruption component *AIP Conference Proceedings, American Institute of Physics*

[15] Malafeyev O, Farvazov K, Zenovich O, Zaitseva I, Kostyukov K, Svechinskaya T 2018 Geopolitical model of investment power station construction project implementation *AIP Conference Proceedings, American Institute of Physics*

[16] Malafeyev O, Awasthi A, Kambekar K, Kupinskaya A 2019 Random walks and Market Efficiency in Chinese and Indian equity markets K S Statistics *Optimization and Information Computing 7(1)* 1-25

[17] Zaitseva I, Malafeyev O, Kolesin I, Ermakova A, Shlaev D 2018 Modeling of the labour force redistribution in investment projects with account of their delay IEEE International Conference on Power, Control, Signals and Instrumentation Engineering ICPCSI 2017 68-70

[18] Pichugin Y A, Malafeyev O A, Rylow D, Zaitseva I 2018 A statistical method for corrupt agents' detection *AIP Conference Proceedings* https://doi.org/10.1063/1.5043758

[19] Malafeyev O, Rylow D, Zaitseva I, Ermakova A, Shlaev D 2018 Multistage voting model with alternative elimination *AIP Conference Proceedings* http://doi.org/ 10.1063/1.5043756

[20] Kriulina E N, Tarasenko N V, Miroshnitchenko N V, Zaitseva I V, Dedyukhina I F 2016 Environmental management in agriculture: Problems and solutions *Research Journal of Pharmaceutical, Biological and Chemical Sciences* **7(3)** 1908-1912