# Preliminary Study of Cognitive Obstacle on the Topic of Finite Integral Among Prospective Teacher

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**ABSTRACT**
Cognitive obstacle are one of the four types of obstacles, three of which are: ontogenic obstacle, didactive obstacle, and epistemological obstacle. Obstacles are all things that hinder students' progress in learning. Cognitive obstacles are part of a students knowledge at a time that is generally reliable in solving a particular problem but this knowledge is then inadequate when dealing with a new problem. Cognitive cbstacles are the knowledge that a person possesses and are generally sufficient in solving a particular problem, settling in the mind, but when faced with a new problem, this knowledge is inadequate and difficult to adapt. Obstacles arises from the fact that certain concepts have a degree of complexity and necessary to recognize them in a particular order. For example, fractions are more complicated than all numbers, the student experience with operations on integers leads to the implicit nature that "multiplications make numbers larger" leading to a cognitive obstacle when encounter multiplications fractions less than one. Cognitive obstacles basically seem to be ontogenetic, epistemological, didactic and even cultural, according to their origins and the way they develop. The cognitive obstacles of Mathematics Education Students of Universitas PGRI Semarang were found when subjects solved the finite integral problem and assumed that the integral was integrated, whereas if the function in the integral is a function that can not be integrated.
*Keywords: obstacles, cognitive obstacle, finite integral, improper integral*

## 1. INTRODUCTION

Solso[1] reveals that thinking is the process of generating new mental representations through information transformation involving complex interactions between mental attributes such as judgment, abstraction, reasoning, imagination and problem solving. Thinking as a mental activity to assist formulate or solve a problem, make a decision, or fulfill the curiosity desire (Ruggiero, 1998)[2]. A mathematical truth is developed based on logical reasoning, but the work of mathematics involves observing, guessing, guessing, making and testing hypotheses, seeking analogies, connecting and communicating, making representations, generalizing, proving theorems, and solving problems. In problem solving activities, a person sometimes front obstacles.

Cornu[3] stated that cognitive obstacle are part of the student's knowledge, this knowledge at one time is generally reliable in solving certain problems but this knowledge is then inadequate when front with new problems. Tall[4] argues that cognitive obstacles are something that results from the tightly held belief in mathematics that is rarely easy to remove from the mind. Obstacles are the knowledge that a person possesses and are generally sufficient in solving a particular problem, settling in the mind, but later, when front with a new problem, this knowledge is inadequate and difficult to adapt. For example, fractions are more complicated than all numbers, the student experience with operations on integers leads to the implicit nature that "multiplications make numbers larger," leading to a cognitive obstacle when individuals encounter fewer than one fractions.

Cognitive obstacle are a product of previous experience and internal processes of students from these experiences and are manifested when students experience difficulty in the learning process, the tendency to rely on misleading intuitive experiences, and the tendency to generalize Cornu[3], Radovan[5] states that cognitive obstacles are situations in which a student is confronted with mathematical content that contains known and unknown sections of material content. Cognitive obstacles are a way of thinking about mathematical structures or objects that fit in a situation but are not suitable in other situations (Tague, 2014)[6]. The cognitive obstacle in this study is that one's previous

experience poses an obstacle to the person in solving the problem.

Herscovics [7] believes that learning difficulties have two basic types: learners' efforts to map new material into existing mental structures are valid in certain domains but do not match the knowledge learned, and the second inherent structure of new material may be like students who does not have a mental structure that allows to assimilate new material. Investigation of cognitive barriers carried out by Yoshida [8] on the concept of a minority of elementary school students. Fraction is a concept that is considered difficult for elementary school students, especially on the topic of fraction order and similarity. Another study conducted by (Nyikahadzoy, 2013) [9] investigated cognitive obstacle in "Upper Level" high school students in understanding the subject matter of equality. Students are given problems sketching graphs y = | x-1 | -2 and y = - | x-1 | It turns out students sketched the graph y = | x-1 | -2 starts by drawing a graph y = | x-1 | resulting from the graph y = | x | then shifting one unit to the left obtains the graph y = | x-1 | -2 students shift down as far as two units.

## 1.1 Obstacle

Stenberg[10] states that cognitive is a mental process or activity of the mind in receiving, learning, remembering and thinking of information. While the definition of obstacle is a state that prevents a person from reaching a problem (Cornu[3], Tall[4], Brousseau[11]) .

Cornu[3] revealed there are four obstacles experienced by a person that are cognitive obstacles occur when students experience difficulty in the learning process; genetic and psychological obstacles occur as a result of a student's personal development; didactical obstacles occur because of the nature of the teaching and/or the teacher; epistemological barriers occur because of the nature of the mathematical concepts themselves.

## 1.2 Cognitive Obstacle

The idea of cognitive obstacles was first introduced in the context of the development of scientific knowledge by Herscovics[7] and Cornu[3]. Cognitive obstacles are the product of prior experience and the students' internal processes of these experiences and manifested when students have difficulty in the learning process (Cornu, 1991)[3] . According to Radovan[5], a situation in which a student is confronted with mathematical content containing known and unknown pieces of material content is a cognitive obstacle. Nyikahadzoy[9] revealed that a cognitive obstacle is a way of thinking about mathematical structures or matched objects in a situation but not appropriate in other situations. So the term cognitive impediment is defined as any cause of stagnation or inaction in finding, finding, knowing, and understanding information / knowledge in solving problems.

To solve a mathematical problem, the student must perform more and more complex mental activities (thinking) than the mental activities he/she performs when solving non math problems. Probem solving in mathematics is an activity to find solutions of mathematical problems encountered by using integratively all the stock of mathematical knowledge

already owned or it can be concluded that problem solving is a multi step process in which the problem solvers must find the connection between past experience and scheme and the problems at hand and then act on solutions.

Cognitive obstacle to problem solving can be assessed under the assimilation and accommodation framework. Eraslan[12] explains that when a person interacts with the environment there will be cognitive processes of assimilation and accommodation. The process of assimilation occurs when a person brings new knowledge into their existing scheme and the accommodation process occurs when a person changes their scheme to match new knowledge (Eraslan, 2005)[12]

## 2. METHODOLOGY

This research is descriptive explorative research and using design qualitative research to explore the obstacle of difficulties associated with integral problems that have upper and lower limits. Explorative design is one of the designs where the main emphasis is on getting ideas and insights (Paul, 2014)[13]. Exploratory research is conducted to provide a better understanding of the situation and is not designed to get an answer or final decision but to generate hypotheses and explanations of what is going on in a situation.
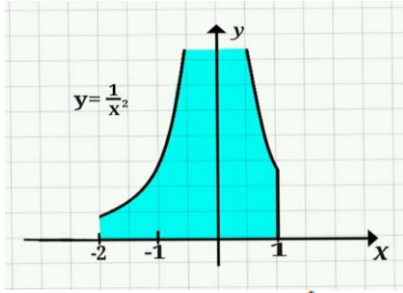
The research subjects were taken from twenty nine students of the Mathematics Education Program at the University of PGRI Semarang who had taken the Integral Calculus course. Cognitive obstacle faced by students in solving integral problems are seen based on students 'written answers and interviews to dig deeper into students' thinking processes. The research instrument used in this study is an Integral problem. During the interview, students will be able to improve their work as long as there is awareness of them to do it. If the verification results are the same as the results of their written work, then this student is chosen as the subject. Indicators of cognitive obstacle in this study are if the ability of students who were previously well controlled, unable or difficult to adapt to solve the given problem. Following are the problems given to students:

If possible, calculate the integral $\int_{-2}^{1} \frac{1}{x^2} \, dx$

## 3. RESULT AND DISCUSSION

Researchers made initial observations on mathematics students who have taken Integral Calculus course in Universitas PGRI Semarang. On the observation, the researchers saw the cognitive obstacle that occurred. Furthermore, researchers continue the research on twenty nine mathematics education students to see how cognitive obstacle faced by students in solving the problem. The researcher gives one problem as the initial observation and and one problem as the research instrument. Students' work results will be analyzed and selected two answers from students who have good communication skills. Here is a problem as an instrument of this research.

If possible, calculate the integral $\int_{-2}^{1} \frac{1}{x^2} \, dx$

Picture 1. Graphic $\frac{1}{x^2}$

The following answers from subjects 1 and 2:



Picture 2. Answer Subject 1



Picture 3. Answer Subject 2

From the answer of the above subject, it appears that the first and second subject does not notice that the given integral is an improper integral because at the inner point (ie zero) the integral is infinite, although the answer to the first subject (S1) seems correct. But at a glance the picture above tells us that the integral value (if it exists) must be a positive number. It can also be seen from the graph that the area that is the

area $\frac{1}{x^2}$ is above the x axis. While the answer of the two subject (S2) made a mistake when calculating the value of x, ie insert value -2.

According to the Integrity Theorem: If $f$ is limited to [a, b] and continuous at that interval except at a finite number of points, then f is integrated in [a, b]. In particular, if f is continuous at all intervals [a, b], then f is integrated in [a, b]. While there is a definition which states that let f be continuous at half-open intervals [a, b) and assume that

$$\lim_{x \to \sim} |f(x)| = ~ \text{So} \int_a^b f(x)\ dx = \lim_{t \to b^-} \int_a^t f(x)\ dx$$

provided that this limit exists and is finite, where we say that the integral converges. Otherwise, we say that the integral is divergent. Thus the given finite integral is an intergal divergent. The knowledge that students have so far is to calculate an integral, of course, the first to do is to integrate, then enter the upper limit value minus the integral lower bound value. Based on the interview summary, both subjects were aware of the mistakes they made. The upper and lower limits on the integrals become obstacles in solving the given problem. This obstacle prevents a person from reaching a solution to a problem.

At the beginning of Integral Calculus learning has been delivered an finite integral definition $\int_a^b f(x)\,dx$ with $a, b \in \mathbb{R}$. According to the Law of Gravity, the force experienced by an object with a mass $m$ located at a distance $x$ (>R) from th center of earth is $(x) = \frac{GMm}{x^2}$. Then the work required to raise the object from the surface of the earth to the height of $\Delta h$ is

$$\int_R^{R+\Delta h} F(x)\ dx \approx \int_R^{R+\Delta h} \frac{GMm}{x^2}\ dx =$$
$$GMm\left[-x^{-1}\right]_R^{R+\Delta h} = GMm\left(\frac{1}{R} - \frac{1}{R+\Delta h}\right)$$

An object separated from the influence of gravity if $R \to \infty$. So the energy needed for it is

$$\lim_{\Delta h \to \infty} \int_R^{R+\Delta h} \frac{GMm}{x^2}\ dx = \lim_{\Delta h \to \infty} GMm\left(\frac{1}{R} - \frac{1}{R+\Delta h}\right) = \frac{GMm}{R} \dots \dots \dots \dots \dots (i)$$

Value of $\lim_{\Delta h \to \infty} \int_R^{R+\Delta h} \frac{GMm}{x^2}\ dx$ in equation (i) is abbreviated as $\int_R^{\infty} \frac{GMm}{x^2}\ dx$ and be named Improper Integral. This integral is called improper Integral because it differs from the previous integral at the beginning of the learning. $\int_a^b f(x)\,dx$ with $a, b \in \mathbb{R}$.

# 4. CONCLUSION

The cognitive obstacles face in the Mathematics Education Study Program students in solving integral problems are when applying the integral formulas they used to use before (the lack of required prerequisite knowledge that is still related to the problem), and the weak ability to extract

information from the problem being integrated is an unnatural integral with infinite limits). Cognitive obstacle experienced by S1 and S2 with different background skills, resulting in different cognitive obstacle. This is in step with the results of (Setiadi , 2017)[14]  that found six learning obstacles. First, the lack of spatial ability of students. Second, the missing of finite integral concept in teaching material and lecturer explanation. Third, the lack of student understands in prerequisites topic or concept. Fourth, sstudents are mistaken in understanding the requirements of functions can be integrated. Fifth, student using solve finite problem only if it was the same with the example problem. Sixth, lecturer did not involve their teaching with constructing concept activity. This study is limited to the exposure of cognitive obstacle faced by S1 and S2 as students who have been and are studying Integral Calculus courses. Further research can be done with different materials and wider subjects.

## ACKNOWLEDGMENT

## REFERENCES

[1]  Solso, Robert 1995 *Cognitive Psychology* Boston Allyn and Bacon.

[2]  Ruggiero, Vincent R. (1998). *The Art of Thinking. A Guide to Critical and  Creative Thought* New York: An Imprint of Addison Wesley Longman, Inc.

[3]  Cornu, B  1991 Apprentissage de la notion de limite: conceptions et obstacles Thèse de 3$^{me}$ Cycle, Grenoble.

[4]  Tall, David 1991   The Psychology of Advanced Mathematical  Thinking  Advanced  Mathematical Thinking,Kluwer: Holland 3-21

[5]  Radovan,   Antonijevic *Cognitive Activities in Solving Mathematical Tasks: The role of a Cognitive Obstacle* Eurasia Journal of Mathematics Science & Technology Education 2016 Vol 12(9) 2503-2515

[6]  [Tague, J., Baker, G. R. 2014. *Flipping the Classroom to Address Cognitive Obstacles* 12$^{st}$ ASEE Meeting Conference and Exposition American Society for Engineering Education, Indianapolis.

[7]  Herscovics, N. 1989. Cognitive obstacles encountered in the learning of Algebra. In: C. Kieran, & S.Wagner (Eds.), Research issues in the learning and teaching of algebra (pp. 60–86). Hillsdale, NJ: Lawrence Erlbaum Associates

[8]  Yoshida H.,   Sawano K. 2002 *Overcoming Cognitive Obstacles in learning fractions: Equal-partitioning and equal-whole* Japanese Psychological Research Vol 44 No 4 183–195

[9]  Nyikahadzoy,   M. 2013 Some Cognitive Obstacles Faced By 'A' Level Mathematics Students in Understanding Inequalities: A  Case Study of Bindura Urban High Schools. International Journal of Academic Research in Progressive Education and Development April 2013 Vol 2 No. 2

[10] Stenberg, Robert, J., 2012, *Cognitive Psychology*, Sixth Edition, Wardsworth, USA.

[11] Brousseau, G. 1997. *Theory of Didactical Situations in Mathematics*. Dordrecht, Netherlands: Kluwer.

[12] Eraslan, Ali. 2005.  A Qualitative Study: Algebra Honor Students' Cognitive Obstacles as They Explore Concepts of Quadratic Functions Florida State University Libraries

[13] Paul, Mutodi. 2014  The Nature of Misconceptions and Cognitive Obstacles Faced by Secondary School Mathematics Students in Understanding Probability: A Case Study of Selected Polokwane Secondary Schools Mediterranean Journal of Social Sciences MCSER Publishing Rome-Italy Mei 2014,Vol.5, No.8.

[14] Setiadi dkk. 2017 *Didactical Design Enrichment of Angle in Geometry* Journal of Physics: Conference Series Vol 895 conference 1 http://iopscience.iop. org/article/10.1088/1742-6596/895/1/012066

[15] Writer's  Handbook. Mill Valley,  CA: University Science, 1989.