

Modeling the Optimal Range of Dairy Products

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ABSTRACT

The use of economic and mathematical models for planning the activities of organizations is a core area of scientific research. One solution to this problem is to use linear economic and mathematical models that take into account a complex of different factors and limitations. This article implements an attempt to determine optimal production and economic parameters of organizations of the agro-industrial complex. As a result, an economic and mathematical model was developed, which can serve as a universal tool for determining the optimal structure of organizations of any regions.

Keywords: *economic-mathematical model, optimum parameters, cost efficiency*

1. INTRODUCTION

Linear programming is used in various areas of economy when make economic and mathematical modeling. So Iranian scientists H. Zohali, B. Naderi and M. Mohammadi use linear programming to build mathematical models at economic lot scheduling problem in limited-buffer flexible flow shops. They developed two new mixed integer linear programming models for the problem. Fruit fly optimization algorithm is developed to effectively solve the large problems and the proposed algorithm is also evaluated by comparing with two well-known algorithms (tabu search and genetic algorithm) in the literature and adaption of three recent algorithms for the flexible flow shop problem. Swedish scientists M. R. Hesamzadeh, O. Galland and D. R. Biggar used linear programming for modeling short-run economic dispatch. Three stages model the state of power system before, during, and after contingency occurred, because the higher the cost of responding to contingencies ex post the greater the need there is to distort the ex ante operation of the power system [1].

Today linear programming at all and least squares method in particular is popular not only in practical research, it is used to manage and value early or multiple exercise real options. In business, it can be used in areas such as production planning to maximize profits, selecting components to minimize costs, selecting an investment portfolio to maximize profitability, optimizing the transport of goods to reduce distances, distributing staff to maximize work efficiency and scheduling work in in order to save time [2].

2. ECONOMIC-MATHEMATICAL MODEL

Let's build a mathematical model and optimize the product range of LLC «Belinskmoloko» taking into account the use of raw materials produced at LLC «Gorodishcheagro» (it is

one of the structural divisions). It should be noted that the optimization of the turnover of the herd and feed base at the holding's enterprises was made beforehand. The results were used in the development of the model under consideration.

The profitability of LLC «Belinskmoloko» depends on many factors, which can be conditionally reduced to two main groups - production and market. The first group includes factors of availability of production resources and efficiency of their use, possible savings in production costs, and some others. Market factors include market conditions and prospects for its dynamics, as well as the state of competition, the development of supply and demand, the price level, and so on.

We use a mathematical model of optimal use of funds of LLC «Belinskmoloko» for the acquisition of resources and production of products according to the maximum profit criterion, taking into account its sale on the market at the specified free-sale prices.

The model for determining the optimal volume of products based on the maximum profit criterion taking into account market restrictions on demand has the following form: [3,4,5]

Full use of milk resources for the production of basic products:

$$\sum a_{ij}x_j = x_{j1} \quad (i M)$$

Restriction on the possible purchase of milk:

$$rx_{ij} = x_{j3}$$

Compliance with balance sheet ratios by definition:

a) the total volume of individual product groups:

$$\sum a_{ij}x_j - x_{j2} = 0 \quad (i M)$$

b) the volume of intermediate raw materials obtained by various technologies:

$$\sum v_{ij}x_j - x_{j2} = 0 \quad (i M1)$$

c) the total volume of intermediate raw materials of each type:

$$\sum x_{j2} - x_{j2} = 0 \text{ (i M1)}$$

Full use of intermediate raw materials for the production of products from it:

$$\sum h_{ij2} x_j - x_{j2} = 0 \text{ (i M2)}$$

Full use of resources for production:

$$\sum x_{j3} \leq B_i \text{ (i M3)}$$

Compliance with restrictions on the production of certain types of dairy products:

$$Q_p \leq x_j \leq Q_j \text{ (i M4)}$$

Nonnegativity of all variables:

$$x_j \geq 0; x_{j1} \geq 0; x_{j2} \geq 0; x_{j3} \geq 0;$$

$$Z = C_j x_j \rightarrow \max,$$

where N is the set of j-type dairy products;
 N2-a set of intermediate raw materials of the j2 type;
 N3-set of monetary resources of the j3 type;
 M-group of restrictions on the use of milk;
 M1 is a group of restrictions on the production of intermediate raw materials;
 M2-group of restrictions on the use of intermediate raw materials;
 M3-group of restrictions on the use of funds;
 M4-group of restrictions on the release of certain types of products;
 X_j – volume of output of j-ro type dairy products (j N).
 X_{j1}-possible volume of milk purchase;
 X_{j2}-possible volume of obtaining intermediate raw materials of the j2 type (j2 N2);
 X_{j3}-expenditure of monetary resources of the j3 type on production (j3 N2);
 a_{ij}-the rate of consumption of milk for the production of a unit of production of the j-type;
 r-purchase price of 1 ton of products;
 V_{ij}-output of J2-type by-products (j2 N2) in the production of j - type products (j N);
 h_{ij}-the rate of consumption of intermediate raw materials j2-type (j2 N2) for the production of a unit of j-type (j N);
 B_i-reserves of monetary resources in the planned period;
 Q_p and Q_j - acceptable upper and lower limit sales volumes of products produced using j-technology.
 C_j-profit per unit of production produced using j-technology.

Here, B_i is considered a given constant. Therefore, the mathematical model can not be used to determine the optimal reserves of resources and, accordingly, the distribution of funds for their acquisition.

The task is to determine what types of dairy products and in what quantity should be produced by LLC «Belinskmoloko» with the available amount of money to obtain the maximum annual profit.

It is necessary to find out what resources and in what quantity to purchase, what types of products and in what volume to produce in order to maximize the annual profit, and the cost of purchasing resources does not exceed the available funds.

According to actual data, the annual profit from the sale of 1 ton of each type of dairy product is known. LLC «Belinskmoloko» does not have information about the dependence of sales volumes of each type of product on its selling price. Therefore, the optimization of selling prices for products has not been studied.

There is information about the minimum and maximum possible guaranteed volumes of annual sales of each type of product at their specified selling prices. The maximum volume of annual sales does not exceed the production capacity of the enterprise.

The actual annual sales volumes and some other indicators are presented in table 1. These types of products were selected for optimization, because they have the largest production volumes at the enterprise and are in constant demand. These data do not characterize the area of uncertainty in the sale of the corresponding product, but the volume of its guaranteed sale.

Analyzing the table, we can say that in general the activity of the enterprise is profitable. Unprofitable among the said range of dairy products is the production of cottage cheese 18% fat, the mass of curd 22% fat, dry skim milk (DSM) and peasant oil.

3. NUMBER ECONOMIC-MATHEMATICAL MODEL

In accordance with the setting of the task of its economic-mathematical model and original information, we use a system of variables presented as follows (table 1).

Annual output:

milk 2.5% - x₁

bilife 2.5% - x₂

jam 2.5% - x₃

sour cream 15% - x₄

2.5% - x₅

cottage cheese 18% - x₆

cheese 8% with raisins - x₇

weight 22% curd «Special» with raisins 0.25 kg - x₈

DSM - x₉

Dutch cheese - x₁₀

peasant oil - x₁₁

low-fat cottage cheese - x₁₂

low-fat kefir - x₁₃

Skimmed milk in oil production - x₁₅

Skimmed milk in sour cream production - x₁₆

Annual resource reserves:

milk for processing - x₁₄

Skimmed milk - total - x₁₇

Skimmed serum - x₁₈

Types of resources: from them: raw materials and basic materials - x₁₉

Auxiliary materials - x₂₀

Transportation costs - x₂₁

Fuel - x₂₂

Cold - x₂₃

S/P OPP-x₂₄

ESN - x₂₅

Table 1 Input data

| Indicators \ Products | milk 2.5%, 1l | bilife 2.5%, 0.5 kg | 2.5%, 0.5 kg | sour cream 15%, 0.2 kg | 2.5 per cent, 0.5kg | cottage cheese 18% | cheese 8%, 0.1 kg | weight 22% curd, 0.25 kg | DSM | Dutch cheese | Peasant oil | Low-fat curd, 0.25 kg | Low-fat kefir, 0.5 kg |
|---|---------------|---------------------|--------------|------------------------|---------------------|--------------------|-------------------|--------------------------|--------|--------------|-------------|-----------------------|-----------------------|
| Annual sales | | | | | | | | | | | | | |
| Production, t | 1219,0 | 468,3 | 193,1 | 94,5 | 547,0 | 46,8 | 114,1 | 126,8 | 163,0 | 66,0 | 560,5 | 166,9 | 761,3 |
| Sales of products for the year | | | | | | | | | | | | | |
| Production volume, t | 1218,9 | 468,2 | 193,1 | 94,48 | 547 | 46,8 | 114,1 | 126,8 | 163 | 65,9 | 560,5 | 166,9 | 761,3 |
| Full cost of 1 t, thousand rubles. | 9,5 | 12,6 | 12,0 | 34,1 | 12,0 | 66,2 | 63,1 | 75,4 | 65,6 | 101,1 | 88,1 | 54,8 | 8,4 |
| Profit (+) Loss (-), 1 t, thousand rubles. | 3,3 | 6 | 6,4 | 33,1 | 6,73 | -2,6 | 15,6 | -1,4 | -14 | 5,4 | -28,1 | 3,3 | 5,2 |
| Output | | | | | | | | | | | | | |
| Actual volume | 1219 | 468,3 | 193,1 | 94,5 | 547 | 46,8 | 114,1 | 126,8 | 163 | 66 | 560,5 | 166,9 | 761,3 |
| | 26,9% | 10,3% | 4,3% | 2,1% | 12,1% | 1% | 2,5% | 2,8% | 3,6% | 1,5% | 12,4% | 3,7% | 16,8% |
| Optimal volume | 1165 | 455 | 190 | 98 | 846,34 | 46 | 115 | 130 | 203,85 | 50 | 315 | 200 | 800 |
| | 25,25% | 9,9% | 4,1% | 2,1% | 18,34% | 1% | 2,5% | 2,8% | 4,4% | 1,1% | 6,8% | 4,3% | 17,3% |

General production costs - x_{26}

General expenditure - x_{27}

Management expenses - x_{28}

Commercial expenses - x_{29} .

Variables are linked by a system of limitations.

Full use of milk resources for production:

$$0,731x_1 + 0,754x_2 + 0,755x_3 + 4,5x_4 + 0,76x_5 + 5,3x_6 + 2,8x_7 + 5,9x_8 + 8,3x_{10} + 22,19x_{11} = x_{14}$$

Variable ratios indicate milk consumption per tonne of production.

Average production capacity per year thousand tons:

$$x_{14} = 11160$$

We calculate the average purchase price of raw materials for processing, taking into account the partial use of raw materials of their own production.

The market price of milk purchase is 6 rubles. 1 litre.

Cost of delivery of own milk plus transportation costs for delivery of milk to the plant - 3.24 p. - 0.42 p.

Average production capacity per year thousand. tone - 11160 l.

Production of milk on the farm per year according to the optimal plan - 2001 liters.

Percentage of own milk in the production structure (2001/11160)

The percentage of milk purchased is 82%.

The average purchase price of milk is 0.18 to 3.66 q 0.82 q 6 and 5.58 p.m. per liter.

Possible volume of milk purchases. $5.58x_{14} \leq 70000$, where variable x_{14} ratios correspond to the purchase price of 1 ton of milk.

Restrictions on the production of skimmed milk:

by production of oil: $19,97x_{11} = x_{15}$;

by production of sour cream: $3,5x_4 = x_{16}$;

Total skim milk:

$$0,19x_5 + 2,14x_7 + 1,5x_8 + 0,51x_{10} + x_{15} + x_{16} = 17.$$

Restriction on the production of serum:

$$6,5x_6 + 2,62x_{10} + 2x_{12} = x_{18}$$

Using skimmed milk:

$$0,269x_1 - 0,246x_2 - 0,248x_3 - 0,24x_5 - 5 - 6 - 6,2x_7 - 4,4x_8 - 12,3x_9 - 2,9 - 10 - 8,03x_{12} - 1,015x_{13} - 17 = x$$

Limit on the consumption of cash resources in the production of dairy products:

Raw materials and basic materials:

$$5,38x_1 + 6,92x_2 + 6,48x_3 + 17,82x_4 + 6,53x_5 + 49,2x_6 + 41,08x_7 + 50,9x_8 + 43,81x_9 + 60,25x_{10} + 62,64x_{11} + 37,38x_{12} + 3,7x_{13} \leq x_{19}$$

auxiliary materials:

$$0,58x_1 + 0,75x_2 + 0,73x_3 + 5,92x_4 + 0,73x_5 + 0,14x_6 + 2,89x_7 + 2,47x_8 + 0,78x_9 + 3,94x_{10} + 0,74x_{11} + 2,17x_{12} + 0,72x_{13} \leq x_{20} \text{ etc.}$$

Limit on annual cash costs:

$$x_{19} + x_{20} + x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} \leq 131765,2$$

The restriction on the production of certain types of products is written according to market research, planned production indicators, company capabilities, such as milk production of 2.5%:

$$x_1 > 1165 \text{ etc.}$$

The target function of the task is the maximum profit.

$$Z = 3.3x_1 + 6x_2 + 6.4x_3 + 33.14x_4 + 6.73x_5 - 2.6x_6 + 15.6x_7 - 1.35x_8 - 14x_9 + 5.4x_{10} - 28.1x_{11} + 3.3x_{12} + 5.2x_{13} \rightarrow \max$$

The mathematical model is a linear programming task that uses the simplex method [6, 7].

4. CONCLUSIONS

Comparative assessment of the options for the actual and optimal use of the company's money shows that without additional costs, provided the range of the dairy plant is

optimized, can be 8402.7 thousand. rub. increase its annual profit from the sale of basic dairy products. Consequently, the economic efficiency of the company as a whole increases. To achieve this goal, the following changes should occur in the structure of the sold products: a decrease in the share of peasant oil by 5.6% or 245.5 tons, as the most unprofitable, an increase in the production volume of 2.5% fat by 6.24% or 299.3 tons, as well as low-fat kefir by 0.5% or 171.2 tons. The allocation of resources for production is presented in Table 2.

Table 2 Distribution of resources to production

| Types of cost | Reserves of resources, thousands rub. | |
|-----------------------------------|---------------------------------------|----------|
| | Actual | Optimal |
| Raw materials and basic materials | 75798,68 | 73636,75 |
| Auxiliary materials | 4515,83 | 4612,6 |
| Transportation costs | 3911,26 | 3999,09 |
| Fuel | 2555,52 | 2280,98 |
| Cold | 2766,76 | 2332,15 |
| W/p OPP | 4709,44 | 4749,46 |
| ESM | 1126,4 | 1133,3 |
| General production costs | 4154,31 | 4144,1 |
| General economic expenses | 10699,44 | 10056,88 |
| Management costs | 1007,72 | 852,4 |
| Commercial expenses | 9292,92 | 8729,29 |
| Total annual costs | 120538,25 | 116566,0 |

It is clear from this table that at the optimal cost of production and sale of dairy products, the company saves 3,972.25 thousand rubles. There is also a saving in the resources of the following items of expenditure: raw materials and basic materials, general economic costs, cold and fuel. The largest reduction in raw materials costs can be explained by the decrease in the optimal volume of oil, which is the most resource-intensive.

The economic performance of the proposed production project is:

The total annual cost of production in 2008 amounted to 120538.25 thousand. rub., and in the project - 116566;

Profit in 2008 was equal to 3,229.3 thousand. ruble, and according to the project should be 11632 thousand rubles;

Cost margins in 2008 were 2.7% and the project's profit ability was 9.98%.

Cost margins at the optimal volume of production increase by 7.28%, which gives a significant economic effect. Therefore, in order to obtain higher financial values, the company needs to change the volume of output and sales of products, as well as the volume of purchases of resources.

Thus, the developed economic and mathematical model allows you to determine the optimal maximum profit criteria:

1) Production volumes of appropriate types of dairy products.

2) The amount of resources needed to implement the optimal dairy production plan.

The results of research using this model have shown that without increasing the cost of dairy production, it is possible to increase the profitability and profitability of the company. This demonstrates the high efficiency of the practical application of the developed economic and mathematical model of the optimal range of dairy products at Belinskamoloko LLC.

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