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Option Pricing Approach and Its Application Based on Abel Equation

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Abstract: Financial derivative is one kind of financial instrument with high leverage ratio, high risk and high returns. Therefore, in it important to help investors to adopt appropriate risk control measures to judge the trend of the price change of financial derivatives. The trend of financial derivatives prices has been analyzed by solving the responding Abel equation. At last, we illustrate the scheme by testing several ETFs in the market for the system.

1. Introduction

Armengol(2012) used an extended complete Chebyshev system to give upper bounds for the number of isolated periodic solutions of some perturbed Abel equations[1].

$$x' = \frac{\cos t}{q-1} x^q + \varepsilon p_n \left(\cos t, \sin t\right) x^p$$

The latest research results belong to Alvarez (2020), he studied the existence of limit cycles for a family of generalized Abel equation[2].

$$x' = A(t)x^m + B(t)x^n, m, n > 2$$

The application of Abel Equation in finance is rare[3-10]. In this paper, we try to apply Abel Equation for the option pricing in financial derivatives with several ETFs in the market.

2. Model

In 2001, Yun Tianquan attempt to describe how the price of options changes by using ordinary differential equation[11]

$$h\dot{v}_0 = m_1 v_0^{-1} - n_1 v_0 + E \ (1)$$

Equation (1) can be turned into Abel equations of the first kind, after reciprocal transformation

$$y = \frac{1}{v_0(t)}$$

$$\dot{y} = a_3 y^3 + a_2 y^2 + a_1 y \ (2)$$

However, Yun Tianquan doesn’t consider the situation $\Delta = a_2^2 - 4a_1a_3 < 0$. In this paper, we discuss this kind of situation and its application.

In the equation (2), $a_1 = \frac{n_1}{h}$, $a_2 = -\frac{E}{h}, a_3 = -\frac{m_1}{h}$

If $\Delta = a_2^2 - 4a_1a_3 < 0$, Then (2) can be rewritten as the following form after formula in elementary mathematics.

$$\frac{1}{a_3} \int \frac{dy}{y\left[\left(y + \hat{s}\right)^2 + d^2\right]} = t + C \ (3)$$

$$\text{Where } \hat{s} = \frac{a_2}{2a_3}, d^2 = -\frac{\Delta}{4a_3^2} > 0 \ (4)$$

Let

$$\frac{1}{y\left[\left(y+\hat{s}\right)^2+d^2\right]}=\frac{B_1}{y}+\frac{B_2 y+B_3}{\left(y+\hat{s}\right)^2+d^2}$$

It is easy to have:

$$B_1=\frac{1}{\hat{s}^2+d^2},\ B_2=-\frac{1}{\hat{s}^2+d^2},\ B_3=-\frac{2\hat{s}}{\hat{s}^2+d^2}$$

Therefore, we obtain the analytic expression of integral (3):

$$\frac{B_1}{a_3}\ln y+\frac{B_2}{2a_3}\ln\left[\left(y+\hat{s}\right)^2+d^2\right]+\rho\arctan\frac{y+\hat{s}}{d}=t+C \quad (5)$$

Notice that $\rho=\left(B_3-B_2\hat{s}\right)/(2a_3 d)$.

3. Applications

The application of Abel equation in the financial field is given in the following special cases:

Case1： ρ=0 and B_1=-2B_2 , $B_2=2a_3$ let C=0 .At this point, the explicit expression of v_0 that can be solved by (5) :

$$v_0(t)=\frac{-\hat{s}+\sqrt{\hat{s}^2+(\hat{s}^2+d^2)(e^t-1)}}{\hat{s}^2+d^2}$$

Obviously, $v_0(t)$ is a monotonically increasing trend. As shown in Fig1(a), which shows the general form in this case.

(a) (b) (c)

(d) (e) (f)

Fig.1 Risen type with volatility

This model is mainly introduced by the fitting CSI 300 ETF of Huatai-Pine Bridge Investments. It was released on April 5, 2012, with a set of 32.97 billion copies. As of the latest trading day, November 1, 2019, closing price is 4.026, the unit net value is 4.0177, the premium rate is 0.21%, the amount of floating shares is 8.33billion, the turnover rate is 3.85% , and the number of listing shares is 9.12 billion. The main assets allocation is stocks accounting for 99.78%, cash accounting for 0.21%, other assets accounting for 0.01%, bonds accounting for 0%. Overall analysis shows that since the established day it has achieved a gain of 65.94%, which is higher than the average level of similar products. From the performance of each range, the fund has performed well this year, with a return of 33.34%, a one-year return of 26.41%, a three-year return of 22.98% and a five-year return of 69.47%. As shown in Fig.1(b), the Fund's five-minute K-line chart changed trend in 2019/10/9

9:35-2019/10/11 15:00, with an overall growth rate of 3.08%, in Fig.1(c) the Fund's sixty-minute K-line chart changed trend in 2019/8/6 10:30-2019/9/9 14:00, the overall change is consistent with the data model. From the perspective of risk control, the fund has a downside risk of 12.29% in the past year, which is higher than the average of the market. The annualized volatility is 19.47%, less than the average of the same kind. By the rank of the fund's downside risk and volatility among its peers shows that the fund has been at average risk over the past year. In addition, selecting some other funds , such as shown in Fig.1(d), CSI 500 ETF of China Southern Securities Investment in 2019/10/08 14:55-2019/10/11 13:55, 5-minute K-line chart, Fig.1(e) Gold ETF of E funds 2015/7/31-2016/7/15, weekly K-line chart, Fig.1(f) Gold ETF of Huaan 2019/3/29 15:00-2019/4/11 15:00, 30-minute K-line chart , the model fit the data very well.

Case2： ρ=0 and B_1=-2B_2 , $B_2=-2a_3$ let C=0 .At this point, the explicit expression of v_0 that can be solved by (5) :

$$v_0(t)=\frac{-\hat{s}+\sqrt{\hat{s}^2+(\hat{s}^2+d^2)(e^{-t}-1)}}{\hat{s}^2+d^2}, \qquad (t<\ln(1+(\frac{\hat{s}}{d})^2))$$

Obviously, $v_0(t)$ is a monotonically decreasing trend. As shown in Fig.2(a), which shows the general form in this case.

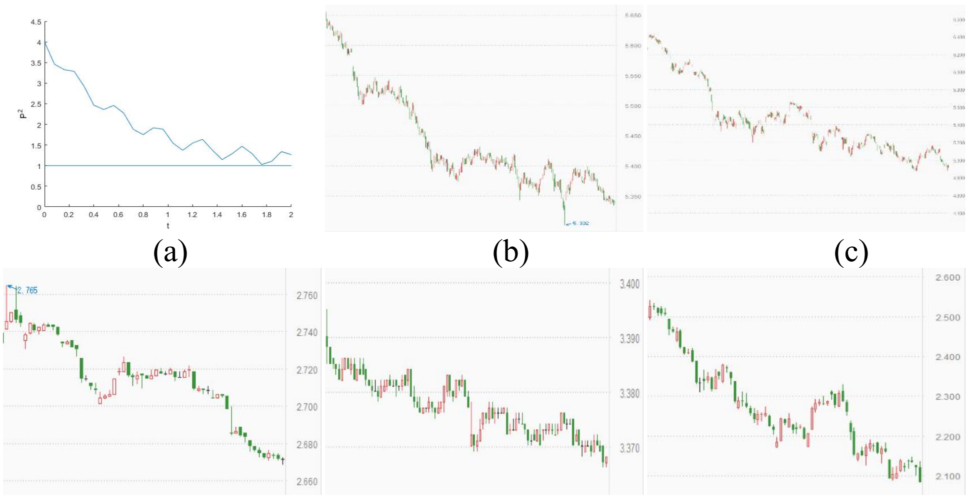

Fig.2 Decline type with volatility

This kind of model is mainly by fitting CSI 500 ETF of China Southern Securities Investment. The fund was issued on January 10, 2013, and established on February 6, 2013. It has a size of 1.15 billion pieces. As of the latest trading day, November 1, 2019, the closing price is 5.366, the unit net value is 5.3639, premium rate is 0.04%, the amount of floating shares is 7.98 billion, the turnover rate is 2.52%, and the number of listing shares is 8.252 billion. The asset allocation is 94.75% for stocks, 3.66% for cash, 1.56% for other assets and 0.04% for bonds. In the past year, the fund's downside risk is 15.02%, higher than the average of the market. The annualized volatility is 23.64%, higher than the average of the same kind. By the rank of the fund's downside risk and volatility among its peers shows that the fund was high risk over the past year. Since its establishment on February 6, 2013, the CSI 500ETF of China southern securities has gained 50.36%, lower than the average of the market. From the performance of each range, the fund has performed poorly this year, with a return of 20.20%, a one-year return of 17.07%, a three-year return of -21.30%, and a five-year return of 7.32%. For example, Fig2.(b) is the K-line chart of 2019/9/25 9:35-2019/10/08 14:20, the overall fluctuation continues to decline. Fig.2(c) is the K-line chart of 2018/5/22 14:00-2018/10/10 15:00 for 60 minutes, both of which are consistent with the characteristics of the data model. Select other ETFs in the market to fit the model, for example, Gold ETF of E funds in Fig2.(d) is the 60-minute K-line chart of 2018/10/25 10:30-2018/11/14 15:00, Fig.2(e) is the 5-minute K-line chart of 2019/9/16 9:35-2019/9/17 14:50, and Fig.2(f) is the daily K-line chart of CSI 300 ETF of Huatai-PineBridge Investmentsin2013/12/4-2014/3/20.It showed that the mathematical model fit well.

Case3： $\rho \neq 0$,and $B_1 = -2B_2$, $B_2 = 2a_3$, $B_1\rho < 0$. At this point, easy to prove by (5), exists $t=t^*$ which makes $v_0(t^*) = v_{\max}$ .That is, the maximum value of $v_0(t)$ is reached at a limited time. As shown in Fig.3, which shows the general form in this case.

(a) (b) (c)

Fig.3 Upward blow-up in a finite interval

Case4： $\rho \neq 0$,and $B_1 = -2B_2$, $B_2 = 2a_3$, $B_1\rho > 0$. At this point, easy to prove by (5), exists $t=t^*$ which makes $v_0(t^*) = v_{\min}$ .That is, the minimum value of $v_0(t)$ is reached at a limited time. As shown in Fig4, which shows the general form in this case.

(a) (b) (c)

Fig.4 Downward blow-up in a finite interval

Fig.3(b) is the K-line chart for 30 minutes in 2019/9/3 11:30-2019/7/29 14:30; Fig.3(c) is the K-line chart for 30 minutes in 2019/3/1 11:30-2019/4/1 14:30; Fig.4(b) is the weekly K-line chart in 2018/01/19-2018/11/23; Fig.4(c) is the daily K-line chart of 2016/9/21-2017/1/20

We mainly fit this model with SSE Pledge able Urban Construction Investment Bond ETF of Fortis Haitong. It was released on October 17, 2014 and the formal establishment date is November 13, 2014 with a total of 6.6878 billion shares. As of November 1, 2019, the latest trading day, the closing price is 96.96, the unit net value is 97.81, the discount is 0.87%, the turnover rate is 0.12%, and the number of listing shares is 0.22 million. The fund has 96.05% of its assets in bonds, 2.66% in other assets and 1.29% in cash. For the risk analysis of the recent 1 year, the fund's downside risk is 0.32%, less than the average of the similar. The annualized volatility is 0.76%, less than the average of the market. By the rank of the fund's downside risk and volatility among similar funds, the fund has been at average risk over the past year. According to Fig.3(b) and Fig.3(c), the change trend of the fund showed an upward blow-up of the price which appeared in the process of the fund fluctuation. Fig.4(b) and Fig.4(c) show a downward mutation point in the process of the fund smooth fluctuations. We find that these images have high simulation accuracy with the model.

4. Conclusions

The basic equation of the market price of the option has been formulated by taking assumptions based on the characteristics of option and similar method for formulating basic equation in solid mechanics. While the predecessors did not make too much explanation for the combination of this basic equation and financial practice, and the application of finance is still very rare. This paper not only fills the theory of the application of the basic equation in the field of option pricing, but also makes good fitting with the market ETF products, and connects the theory with practice, which provides a new approach of studying option pricing. The paper also has far-reaching significance for quantitative option investment analysis and the practical application of financial level.

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