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# Optimal Walking Gait Generator for Biped Robot Using Modified Jaya Optimization Technique

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## ABSTRACT

This paper treats the optimization of the biped walking trajectory that can be used as a reference trajectory for control. The biped robot is modeled as a kinetic chain of 11 links connected by 10 joints. The inverse kinematics of the biped is derived for the specified positions of the hips and feet. The objective is to optimize the biped robot able to stably and naturally walking with preset foot-lift magnitude (or preset hip-shift, or preset step-length). The stability of the biped robot is quantified by the distance between the ZMP and the foot center in the step cycle, which represents the first objective function. Additionally, for the biped robot to follow the preset foot-lift value, the difference between the magnitude of foot-lift value and the foot-lift preset value represents the second objective function. Specifically, we minimize the value of the two objective functions by considering the gait parameters of biped robot as variables. The new Jaya optimization algorithm is innovatively applied to optimize the biped gait parameters as to ensure the biped robot walking robustly and steadily. The efficiency of the proposed Jaya-based identification method is compared with the Genetic Algorithm (GA), the Particle Swarm Optimization (PSO), the Central Force Optimization (CFO) and improved Differential Evolution (DE) [Modified Differential Evolution (MDE)] algorithms. The simulation results tested on the real small-sized biped robot system HUBOT-4 demonstrate that the novel proposed algorithm offers an efficient and stable gait for biped robot with precise foot-lift value.

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## 1. INTRODUCTION

The advanced biped robot walking mechanism is increasingly researched and developed in various aspects. Novel standards have been verified and applied to humanoid robots as to ensure more stable and natural gait gesture. Static stable walking is the first applied principle, in which the vertical projection of center of mass (CoM) on the ground is always to be kept within the supporting foot surface. In other words, humanoid robots can stop and move at any times during walking without falling apart. Based on its very simple manner, this principle is only successfully applied to slow-speed biped robots in which dynamic effects can be ignored [1,2]. Researchers then began to focus on developing dynamic walking [3]. This method allows the humanoid robot to speed up the pace. However, during locomotion process, the biped may fall due to environmental interference and cannot stop abruptly. Therefore, Zero Moment Point (ZMP)-based biped walking was proposed to help control and to handle biped inertia [4,5].

Numerous interesting researches have focused on improving the performance of humanoid robot walking gesture. Huang in [6]

proposed a stable gait generator based on a newly proposed gait pattern in using the interpolation function. The method developed by D. Huan, through the Genetic Algorithm (GA), optimized the gait generator parameters as to help robot move steadily with the least amount of energy [7]. Dip *et al.* [8] exhibited steady gait for biped at constant velocity using the sine wave generator. Maximo *et al.* [9] introduced a new stable and quick model-free gait with specific arms movement for biped robots. Khusainov *et al.* [10] successfully combined kinematics and dynamics approaches in gait optimization for humanoid robot locomotion. The fact is that, nowadays, intelligent algorithms are being increasingly applied in this domain to optimize the gait generator parameters for humanoid robots such as GA [8], Particle Swarm Optimization (PSO) algorithm [11], Modified Differential Evolution (MDE) algorithm [12], Central Force Optimization (CFO) algorithm [13]. Shaffi in [14] introduced the humanoid robot achieved a stable gait by using the Fourier series-based gait generator. Furthermore novel approaches continually applied different intelligent algorithms to develop human robot stable and nature walking movement, such as bee swarm algorithm [15], T-S fuzzy controller [16], evolution techniques [17], recurrent neural networks optimized by Ant-Colony Optimization (ACO) [18] and so on. These intelligent algorithms have shown

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their effectiveness in gait generator for humanoid robot. However, there are still numerous areas for further improvement, especially those related to the computational cost of meta-heuristic algorithms.

Recently among newly proposed meta-heuristic optimization approaches, the powerful potential of the Jaya optimization algorithm has not yet being applied to optimize the biped robot walking gait generator. An enhanced optimization method, namely Jaya, initiatively introduced by Rao [19], proves compact and efficient to use with great benefit related to no need any particular control coefficients. Thus, it has been quickly used and improved for optimally solving different problems in various sectors such as mechatronics (Kumar *et al.* [20]; Rao *et al.* [21]), neural model identification (Suraj and Ghosh [22]), electronics (Bhoye *et al.* [23]), optimal civil problems (Venkata Rao and Saroj [24]; Du *et al.* [25]; Cong *et al.* [26]), control system implementation (Rao and More [27]; Rao and Saroj [28]) among others.

Enhancing with promising applications of the Jaya optimization algorithm, this paper intends to improve the modified Jaya approach for proposing a new approach of optimal nature-walking gait applied to biped robot.

In this paper, a new nature-walking pattern generation (*n*-WPG) is proposed which consists of gait generator for the two-foot trajectory, hip trajectory and the inverse kinematics in the full walking periodic sequence which can be seen as combination of three sub-sequences: *Starting step*, *Periodic steps* and *Ending step*. A combined objective function is newly introduced by linking two separate objective functions for humanoid robots that allows stable stepping with preset foot-lifting magnitude. The first one is based on ZMP criterion to stable stepping and the second is based on the difference between the magnitude of the foot-lift parameter and the foot-lift preset value. The Jaya algorithm is used to optimally identify the gait parameters for the biped as to reach steady nature-walking gait with accurate preset foot-lifting magnitude.

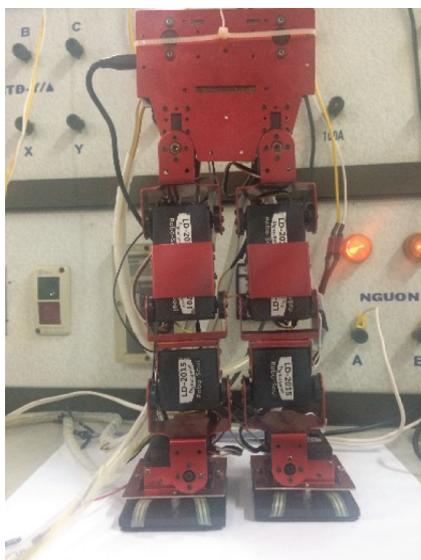
The simulation and experiment results of proposed algorithm applied on the small-sized biped robot demonstrate the performance of the novel algorithm allowing the biped robot to move steadily and stably with an effectively reduced training time. The optimal nature-walking gait results attained by the proposed algorithm are comparatively tested with those obtained by well-known optimization approaches such as Modified Differential Evolution (MDE), CFO, GA and PSO algorithms.

The rest of this paper is arranged as follows: Section 2 fully presents the model design and implementation of the biped robot. In Section 3, inverse kinematics applied in biped optimized WPG parameters is presented. Section 4 presents the optimization results of the biped robot gait trajectory using Jaya technique in comparison with other four EAs algorithms. Section 5 presents the three Case Studies results. Finally, concluding remarks are given in Section 6.

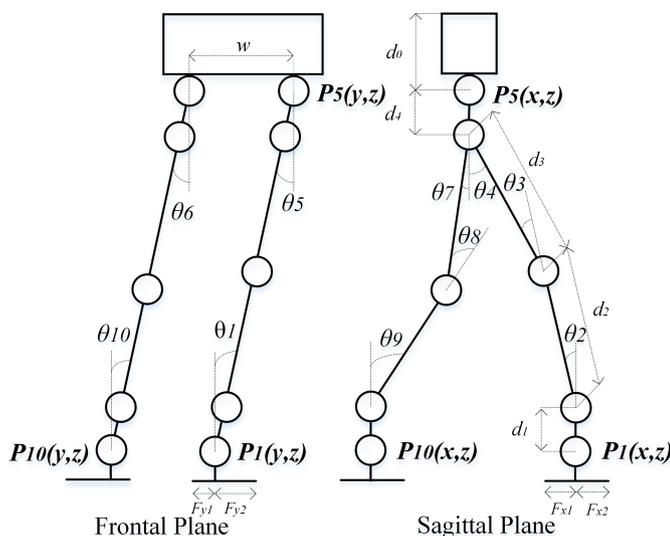
## 2. BIPED MODEL IMPLEMENTATION

The analysis described in this research is based on a biped robot model shown in Figure 1. The origin of the fixed base reference coordinate B is located under Leg 1 on the ground plane. The x-axis points forward, the z-axis points upward and the y-axis is the cross product of z and x axes. The biped robot can be modeled as a kinetic chain of  $(2n + 1)$  links connected by  $2n$  revolute joints, where  $n = 5$  is the number of degrees of freedom of each leg. Figure 1 shows the link and joint coordinate assignments and the values of physical parameters of the biped robot model are shown in Table 1.

The joint angle  $i$  is the rotation of frame  $i$  relative to frame  $(i-1)$  about axis  $z_i$ . Each link has its mass  $m_i$  concentrated at its CoM  $P_i = (P_{ix}, P_{iy}, P_{zi})$ . The position of each link is computed from the 10 angles of rotation at one time in one step with the original coordination at the center of the supporting foot by direct kinematics, as Equations (1–3).



(a)



(b)

Figure 1 | Biped robot model and coordinate configuration.

**Table 1** | Physical parameters of biped robot (HUBOT-4).

Parameters	Value
$d_0$	6.0 cm
$d_1$	4.5 cm
$d_2$	6.0 cm
$d_3$	6.0 cm
$d_4$	4.5 cm
$w$	6.6 cm
$F_{y1}$	2.0 cm
$F_{y2}$	4.0 cm
$F_{x1} = F_{x2}$	2.4 cm
$\begin{cases} m_i \\ i = 1 \dots 11 \end{cases}$	70 g

$$\begin{cases} P_{1xt}(t) = 0, P_{1yt}(t) = 0, P_{1zt}(t) = 0, \\ P_{2xt}(t) = P_{1xt}, P_{2zt}(t) = d_1 \cos[\theta_1(t)], \\ P_{2yt}(t) = P_{2zt}(t) \sin[\theta_1(t)], \\ P_{3xt}(t) = d_2 \sin[\theta_2(t)], \\ P_{3yt}(t) = P_{3zt}(t) \sin[\theta_1(t)], \\ P_{3zt}(t) = P_{2zt}(t) + d_2 \cos[\theta_2(t)] \cos[\theta_1(t)], \\ P_{4xt}(t) = P_{3xt}(t) + d_3 \sin[\theta_2(t) - \theta_3(t)], \\ P_{4yt}(t) = P_{4zt}(t) \sin[\theta_1(t)], \\ P_{4zt}(t) = P_{3zt}(t) + d_3 \cos[\theta_2(t) - \theta_3(t)] \cos[\theta_1(t)], \\ P_{5xt}(t) = P_{4xt}(t) + d_4 \sin[\theta_2(t) - \theta_3(t) + \theta_5(t)], \\ P_{5yt}(t) = P_{5zt}(t) \sin[\theta_1(t)], \\ P_{5zt}(t) = P_{4zt}(t) + d_4 \cos[\theta_2(t) - \theta_3(t) + \theta_5(t)] \cos[\theta_1(t)] \end{cases} \quad (1)$$

$$\begin{cases} P_{5zt}(t) = P_{4zt}(t) + d_4 \cos[\theta_2(t) - \theta_3(t) + \theta_5(t)] \cos[\theta_1(t)], \\ P_{6xt}(t) = P_{5xt}(t), P_{6yt}(t) = P_{5yt}(t) - w, P_{6zt}(t) = P_{5zt}(t), \\ P_{7xt}(t) = P_{6xt}(t), P_{7zt}(t) = P_{6zt}(t) - d_4 \cos[\theta_6(t)], \\ P_{7yt}(t) = P_{6yt}(t) - [P_{6zt}(t) - P_{7zt}(t)] \sin[\theta_6(t)], \\ P_{8xt}(t) = P_{7xt}(t) + d_3 \sin[\theta_7(t)], \\ P_{8zt}(t) = P_{7zt}(t) - d_3 \cos[\theta_7(t)] \cos[\theta_6(t)], \\ P_{8yt}(t) = P_{7yt}(t) - [P_{7zt}(t) - P_{8zt}(t)] \sin[\theta_6(t)], \\ P_{9xt}(t) = P_{8xt}(t) + d_2 \sin[\theta_7(t) - \theta_8(t)], \\ P_{9zt}(t) = P_{8zt}(t) - d_2 \cos[\theta_7(t) - \theta_8(t)] \cos[\theta_6(t)], \\ P_{9yt}(t) = P_{7yt}(t) - [P_{7zt}(t) - P_{9zt}(t)] \sin[\theta_6(t)], \\ P_{10xt}(t) = P_{11xt}(t) + d_1 \sin[\theta_7(t) - \theta_8(t) + \theta_1(t)], \\ P_{10zt}(t) = P_{9zt}(t) - d_1 \cos[\theta_7(t) - \theta_8(t) + \theta_9(t)] \cos[\theta_6(t)], \\ P_{10yt}(t) = P_{7yt}(t) - [P_{7zt}(t) - P_{10zt}(t)] \sin[\theta_6(t)] \end{cases} \quad (2)$$

$$\begin{cases} P_{0xt}(t) = \frac{P_{5xt}(t) + P_{6xt}(t)}{2}, \\ P_{1yt}(t) = \frac{P_{5yt}(t) + P_{6yt}(t)}{2}, \\ P_{0zt}(t) = P_{5zt}(t) + \frac{d_0}{2} \end{cases} \quad (3)$$

in which,  $d_0, d_1, d_2, d_3$  and  $d_4$  are fully illustrated in Figure 1.

The balancing of the biped system can be simplified at an assigned ground point, called the ZMP ( $x_{ZMP}, y_{ZMP}, z_{ZMP} = 0$ ), where the resultant moment at the ground plane is zero. The location of the ZMP [5] is

$$\begin{cases} x_{ZMP} = \frac{\sum_{i=1}^n m_i (\ddot{z}_i + g) x_i - \sum_{i=1}^n m_i \dot{x}_i z_i}{\sum_{i=1}^n m_i (\ddot{z}_i + g)} \\ y_{ZMP} = \frac{\sum_{i=1}^n m_i (\ddot{z}_i + g) y_i - \sum_{i=1}^n m_i \dot{y}_i z_i}{\sum_{i=1}^n m_i (\ddot{z}_i + g)} \end{cases} \quad (4)$$

Constraints on the motion of the biped robot are that the location of the ZMP must be inside the convex hull of the support foot or feet. We assume the frictional coefficient is large enough so that the frictional constraint is negligible.

### 3. INVERSE KINEMATICS

To describe the specific configuration of the biped robot, we assign the motions of link 1 (foot 1), link 6 (body) and link 11 (foot 2). By fixing both the position and the orientation of these three links, the configuration of the biped robot is established. For simplicity, we assume that no rotational motion occurs for these three links during walking. In order to compute the joint motion, the inverse kinematic model is needed for given  $P_1 = (P_{1x}, P_{1y}, P_{1z})$ ,  $P_5 = (P_{5x}, P_{5y}, P_{5z})$  and  $P_{10} = (P_{10x}, P_{10y}, P_{10z})$  where  $P_i$  is the reference point of link  $i$  as shown in Figure 1. Because there is no rotation, the joint angles can be expressed by,

$$\begin{cases} \theta_1(t) = \arctan\left(\frac{y_l(t)}{z_l(t)}\right), \quad \theta_5(t) = -\theta_1(t), \\ \theta_{10}(t) = \arctan\left(\frac{y_r(t)}{z_r(t)}\right), \quad \theta_6(t) = -\theta_{10}(t), \\ \theta_3(t) = \pi - \theta_A(t), \quad \theta_8(t) = \pi - \theta_C(t), \\ \theta_4(t) = \frac{\pi}{2} - \theta_A(t) + \theta_B(t) - \arcsin\left(\frac{x_l(t)}{l_l(t)}\right), \\ \theta_7(t) = \frac{\pi}{2} - \theta_C(t) + \theta_D(t) - \arcsin\left(\frac{x_r(t)}{l_r(t)}\right), \\ \theta_2(t) = \theta_3(t) - \theta_4(t), \quad \theta_9(t) = \theta_8(t) - \theta_7(t) \end{cases} \quad (5)$$

where

$$\begin{cases} x_l = P_{5x} - P_{1x}, \quad y_l = P_{5y}, \quad z_l = P_{5z} - P_{1z}, \\ l_l = \sqrt{(P_{4x} - P_{2x})^2 + (P_{4y} - P_{2y})^2 + (P_{4z} - P_{2z})^2}, \\ x_r = P_{6x} - P_{10x}, \quad y_r = P_{5y}, \quad z_r = P_{6z} - P_{10z}, \\ l_r = \sqrt{(P_{7x} - P_{9x})^2 + (P_{7y} - P_{9y})^2 + (P_{7z} - P_{9z})^2} \\ \theta_A = \arccos\left(\frac{d_2^2 + d_3^2 - l_l^2}{2d_2d_3}\right), \quad \theta_B = \arccos\left(\frac{d_3 \sin(\theta_A)}{l_l}\right), \\ \theta_C = \arccos\left(\frac{d_2^2 + d_3^2 - l_r^2}{2d_2d_3}\right), \quad \theta_D = \arccos\left(\frac{d_3 \sin(\theta_C)}{l_r}\right) \end{cases} \quad (6)$$

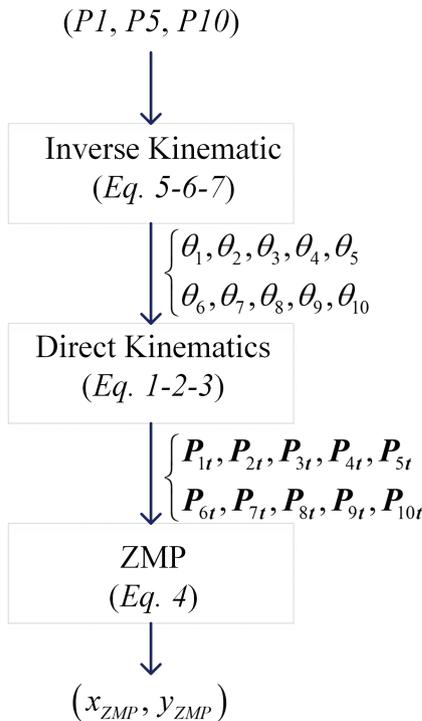
The coordination  $P_6(x, y, z)$  is computed using  $P_5(x, y, z)$ , and the coordination of  $[P_2(x, y, z), P_4(x, y, z), P_7(x, y, z), P_9(x, y, z)]$  is computed using  $[P_1(x, y, z), P_5(x, y, z), P_6(x, y, z), P_{10}(x, y, z)]$  and the rotary angle  $[\theta_1, \theta_5, \theta_6, \theta_{10}]$ . Equation (6) below are used to determine  $P_2, P_4, P_6, P_7, P_9$ .

$$\begin{cases} P_{2x} = P_{1x}, P_{2z} = d_1 \cos(\theta_1), P_{2y} = P_{2z} \sin(\theta_1), \\ P_{4x} = P_{5x}, P_{4z} = P_{5z} - d_4 \cos(\theta_1), \\ P_{4y} = P_{5y} - (P_{5z} - P_{4z}) \sin(\theta_1), \\ P_{6x} = P_{5x}, P_{6y} = P_{5y} - w, P_{6z} = P_{5z}, \\ P_{7x} = P_{6x}, P_{7z} = P_{6z} - d_4 \cos(\theta_{10}), \\ P_{7y} = P_{6y} - (P_{6z} - P_{7z}) \sin(\theta_{10}), \\ P_{9x} = P_{10x}, P_{9z} = P_{10z} + d_4 \cos(\theta_{10}), \\ P_{9y} = P_{10y} + (P_{9z} - P_{10z}) \sin(\theta_{10}) \end{cases} \quad (7)$$

In that,  $d_0$  represents the length of the upper body,  $d_1$  is the distance between two points  $P_1$  and  $P_2$ ,  $d_2$  is the distance between two points  $P_2$  and  $P_3$ ,  $d_3$  is the distance between two points  $P_3$  and  $P_4$ ,  $d_4$  represents the distance between two points  $P_4$  and  $P_5$ ,  $w$  represents the distance between 2 legs. Therefore, the biped motion can be expressed solely by positions  $P_1, P_5$  and  $P_{10}$ . A flowchart for computing the ZMP from the trajectories of  $P_1, P_5$  and  $P_{10}$  is shown in Figure 2.

#### 4. OPTIMIZATION OF THE BIPED ROBOT TRAJECTORY

The trajectory optimization is based on an objective functional, involving the biped robot able to stably and naturally walking with



**Figure 2** | Calculation of the zero moment point.

preset foot-lift magnitude, with the gait parameters of biped robot as variables. Specifically, we define the optimization problem as follows:

The goal of biped robot is to achieve a stable gait with preset foot-lift value. For this purpose, the ZMP point is to always be ensured within the foot area [6]. If the ZMP is within the area of the supporting leg, the robot does not fall [7]. The calculation of the ZMP of biped robots in walking is shown in Figure 2. The stability of the humanoid robot is quantified by the distance between the ZMP and the foot center in the step cycle. Walking gait with maximum stability is obtained by minimizing the function  $f_1$  in Equation (8) [1]:

$$f_1 = \int_0^T \sqrt{x_{ZMP}^2 + y_{ZMP}^2} dt \quad (8)$$

where  $T$  denotes stepping cycle and  $(x_{ZMP}, y_{ZMP})$  denotes the coordination of ZMP point in the robot's process of stepping which is away from the quadrant in the center of the foot. In this paper the Equation (8) represents the first objective function.

Additionally, for the humanoid robot to follow the preset foot-lifting height value -  $H_{ref}$ , the difference between the magnitude of the foot-lift value  $H$  and the foot-lift preset value -  $H_{ref}$  (see Equation (9)) represents the second objective function.

$$f_2 = |H_{ref} - H| \quad (9)$$

Thus, in order for biped robot to obtain a steady gait with the foot-lift set up in advance, we find the minimum value of the two objective functions  $f_1$  and  $f_2$ , or similarly to find the minimum of the function  $f$  as

$$\begin{cases} f = \lambda \cdot \left( \int_0^T \left( \sqrt{x_{ZMP}^2 + y_{ZMP}^2} \right) .dt \right) + (1 - \lambda) \cdot |H_{ref} - H| \\ -F_{x1} \leq x_{ZMP} \leq F_{x2}; -F_{y1} \leq y_{ZMP} \leq F_{y2} \end{cases} \quad (10)$$

in which,  $F_{x1} + F_{x2}$  and  $F_{y1} + F_{y2}$  is the length and width of the biped robot foot,  $\lambda$  ( $0 \leq \lambda \leq 1$ ) can be used to satisfactorily select the priority between the walking stability ( $\lambda$  increase) and the variance with the desired foot-lifting magnitude ( $\lambda$  decreased).

To solve the problem, algorithm diagram of optimal walking gait for biped robots is shown in Figure 3. The  $n$ -WPG composes of the gait generator of the two-foot trajectory, the hip trajectory and the biped inverse kinematics (see additional works at [4]). The meta-heuristic optimization algorithms are used to create 4 gait parameters ( $S, H, h, n$ ) that provide the  $n$ -WPG in which outputs of the  $n$ -WPG are to create 10 angles for biped robot. The output of biped robot is the location of ZMP points as described in Figure 2. The optimal target function identified by the meta-heuristic optimization algorithm is a biped robot that walks steadily with the desired foot-lift, as presented in Equation (9). The following difference approximation is used to calculate the derivatives:

$$\begin{cases} \dot{x}_{i+1} = \frac{x_{i+1} - x_i}{\Delta t}; \ddot{x}_{i+1} = \frac{\dot{x}_{i+1} - \dot{x}_i}{\Delta t} \\ \dot{y}_{i+1} = \frac{y_{i+1} - y_i}{\Delta t}; \ddot{y}_{i+1} = \frac{\dot{y}_{i+1} - \dot{y}_i}{\Delta t} \\ \dot{z}_{i+1} = \frac{z_{i+1} - z_i}{\Delta t}; \ddot{z}_{i+1} = \frac{\dot{z}_{i+1} - \dot{z}_i}{\Delta t} \end{cases} \quad (11)$$

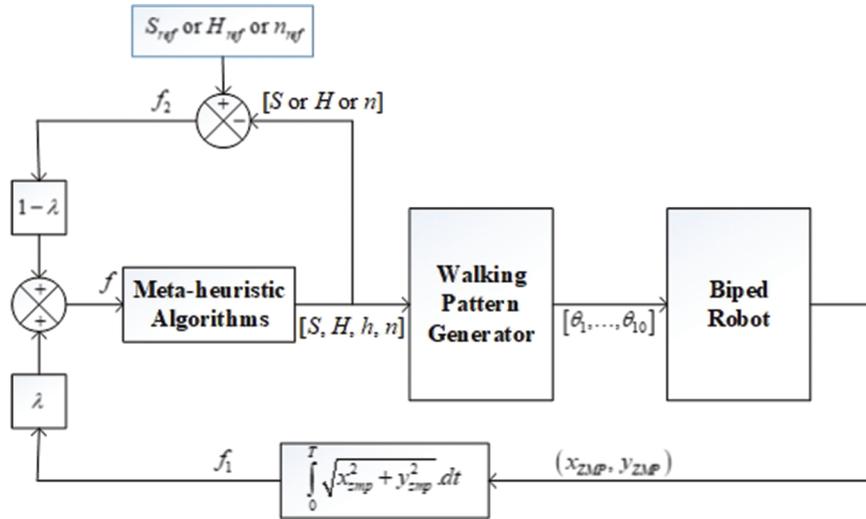


Figure 3 | Algorithm diagram of optimal walking gait for biped robot.

In the simulation, ( $P_1$ ,  $P_5$  and  $P_{10}$ ) are chosen intuitively.  $P_1$  and  $P_{10}$  are selected according to the desired leg movement for a specific activity. The choice of  $P_5$  depends on the mechanical structure of the biped as it involves shifting of ZMP from one foot to another. In a nature-walking sequence, which can be seen as combination of three sub-sequences: *Starting step*, *Periodic steps* and *Ending step*, ( $0 \leq t \leq 3T$ ),  $P_1$  and  $P_{10}$  for straight walking are provided by Equations (12) and (13) while  $P_5$  for straight walking is given by Equation (14).

$$\begin{cases}
 P_{1x}(t) = \begin{cases} 0 & , 0 \leq t < T \\ \frac{S}{2} \times \left(1 - \sin\left(\frac{w}{2}t - \frac{\pi}{2}\right)\right) & , T \leq t \leq 2T \\ S & , 2T < t \leq 3T \end{cases} \\
 P_{1y}(t) = 0, \quad 0 \leq t \leq 3T \\
 P_{1z}(t) = \begin{cases} 0 & , 0 \leq t < T \\ H \times \sin\left(\frac{\pi}{2} \times \left(\sin\left(\frac{w}{2}t - \frac{\pi}{2}\right) + 1\right)\right) & , T \leq t \leq 2T \\ 0 & , 2T < t \leq 3T \end{cases}
 \end{cases} \quad (12)$$

$$\begin{cases}
 P_{10x}(t) = \begin{cases} \frac{S}{4} \times \left(1 + \sin\left(\frac{w}{2}t - \frac{\pi}{2}\right)\right) & , 0 \leq t < T \\ \frac{S}{2} & , T \leq t \leq 2T \\ \frac{S}{4} \times \left(3 + \sin\left(\frac{w}{2}t - \frac{5\pi}{2}\right)\right) & , 2T < t \leq 3T \end{cases} \\
 P_{10y}(t) = -w, \quad 0 \leq t \leq 3T \\
 P_{10z}(t) = \begin{cases} H \times \sin\left(\frac{\pi}{2} \times \left(\sin\left(\frac{w}{2}t - \frac{\pi}{2}\right) + 1\right)\right) & , 0 \leq t < T \\ 0 & , T \leq t \leq 2T \\ H \times \sin\left(\frac{\pi}{2} \times \left(\sin\left(\frac{w}{2}t - \frac{5\pi}{2}\right) + 1\right)\right) & , 2T < t \leq 3T \end{cases}
 \end{cases} \quad (13)$$

$$\begin{cases}
 P_{5x}(t) = \begin{cases} \frac{S}{8} \times \left(1 + \sin\left(\frac{w}{2}t - \frac{\pi}{2}\right)\right) & , 0 \leq t < T \\ \frac{S}{2} \times \left[1 + \frac{1}{2} \sin\left(\frac{w}{2}t - \frac{3\pi}{2}\right)\right] & , T \leq t \leq 2T \\ \frac{S}{8} \times \left(7 + \sin\left(\frac{w}{2}t - \frac{5\pi}{2}\right)\right) & , 2T < t \leq 3T \end{cases} \\
 P_{5y}(t) = \begin{cases} 0 \leq t < T \\ n \times \sin\left(\frac{\pi}{2} \times \left(\sin\left(\frac{w}{4}t - \frac{\pi}{2}\right) + 1\right)\right) \times \sin\left(\frac{w}{2}t\right), & T \leq t \leq 2T \\ n \times \sin\left(\frac{\pi}{2} \times \left(1 - \sin\left(\frac{w}{4}t\right)\right)\right) \times \sin\left(\frac{w}{2}t\right), & 2T < t \leq 3T \end{cases} \\
 P_{5z}(t) = \begin{cases} k_1 + h \times \sin\left(\frac{\pi}{2} \times \sin\left(\frac{w}{4}t + 1\right)\right) \times \sin\left(\frac{w}{2}t + \frac{\pi}{2}\right), & 0 \leq t < T \\ k_1, & T \leq t \leq 2T \\ k_1 + h \times \sin\left(\frac{\pi}{2} \times \sin\left(\frac{w}{4}(3T - t) + 1\right)\right) \times \sin\left(\frac{w}{2}(3T - t) + \frac{\pi}{2}\right), & 2T < t \leq 3T \end{cases}
 \end{cases} \quad (14)$$

in which,  $T$  represents the time to perform a step of the humanoid robot and  $k_1 = d_1 + d_2 + d_3 + d_4 - h$ .

The biped robot HUBOT-4 will walk stably with the preset foot-lift value. The coefficient  $\lambda$  used in the objective function of Equation (10) is optimally selected equal 0.4, and this  $\lambda$  value will be used through the comparative testing process using GA, PSO, CFO, MDE and proposed Jaya optimization algorithms. For optimal test based on GA, PSO, CFO, MDE and Jaya optimization algorithms, the four key gait parameters derived from the biped HUBOT-4 are bounded as given in Table 2.

Simulation results are carried with MATLAB platform run with 2.49 GHz CPU & 8 GB RAM. Every method is tested 30 times.

Table 3 eventually presents the GA, PSO, CFO, MDE and Jaya optimally selected parametric values.

Specify the foot-lifting height of biped HUBOT-4 being  $H_{ref} = 2cm$ . Survey the performance of Jaya algorithm in optimal walking gait for biped robot by a full comparison to GA, PSO, CFO and MDE. The parameter of the algorithms is described in Table 3, selecting NP = 32 and GEN = 200. Table 4 gives the comparative performance of the algorithms on the optimal procedure. Figure 4 presents the convergence rates of the algorithms using the average results of objective function in the optimal procedures. The comparative results in Table 4 show that the merit of Jaya algorithm has strongly better than GA, PSO, CFO and MDE.

The optimum set of 4 key parameters for biped HUBOT-4 conformed to the objective derived from 30 runs per GA, PSO, CFO, MDE and proposed Jaya algorithms is shown in Table 5.

Based on the results described in Figure 4, it is important to notice that the Jaya algorithm searches for an optimal solution with an average value of 101.856724000000 after about 55 generations, while the MDE algorithm is approximately 134 generations after the search, finding an optimal solution obtained an average value of 101.856724300000, while the CFO algorithm must need around

**Table 2** Bounds for four key walking gait parameters of humanoid HUBOT-4.

Parameters	Lower Bound (cm)	Upper Bound (cm)
S-Step length	8	12
H-foot lifting	0.1	5
h-kneeling	1.1	1.5
n-hip swinging	0.1	6

**Table 3** Parameters of CFO, MDE and Jaya algorithm.

Method	Parameters	Value
GA	Mutating rate	0.2
	Crossover rate	0.7
PSO	Accelerated factor 1	2
	Accelerated factor 2	2
CFO	Inertia factor	0.4
	Alpha	0.25
	Beta	0.35
	Gamma	0.95
	Frep	0.5
MDE	deltaFrep	0.05
	Mutation value (F)	Random [0.4; 1.0]
Jaya	Crossover probability (CR)	Random [0.7; 1.0]
	No parameters	No

GA, Genetic Algorithm; PSO, Particle Swarm Optimization; CFO, Central Force Optimization; MDE, Modified Differential Evolution.

158 generations to find the optimal solution with an average value of 102.626134800000, while the PSO algorithm is approximately 41 generations after the search, finding an optimal solution obtained an average value of 101.885709700000, while the GA algorithm must need around 190 generations to find the optimal solution with an average value of 102.291673400000. These results demonstrate that the Jaya algorithm outperforms GA, PSO, CFO and MDE in terms of convergence speed with the best target function value.

Table 5 shows the comparative optimized results of GA, PSO, CFO, MDE and proposed Jaya. The difference between the magnitude of the foot-lift value -  $H$  and the foot-lift preset value  $-H_{ref} = 2cm$  of proposed Jaya proves better than GA, PSO and about - 0.5. Using comparative results tabulated in Table 5, it is evident to conclude that the proposed Jaya algorithm proves the best precise and robust optimization algorithm in comparison with the GA, PSO, MDE and CFO ones. Figure 5 shows the resulted ZMP trajectories when biped HUBOT-4 steps along with a nature-walking sequence ( $0 \leq t \leq 3T$ ) using GA, PSO, CFO, MDE and JAYA techniques.

The comparatively resulted ZMP trajectories using the five techniques illustrated in Figure 5 show that they surely keep in the supporting foot surface and it proves that biped allowing steadily robust stepping.

## 5. CASE STUDIES

The optimum identified solutions based on four principal biped WPG coefficients using proposed Jaya technique illustrated in Tables 6–8 which will strongly demonstrate that the eventual purpose of this study is completed with respect to the three following case studies which are investigated regarding to the preset foot-lift, the preset hip-shift and the preset step-length magnitudes, respectively.

### 5.1. Case Study-1: Biped Optimal Stable Walking with Preset Foot-lift $H_{ref}$ Magnitudes

Table 6 demonstrates the optimized value of the walking gait parameters to ensure the biped HUBOT-4 to walk steadily with both cases corresponding to different preset foot-lift magnitudes. ( $H_{ref} = 2cm$  and  $H_{ref} = 3cm$ ) optimized by Jaya algorithm. Figure 6 presents the convergence rates of the proposed Jaya algorithm using the average results of objective function in the optimal procedures.

Continually Figure 7a and 7b illustrates the 2D gait in the X-Z plane of the HUBOT-5, corresponding to two cases with different preset foot-lift amplitudes. The results from Table 6 and Figure 7 show

**Table 4** Performance comparisons in optimal fitness function.

No	Optimal Objective Function				Run (Second)
	Best	Worst	Mean	Variance	
Jaya	101.856724029548	1.01856724e+02	1.018567240000001e+02	3.3425940701226e-27	157
MDE	101.856724029550	1.01856724e+02	1.0185672600000000e+02	3.5178160847427e-13	153
CFO	101.889479300000	1.027262942e+02	1.0263177384000000e+02	0.606345370124322	159
PSO	101.856724029549	1.027262942e+02	1.018857096733334e+02	0.025205077757601	152
GA	101.927584300000	1.030504002e+02	1.022916734133333e+02	0.057276108848447	158

GA, Genetic Algorithm; PSO, Particle Swarm Optimization; CFO, Central Force Optimization; MDE, Modified Differential Evolution.

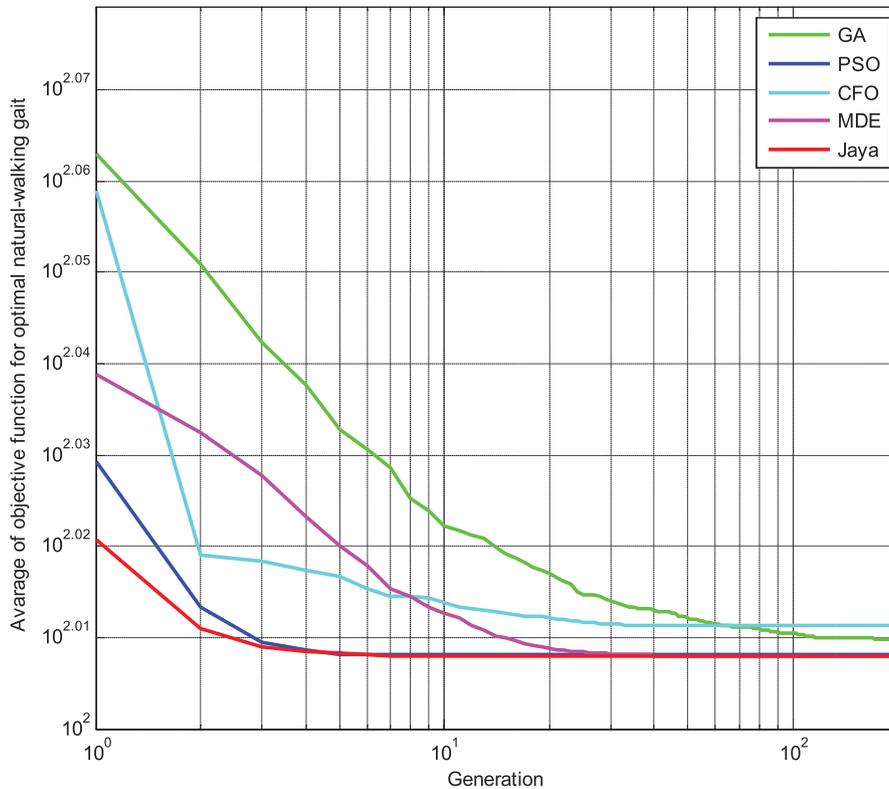


Figure 4 | Convergence rate of the comparative algorithms in the optimal process.

Table 5 | Resulted parametric set for five comparative algorithms.

$H_{ref} = 2 \text{ cm}$					
Algorithms	Walking Gait Parameters Value				Best Fitness Value $f(\text{cm})$
	S (cm)	H (cm)	h (cm)	n (cm)	
GA	11.99	1.99	1.1	5.97	101.927584300000
PSO	12.00	1.99	1.1	6.00	101.856724029549
CFO	11.97	1.99	1.1	5.97	101.889479300000
MDE	12.00	2.00	1.1	5.99	101.856724029550
Jaya	12.00	2.00	1.1	6.00	101.856724029548

GA, Genetic Algorithm; PSO, Particle Swarm Optimization; CFO, Central Force Optimization; MDE, Modified Differential Evolution.

Table 6 | Optimal parameter set.

$H_{ref}(\text{cm})$	Jaya Optimization Results			
	S (cm)	H (cm)	h (cm)	n (cm)
<b>2.0</b>	12.0000	2.0000	1.1000	6.0000
<b>3.0</b>	12.0000	3.0000	1.1000	6.0000

The bold values represent the reference values based on them to optimize the parameter set using Jaya.

Table 7 | Optimal parameter set.

$n_{ref}(\text{cm})$	Jaya Optimization Results			
	S (cm)	H (cm)	h (cm)	n (cm)
<b>4.0</b>	12.0000	1.5000	1.1000	4.0000
<b>5.0</b>	12.0000	1.5000	1.1000	5.0000

The bold values represent the reference values based on them to optimize the parameter set using Jaya.

that the biped HUBOT-4 attains a pickup lift in term of the preset foot-lift value.

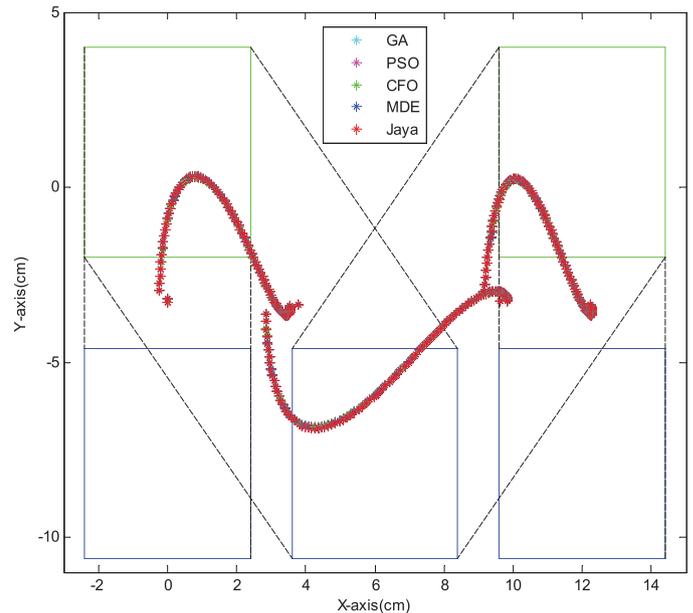


Figure 5 | Resulted comparative Zero Moment Point (ZMP) survey.

Table 8 | Optimal parameter set.

$S_{ref}(\text{cm})$	Jaya Optimization Results			
	S (cm)	H (cm)	h (cm)	n (cm)
<b>9.0</b>	9.0000	2.0000	1.1000	6.0000
<b>10</b>	10.0000	1.0000	1.1000	6.0000

The bold values represent the reference values based on them to optimize the parameter set using Jaya.

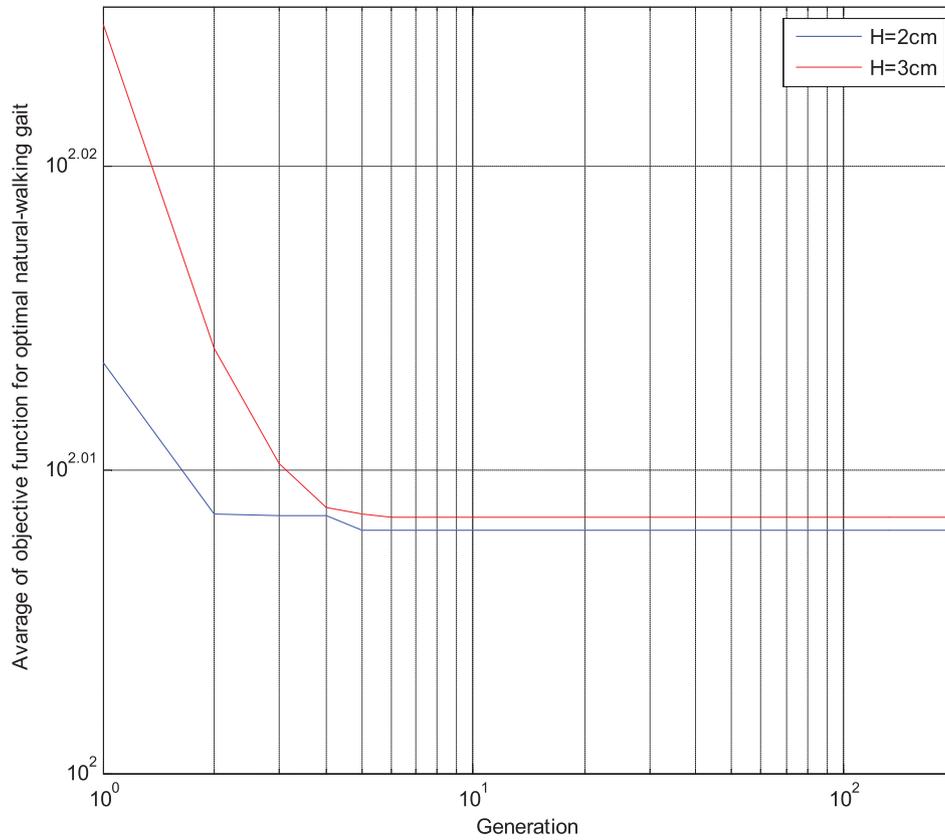


Figure 6 | Convergence rate of the proposed Jaya algorithm in the optimal process.

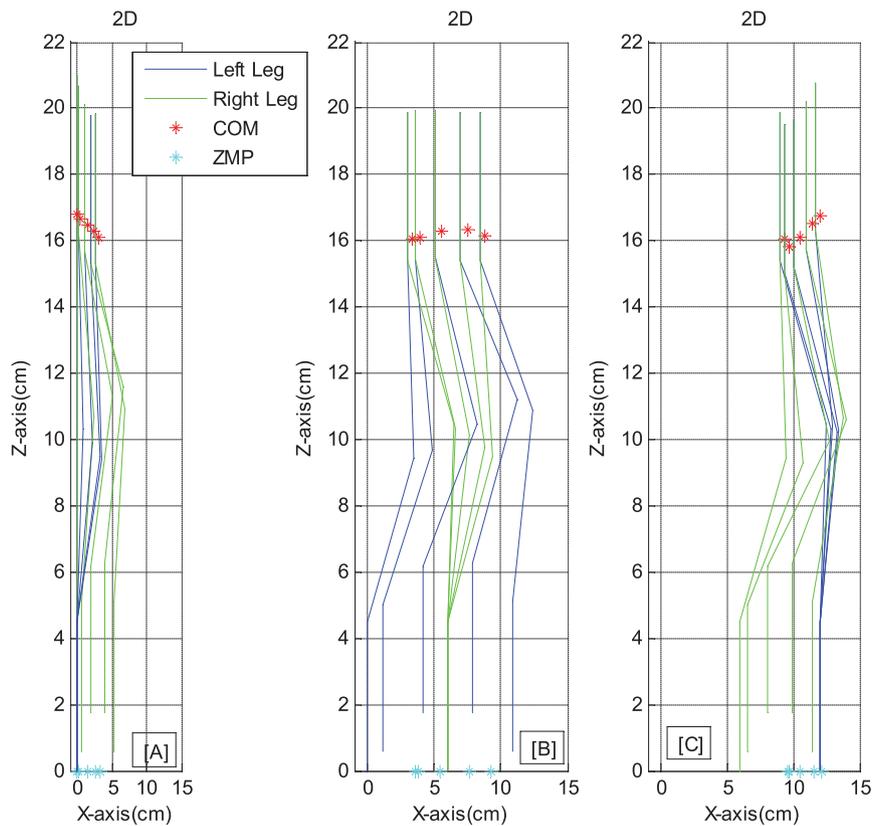
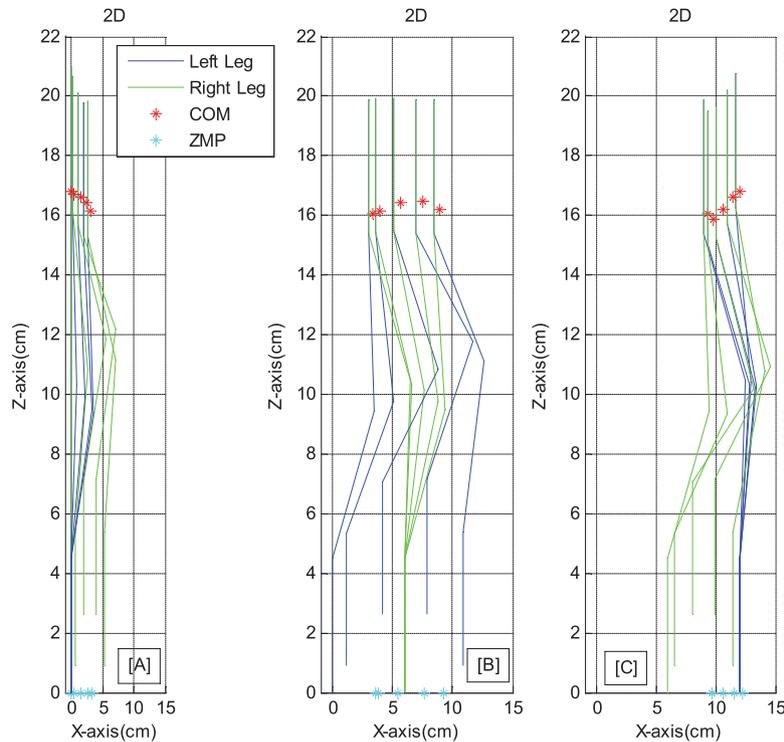


Figure 7a | Stick diagram of biped robot for a nature-walking sequence in the x-z plan. [A]: Starting step. [B]: Periodic steps. [C]: Ending step.



**Figure 7b** | Stick diagram of biped robot for a nature-walking sequence in the x-z plan. [A]: Starting step. [B]: Periodic steps. [C]: Ending step.

Furthermore Figure 8a and 8b illustrates the resulted ZMP point trajectory and the projection of Groundd-Center-Of-Mass (GCOM) trajectory for two different preset foot-lifting amplitudes. This result shows that the ZMP point is always in the supporting foot area and then it ensures that the HUBOT-4 biped robot surely keeps stable walking.

Figure 9 below demonstrates the 10 rotary angular trajectories in one stepping cycle of the two legs of biped HUBOT-4 ( $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}$ ) during walking with two optimally resulted sets of gait parameters with respect to two different preset foot-lift values (tabulated in Table 6).

Based on the results of the optimization and simulation shown in Table 6, Figures 7–9, which convincingly demonstrate that the work of preset foot-lift parameter -  $H_{ref}$  and four optimally selected parameters ( $S$ -step length,  $H$ -foot lifting,  $h$  – kneeling and  $n$  – hip swinging) ensuring the HUBOT-4 biped robot to steadily walking without falling apart and keeping pace with desired foot-lift amplitude. The proposed algorithm with gait parameters optimized by Jaya algorithm is convincingly feasible.

## 5.2. Case Study-2: Biped Optimal Stable Walking with Preset Hip-Shift $n_{ref}$ Values

Table 7 demonstrates the optimum identified value of proposed WPG to guarantee the biped robot to step stably in two different cases regarding to various preset hip-shift values ( $n_{ref} = 4cm$  and  $n_{ref} = 5cm$ ) optimized by Jaya algorithm, and Figure 10 presents the convergence rate optimized by the proposed Jaya algorithm using the average results of objective function in the optimal procedures.

Furthermore Figure 11a and 11b below presents the resulted 2D gait in the Y-Z co-ordinate of the biped, with respect to three different cases of preset hip-shift values. Results from Table 7 and Figure 11 strongly ensure that the humanoid robot successfully obtains a desired hip-shift value in control process.

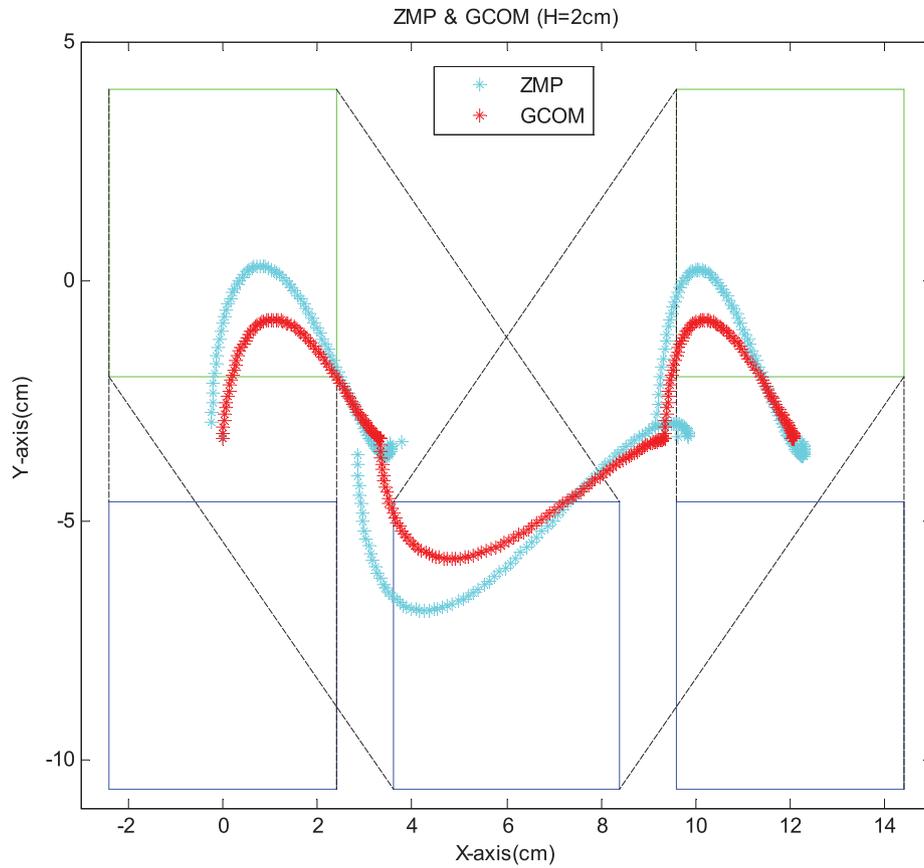
Figure 12 illustrates the resulted ZMP values for two preset hip-shift magnitudes. These results confirm that ZMP trajectory is always to be kept inside the supporting footprint surface and thus guarantees humanoid robot walking efficiently and robustly.

Figure 13 demonstrates the 10 rotary angular trajectories in one stepping cycle of the two legs of biped HUBOT-4 ( $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}$ ) during walking with two optimally resulted sets of gait parameters with respect to two different preset hip-shift values and tabulated in Table 8.

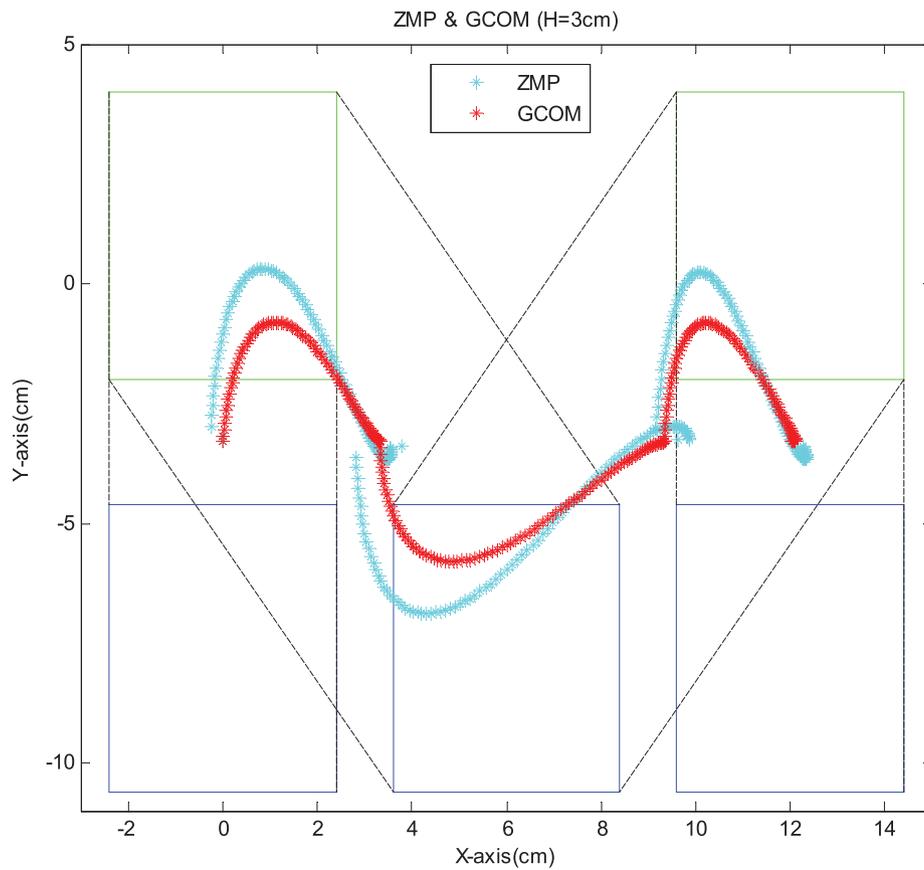
In summary based on the results of the optimization and simulation shown in Table 7, Figures 11–13, which once more demonstrate that the work of preset hip-shift parameter -  $n_{ref}$  and four optimally selected parameters ( $S$ -step length,  $H$ -foot lifting,  $h$  – kneeling and  $n$  – hip swinging) ensuring the HUBOT-4 biped robot to steadily walking without falling apart and keeping pace with desired hip-shift amplitude. The proposed algorithm with gait parameters optimized by Jaya algorithm is convincingly feasible.

## 5.3. Case Study-3: Biped Optimal Stable Walking with Preset Step-Length $S_{ref}$ Values

Table 8 demonstrates the optimum identified value of proposed WPG to guarantee the humanoid robot to step stably in two



**Figure 8a** | Resulted Zero Moment Point (ZMP) và Grounded-Center-Of-Mass (GCOM) trajectories.



**Figure 8b** | Resulted Zero Moment Point (ZMP) và Grounded-Center-Of-Mass (GCOM) trajectories.

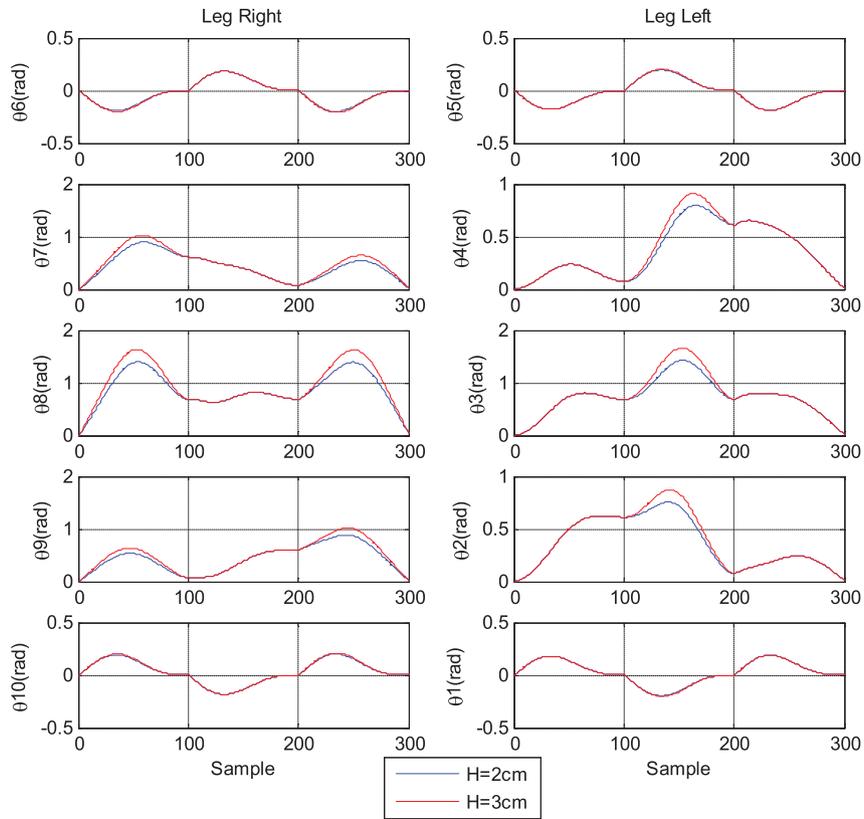


Figure 9 Trajectories of the ten joint angles located at two legs of biped HUBOT-4.

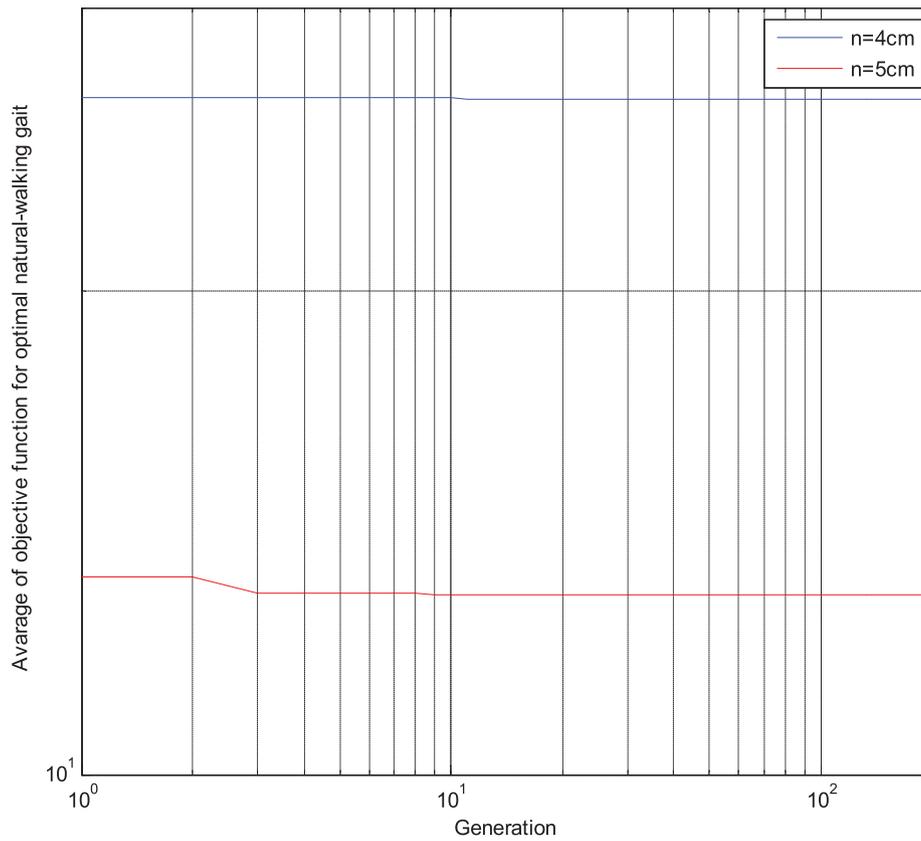
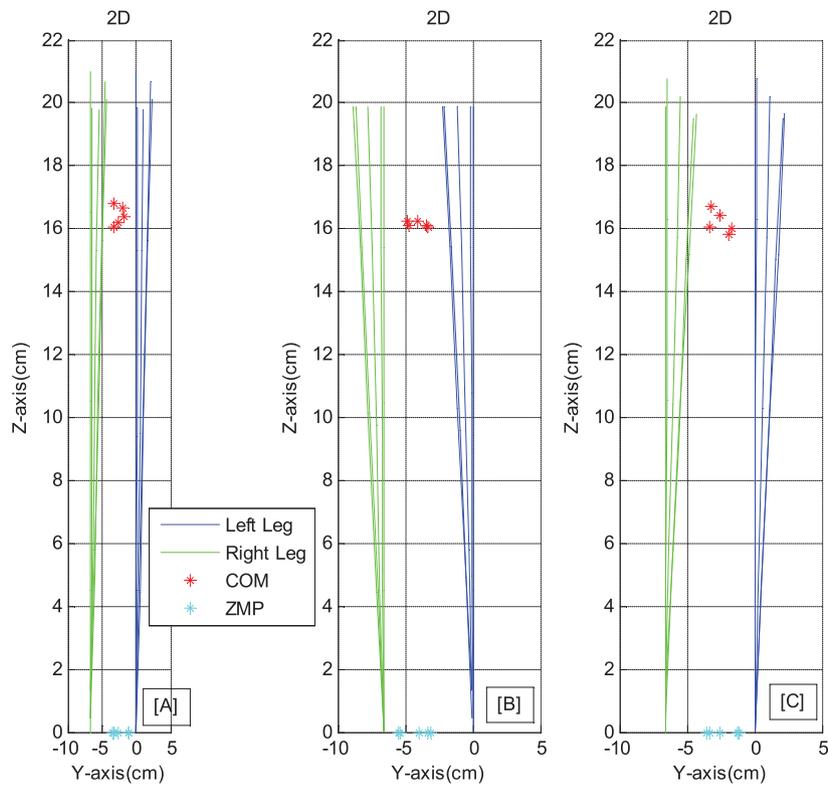
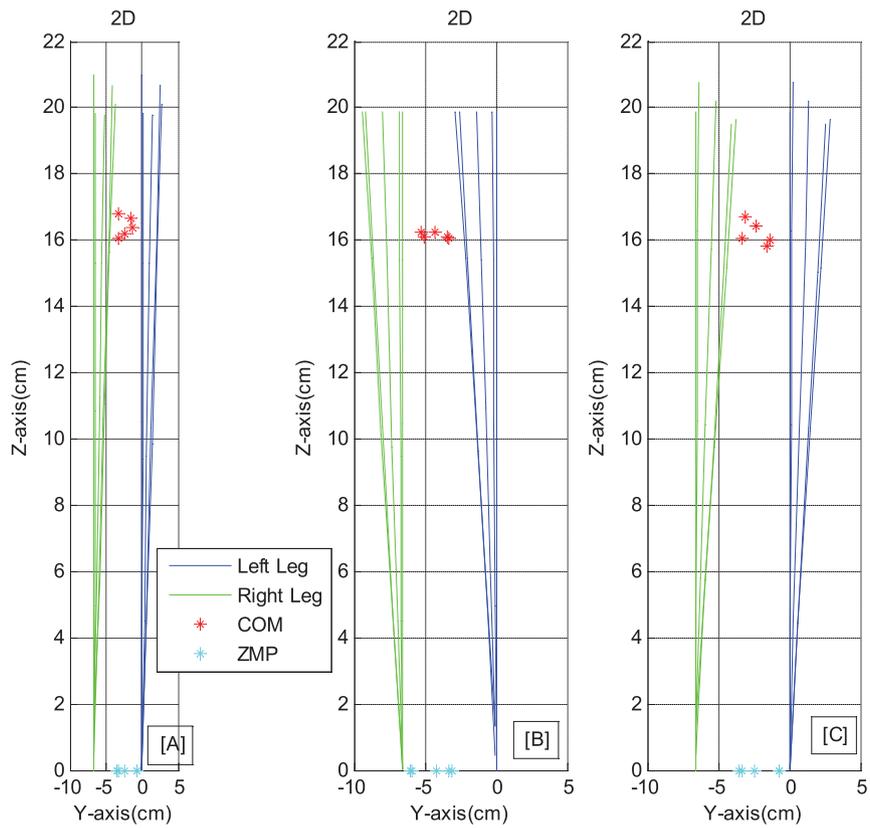


Figure 10 Convergence rate using the proposed Jaya algorithm in the optimal procedure.



**Figure 11a** Resulted 2D gait of humanoid robot with  $n_{ref} = 4\text{cm}$ . [A]: Starting step. [B]: Periodic steps. [C]: Ending step.



**Figure 11b** Resulted 2D gait of humanoid robot with  $n_{ref} = 5\text{cm}$ . [A]: Starting step. [B]: Periodic steps. [C]: Ending step.

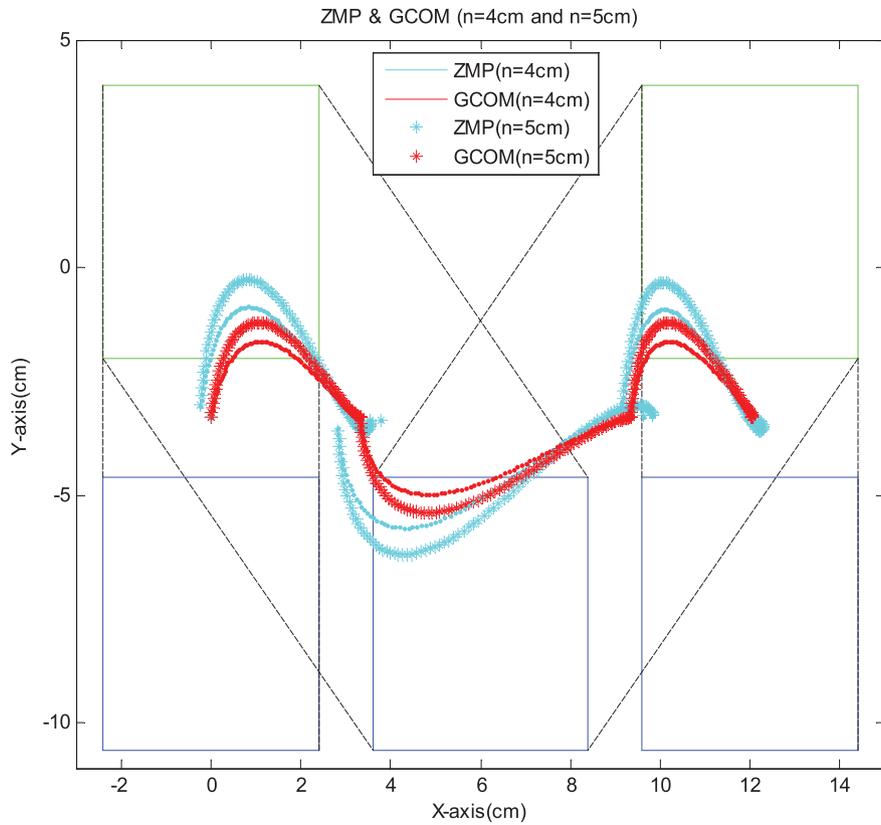


Figure 12 | Resulted Zero Moment Point (ZMP) trajectories.

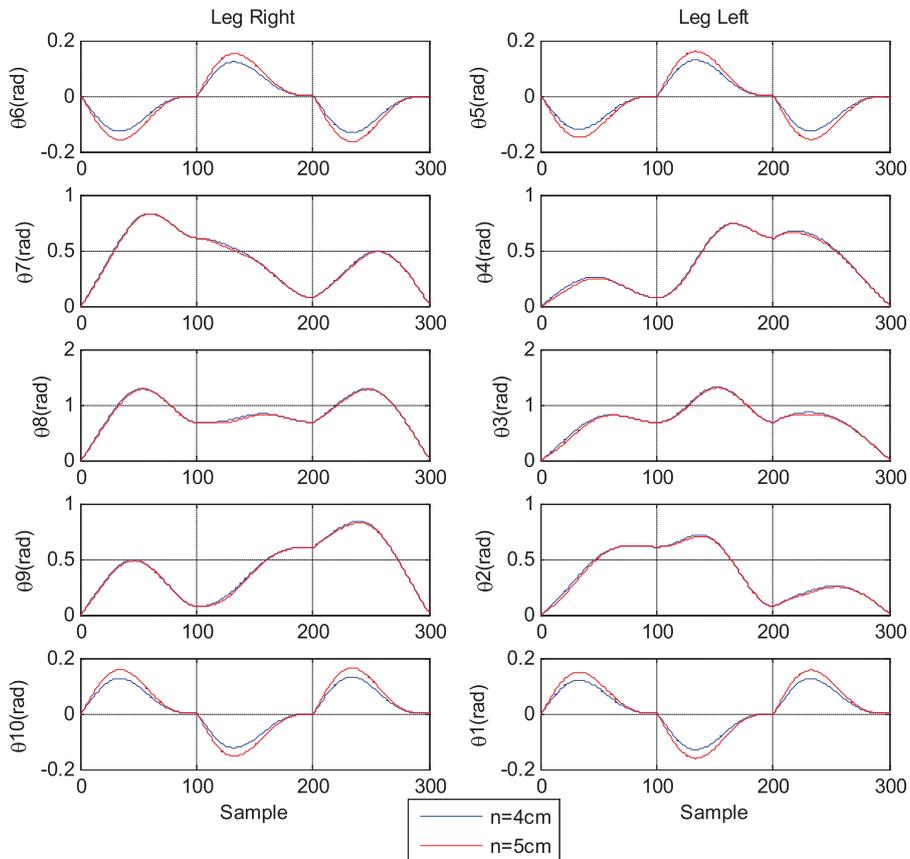
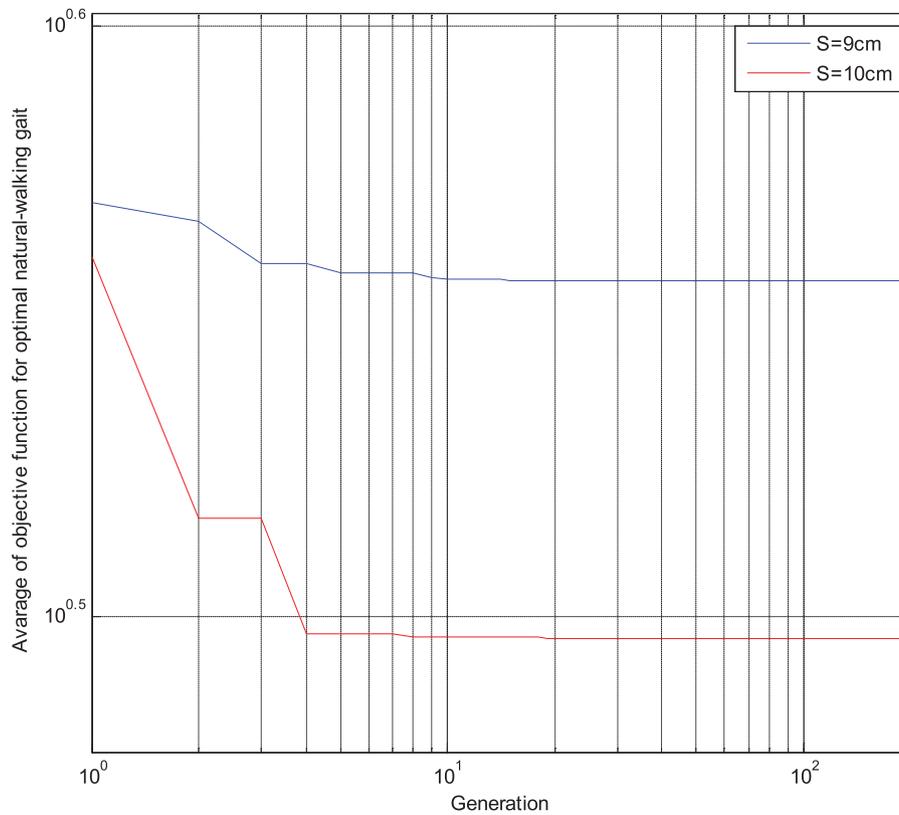


Figure 13 | Trajectories of the ten joint angles located at two legs of biped HUBOT-4.



**Figure 14** | Convergence rates optimized with proposed Jaya algorithm in the optimal procedure.

different cases regarding to various preset step-length values ( $S_{ref} = 9cm$  and  $S_{ref} = 10cm$ ) optimized by Jaya algorithm, and Figure 14 presents the convergence rates using the proposed Jaya algorithm based on the average results of objective function in the optimal procedures.

Continually Figure 15a and 15b presents the resulted 2D gait in the X-Z co-ordinate of the biped, with respect to different cases of preset step-length values. Results from Table 8 and Figure 15 strongly ensure that the biped robot successfully obtains a desired step-length value in control process.

Then Figure 16a and 16b illustrates the resulted ZMP values for two preset step-length magnitudes. These results confirm that ZMP trajectory keeps inside the supporting footprint surface and thus guarantees humanoid robot walking efficiently and robustly.

Consequently results from Figure 17 demonstrate the 10 rotary angular trajectories in one stepping cycle of the two legs of biped HUBOT-4 ( $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}$ ) during walking with two optimally resulted sets of gait parameters with respect to two different preset step-length values and tabulated in Table 8.

In summary based on the results of the optimization and simulation shown in Table 8, Figures 15–17, which once more strongly demonstrate that the work of preset step-length parameter -  $S_{ref}$  with four optimally selected parameters ( $S$ -step length,  $H$ -foot lifting,  $h$  - kneeling and  $n$  - hip swinging) ensuring the HUBOT-4 biped robot to steadily walking without falling apart and keeping

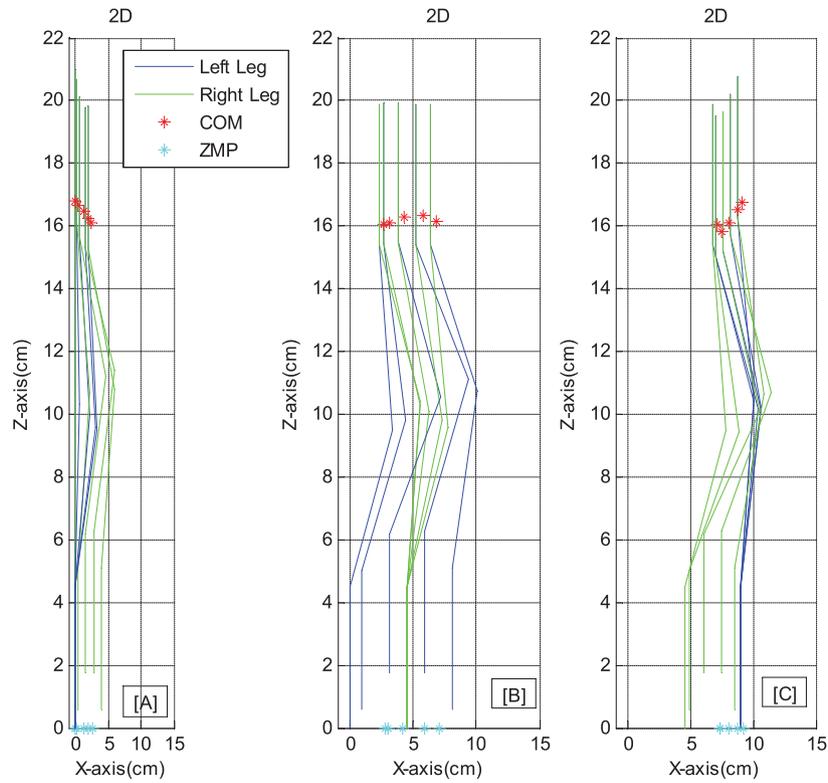
pace with desired step-length amplitude. The proposed algorithm with gait parameters optimized by Jaya algorithm proves convincingly feasible.

## 6. CONCLUSIONS

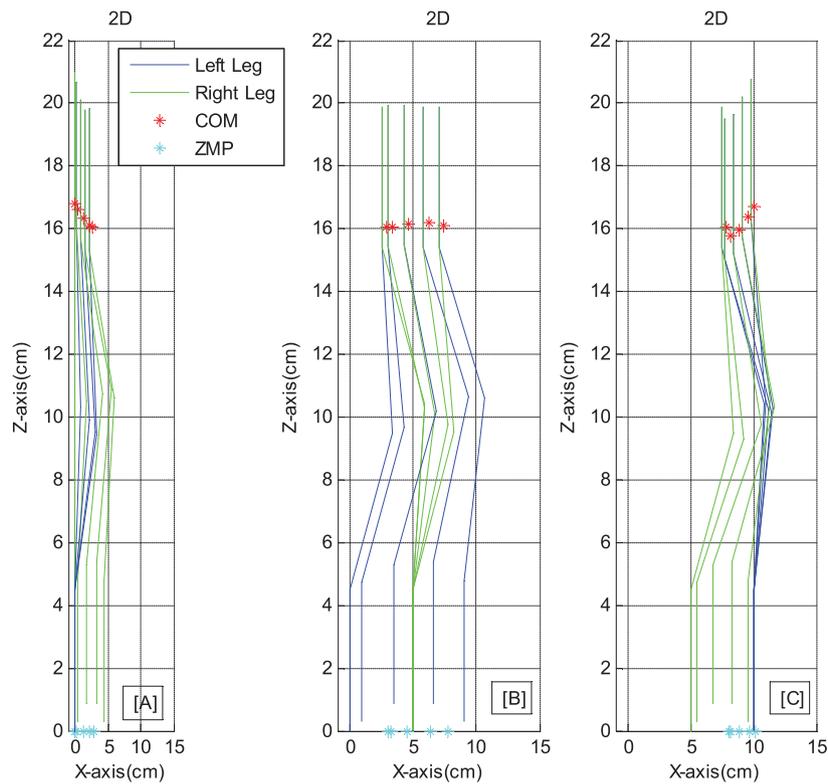
In this paper the optimization of the biped walking gait trajectory available to stably and naturally walking with preset step-length magnitude, has been obtained by quantifying the distance between the ZMP and the foot center in the step cycle and optimally quantifying the difference between the magnitude of step-length value and the step-length preset one. We applied Jaya optimization algorithm to minimize them by treating the gait parameters of biped robot as variables. This innovative concept has been verified by a computer simulation of a real biped robot HUBOT-4. Since the optimization process is performed for each time interval, the proposed technique can generate the biped gait trajectory in real-time.

## CONFLICT OF INTEREST

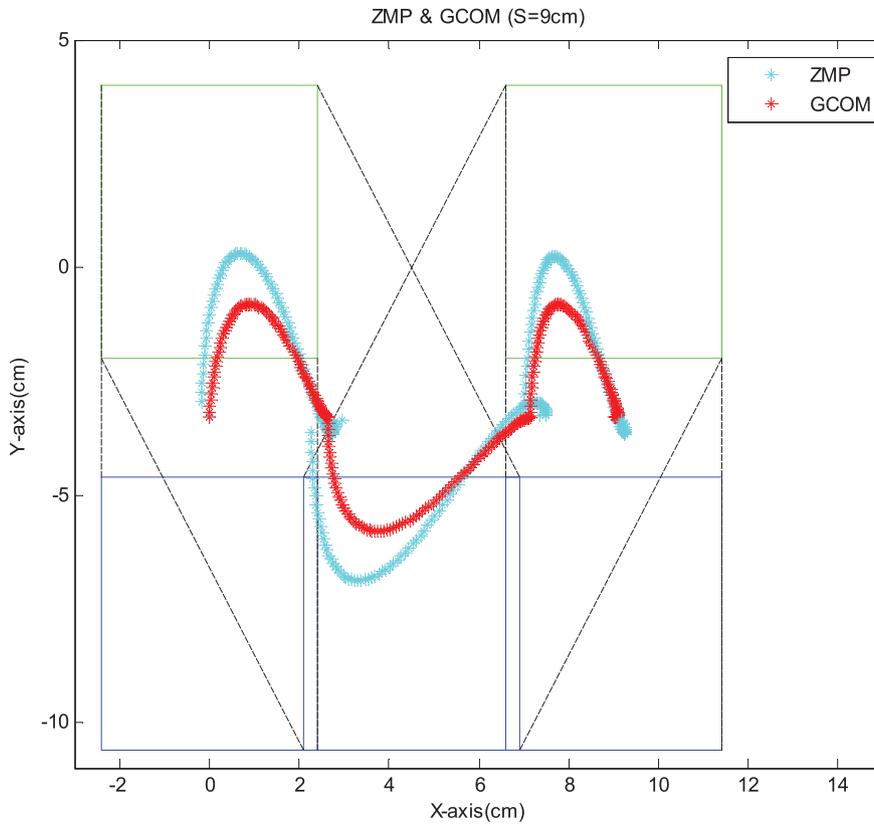
Authors confirm that there is no any conflict of interest in this paper.



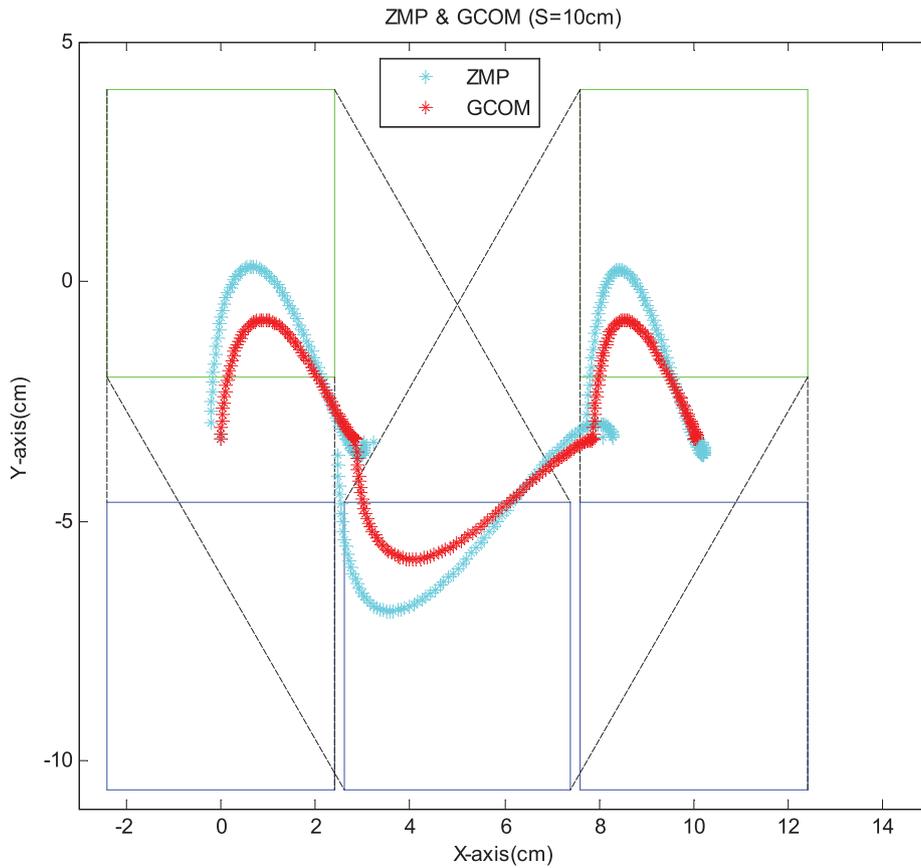
**Figure 15a** | Resulted 2D gait of biped robot with step-length  $S_{ref} = 9cm$ . [A]: Starting step. [B]: Periodic steps. [C]: Ending step.



**Figure 15b** | Resulted 2D gait of biped robot with  $S_{ref} = 10cm$ . [A]: Starting step. [B]: Periodic steps. [C]: Ending step.



**Figure 16a** | Resulted Zero Moment Point (ZMP) trajectories with  $S_{ref} = 9cm$ .



**Figure 16b** | Resulted Zero Moment Point (ZMP) trajectories with  $S_{ref} = 10cm$ .

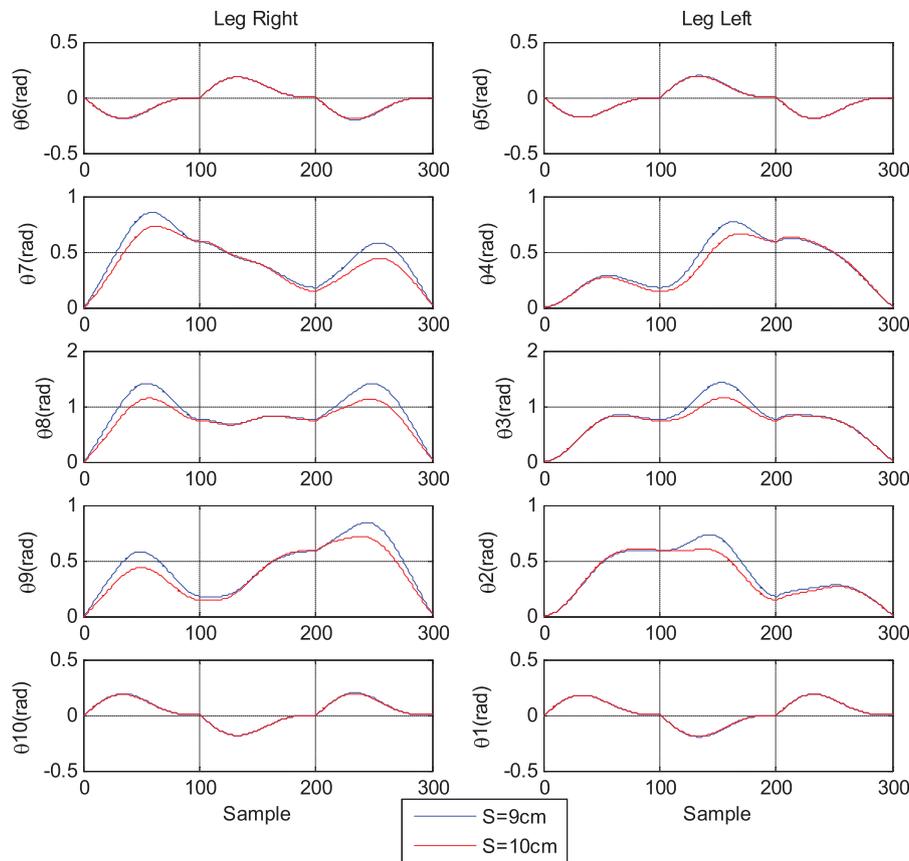


Figure 17 Trajectories of the ten joint angles located at two legs of biped HUBOT-4.

## AUTHORS' CONTRIBUTIONS

Anh H.P.H.: conception, final rewriting, corresponding. Huan T.T.: software, draft writing, data collecting and checking.

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