# The Critical Level of Intervention in Controlling Militant Group Related to Terrorism

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**ABSTRACT**
Nowadays the word terrorism is often heard and becomes popular in many societies. It is connected with a militant group in human population. Terrorism is "the use of force or threat to weaken morals, intimidate and conquer". In broad terms, this is considered as the use of deliberate discriminatory violence as a means of creating fear among people to achieve financial, political, religious or ideological goals. In the case of ideological goals, spreading the ideology is considered among the main course that should be done to achieve those goals. Mathematically, spreading ideology can be viewed analogously as a process of infection or disease transmission in epidemiology. In this paper we review a current model of this ideology transmission from a person to another. We define the critical level of intervention that should be done derived from the basic reproduction number of the transmission process. Some numerical examples are presented and reveal some insight on how to control the number of the militans based on plausible assumptions.
*Keywords: terrorism, mathematical model, equilibrium solution, basic reproduction number, CLoI intervention*

## 1. INTRODUCTION

We give a brief review of a SEIR model for the growth of terrorist population. Recently [1] proposes a mathematical model of terrorism. He discusses the dynamical behavior of a mathematical model specifically on the network of militants. The model is governed by a system of differential equations describing the growth of four human subpopulations. The subpopulations are the potential class militants (with the size at time $t$ is denoted by $P(t)$), the occasional militants (with the size at time $t$ is denoted by $O(t)$), the militants (with the size at time $t$ is denoted by $M(t)$), and the number of quit militans at this time (with the size at time $t$ is denoted by $Q(t)$). Using these variables, the model is given by

$$\frac{dP(t)}{dt} = \lambda - aP(t)M(t) - \alpha P(t), \quad (1)$$

$$\frac{dO(t)}{dt} = aP(t)M(t) - (\beta + b)O(t), \quad (2)$$

$$\frac{dM(t)}{dt} = bpO(t) - (\gamma + c)M(t), \quad (3)$$

$$\frac{dQ(t)}{dt} = b(1-p)O(t) + cM(t) - \delta Q(t) \quad (4)$$

with the description of parameters are given in Table 1.

**Table 1 Symbols And Descriptions**

| Symbol | Description |
|---|---|
| $a$ | transmission coefficient from potential class (the posit value of a increases by increasing the activities of militants) |
| $b$ | transition rate from the occasional class militants in which the flow from occasional class to militants is $bpO$ and to quit ones is $b(1 - p)O$ |
| $c$ | rate at which militants quit militancy |
| $p$ | proportion of transition rate from the occasional class militants to militants, $p \in [0; 1]$ |
| $\alpha$ | natural death rates of potential class |
| $\beta$ | natural death rates of occasional class |

| Symbol | Description |
|--------|-------------|
| $\gamma$ | natural death rates of militants class |
| $\delta$ | natural death rates of quit class |
| $\lambda$ | the constant rate at which the potential class increases due to births and migration |

[a.] Taken from Hussain (2019)

The model resembles the spead of disease in epidemiology of SEIR type - susceptible, exposed, infected, and recovered [2], giving up smoking dynamic [3], and the spread of malware in computers [4, 5] The author of the model [1] obtains the basic reproductive number, which is given by

$$R_0 = \frac{ab\lambda p}{\alpha(\beta + b)(\gamma + c)} \qquad (5)$$

and analyze the resulting equlibrium points in relation to this basic reproduction number. An equilibrium solution is a solution whose derivative is zero everywhere. Visually an equilibrium solution looks like a horizontal line. He found two equilibrium solutions: the trivial solution and the non-trivial one. The trivial equilibrium solution that he called a "*militant free stage*", given by $E_0 = \left(\frac{\lambda}{\alpha}; 0; 0; 0\right)$, and the non-trivial equilibrium point $E_1 = \left(P^*; O^*; M^*; Q^*\right)$ in term of the basic reproductive number, given by

$$E_1 = \left(\frac{\lambda}{\alpha R_0}; \frac{\alpha(\gamma+c)}{abp}(R_0 - 1); \frac{\alpha}{a}(R_0 - 1); \frac{\alpha(\gamma + c - \gamma p)}{\delta ap}(R_0 - 1)\right).$$
$$(6)$$

The author also identifies conditions on the parameters of the model in which any intervention managed by the policy makers would minimize the number of militants. In this paper we propose another measure that would guarantee the extinction of the the militants based on the basic reproduction number of the model.

## 2. THE CRITICAL LEVEL OF INTERVENTION

By observing the model proposed by the author in [1] in epidemiological context, it is interesting to rise a question like "to what extent a vaccination effort is needed to eliminate an infected group?" (an analogous question in terrorism is "to what extent an intervention is needed to eliminate the militants group?"). The authors in [6] propose the critical level of vaccination based on the fundamental theorem in epidemiology related to the basic reproduction number. In this paper we show the existence of a "*Critical Level of Intervention*" (*CLoI*) that enables to eliminate the militants group is given by the relation

$$CLoI > \frac{ab\lambda p - \alpha(\beta + b)(\gamma + c)}{ab\lambda p} = v^* \qquad (7)$$

The relation can be easily obtained by looking at the formula in the "*militant occurence stage*" $E_1$. All the components except the first one are the function of the basic reproduction number in the form of $R_0 - 1$. Certainly all the components are positive provided $R_0 > 1$. Other argument is by assuming that if there is an intervention at the rate of v then the effect is the reduction of the rate of transmission from $aP(t)M(t)$ to $a(1-v)P(t)M(t)$. by assigning $M^* \leq 0$ and doing some algebraic manipulation to solve $v$, we will arrive to the critical level of intervention $v^*$. Furthermore we explore this critical level of intervention in various ways and found some insight on how to manage the population to the desired state.

## 3. NUMERICAL EXAMPLES

In this section we present some numerical examples regarding the solution of the above system of equations. We will consider the following data sets: Data set 1 is taken from Hussain (2019) and data set 2 is the modification of the data set 1 for certain reason.
Data set 1:
$a = 1.5765 \times 10^{-5}; b = 0.20635; c = 0.0022; \lambda = 0.015; \alpha = \beta = g = \delta = 0.008;$
$p = 0.7; P(0) = 439,140; O(0) = 9,000; M(0) = 1,300; Q(0) = 560$.
Data set 2:
$a = 1.5765 \times 10^{-3}; b = 0.20635; c = 0.0022; \lambda = 0.015; \alpha = 0.08;$
$\beta = g = \delta = 0.0008; p = 0.7; P(0) = 500; O(0) = 100; M(0) = 10; Q(0) = 0$
.

The solution of the system is sought by the third order Runge-Kutta method. Figures 1 to 3 show the solution from the data set 1. The transient solution is shown in Figure 1. The long-term solution in Figure 3 shows that all subpopulations almost reach the equilibrium solution. In this case, the data set 1 gives rise to $R_0 = 0.02$ which predicts that the militant group will dissapear eventually as shown in the figure.
The resulting critical level of intervention (equation 5) for this data set is -511. The negative sign means that there is nothing to do to lower the basic reproduction number, since it has already below one. The low basic reproduction

number indicates the eventual dissapearence of the militants as shown in Figure 3.

Looking at only long-term solution can be misleading, since prior to the dissapearence of the militants, there might be a huge number of them around in the system as shown in Figure 2. The initial number of militants is 1,300 and the eventual number of them (without any government intervention) is nearly zero. However, at certain time there is a peak of militants with more than 250,000 peoples. This is among the weaknesses of the critical intervention level. We will address this weakness elsewhere. In this paper we will only focus on the effect of this critical intervention level in the long-term behaviour of the system.
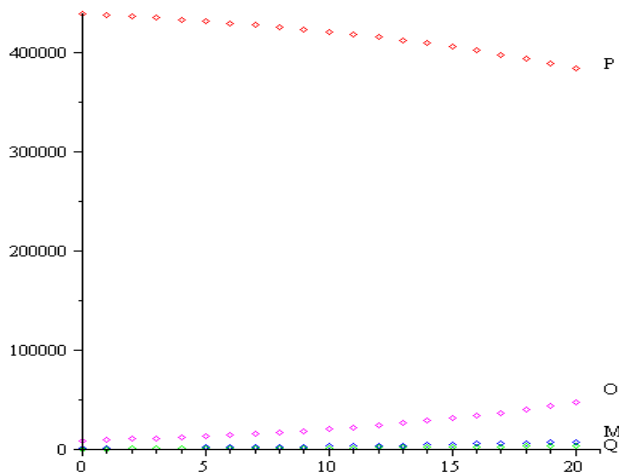


**Figure 1** Plot of subpopulation sizes P, O, M, and Q with data set 1 and time horizon 20 unit time. In this figure and in the subsequent figures the horizontal axes is time and the vertical axes is the number of individuals in the respective subpopulation.
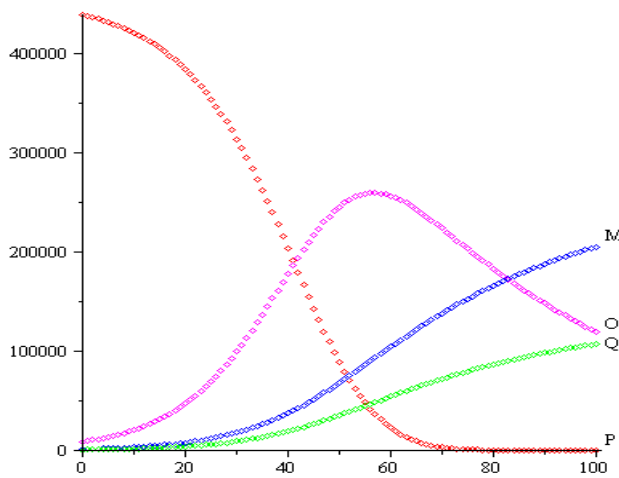


**Figure 2** Plot of subpopulation sizes P, O, M, and Q with data set 1 and time horizon 100 unit time.
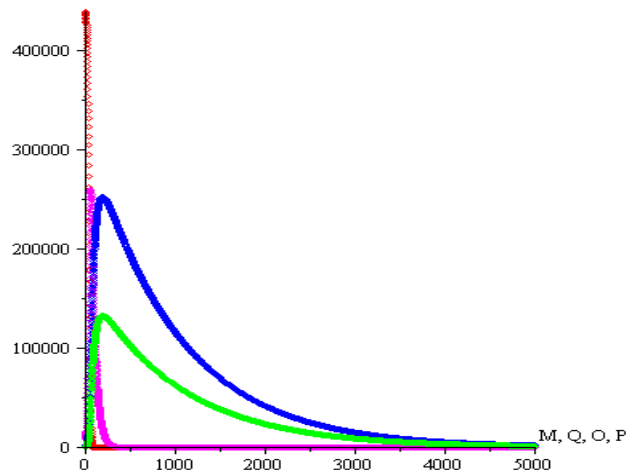


**Figure 3** Plot of subpopulation sizes P, O, M, and Q with data set 1 and time horizon 5,000 unit time. All the subpopulations approach the equilibrium points.

The data set 2 is a hypothetical example just to illustrate the method. The data is created in purpose to produce the large basic reproduction number so that it is larger than one. The basic data is from data set 1 with some parameters are modified for that purpose. The resulting basic reproduction number is $R_0 = 6.87$, roughly says that one militant produces almost 7 militants during his/her life time.

Considering the behaviour of the solution, dynamical system theory points out that eventually there always some number of militants exist in the system, due to the stability of the non-trivial equilibrium state. In this case there will be nearly 30 militants in the long-term (see equation 6 for $M^*$). The significance of our formula in (7) is directed to reduce this number of militants in the long-term down to zero. Theoretically this is possible regardless the time needed to achieve it.
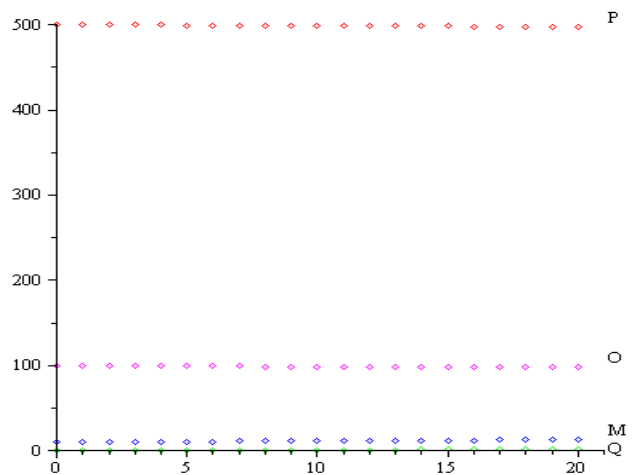


**Figure 4** Plot of subpopulation sizes P, O, M, and Q with data set 2 and time horizon 20 unit time.
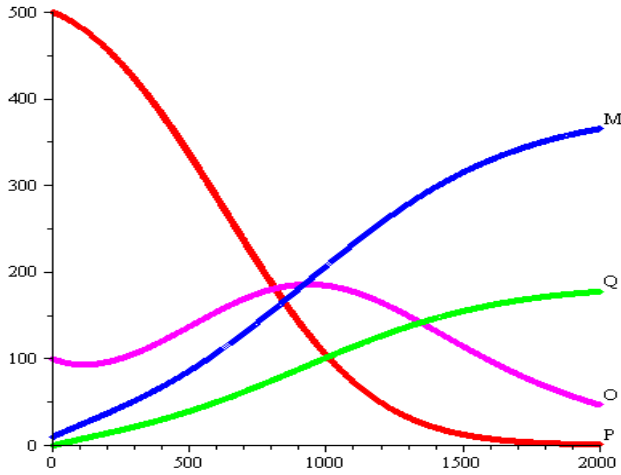
**Figure 5** Plot of subpopulation sizes P, O, M, and Q with data set 2 and time horizon 2,000 unit time.
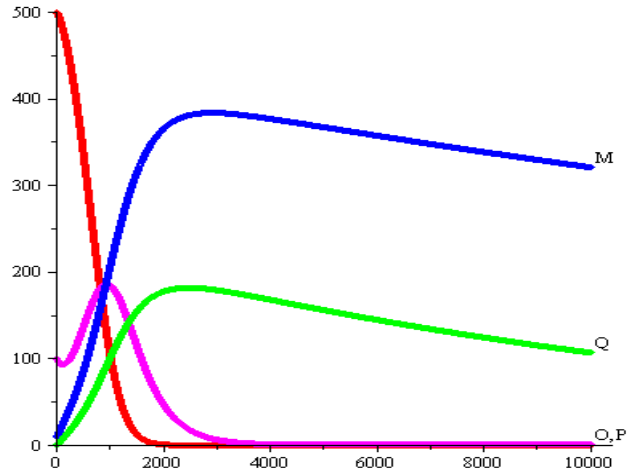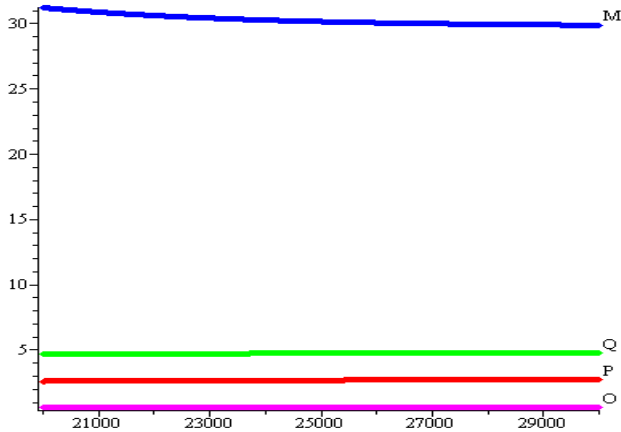


**Figure 6** Plot of subpopulation sizes P, O, M, and Q with data set 2 and time horizon 30,000 unit time.

The critical intervention level that guaranted to eliminate them (say making them quit from the terrorism activity) is 0.854 (see equation 7). The number can be interpreted as a nonviolence intervention, such as 86% (larger than 0.854) of general population should be exposed to a governmental campaign or education regarding anti-terrorisms. We have done a simulation on how to reduce this population number by applying this 86% intensity of intervention. The results are presented in Figures 4 to 10.

Figures 4, 5, and 6 show the short-term, middle-term, and long-term solution of the system without intervention for data set 2. Figure 6 shows that there will be about 30 militants eventually. Figures 7 to 9 show the effect of intervention in reducing the eventual number of militants. We redraw the graph as in the case of Figure 5 but with the window from t=0 to 10,000 in Figure 7. In the case of data set 2 as used in Figure 7, the basic reproduction number $R_0 = 6.87$ and the $CLoI = 0.854$.
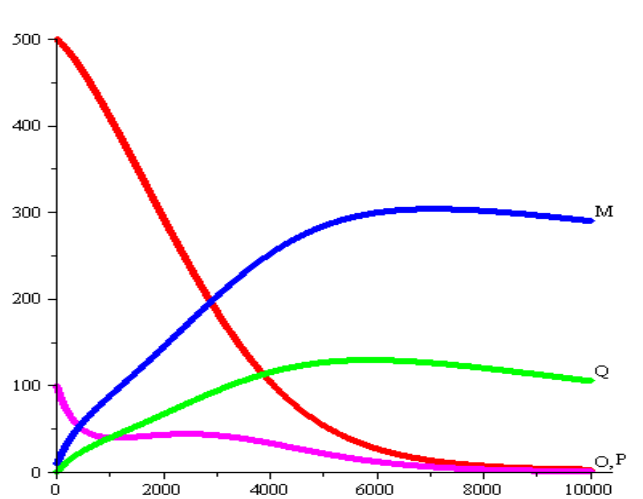


**Figure 7** Plot of subpopulation sizes P, O, M, and Q with data set 2 and time horizon 10,000 unit time without any intervention.



**Figure 8** Plot of subpopulation sizes P, O, M, and Q with data set 2 and time horizon 10,000 unit time with the intervention 0.86 (larger than the CLoI=0.854).
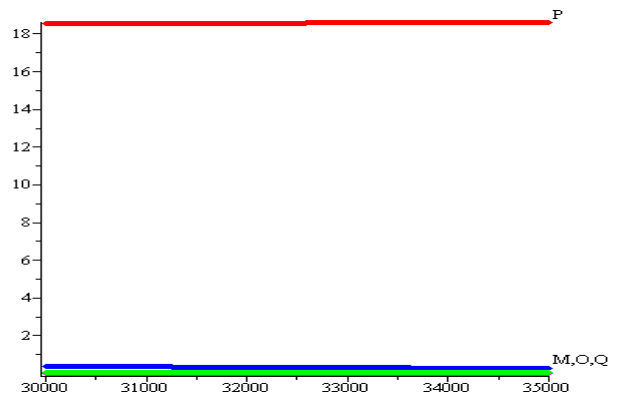


**Figure 9** Plot of subpopulation sizes P, O, and Q with data set 2 and time horizon 35,000 unit time with the intervention 0.86 (larger than the CLoI=0.854).
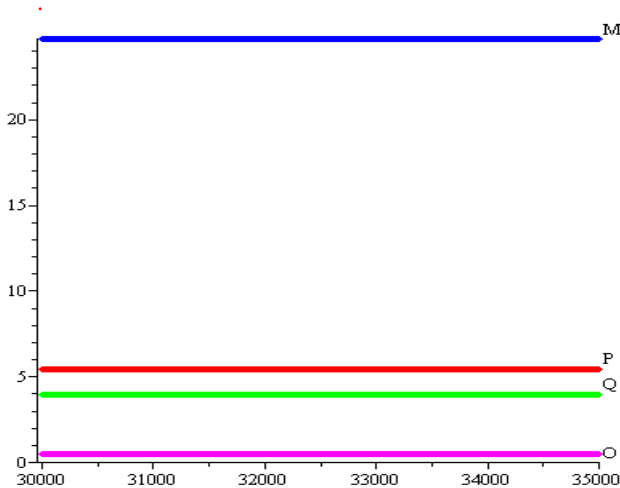
**Figure 10** Plot of subpopulation sizes P, O, M, and Q with data set 2 and time horizon 35,000 unit time with the intervention  0.5b(smaller than the CLoI=0.854).

Figure 8 shows the solution when 86% of general population (larger than the CLI=0.854) are exposed to an intervention, such as governmental campaign or education regarding anti-terrorisms. Compared to Figure 7, the peak of the militant subpopulation is lower and eventually the population goes to zero (Figure 9). However if only 50% are exposed to the intervention, then Figure 10 shows that the militants subpopulation is endemic. Those figures reveal the effects of the critical level of intervention in the transmission of terrorism ideology.

## 4.  CONCLUSION
In this paper we discussed a model of millitancy transmission within a population. The model was inspired by the SEIR model in Mathemaatical Epidemiology. We proposed a critical level of intervention (CLoI) that theoretically enable to reduce the number of millitants down to zero. For other models of transmission, reader may see at [7]. The drawback of the theory is that the *CLoI* based on the long-term solution, the number of militants indeed tends to zero regardless the time needed to approach this zero equilibrium solution. In the case of a quick solution needed, the method should be modified. Other method such as optimal control theory can also be explored if cost of intervention is considered.

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