

Research Article

# Distributed Rotating Encirclement Control of Strict-Feedback Multi-Agent Systems using Bearing Measurements

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## ABSTRACT

This paper focuses on the distributed multi-target rotating encirclement formation problem of strict-feedback multi-agent systems using bearing measurements. To this end, an estimator is presented to localize agents' neighbor targets. Then, based on the trajectory planning method, a reference trajectory is constructed by three estimators, which are utilized to obtain the targets' geometric center, the reference rotating radius and angular. Finally, an adaptive neural dynamic surface control scheme is proposed to drive all agents to move along their reference trajectories, which satisfy the multi-target rotating encirclement formation conditions. A numerical simulation is provided to verify the correctness and effectiveness of our proposed control scheme.

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## 1. INTRODUCTION

Recent years have witnessed considerable attention on the rotating encirclement formation problem of multi-agent systems due to its significant potential applications in both military and civilian areas such as surveillance, search-and-rescue, reconnaissance, etc. [1]. Many interesting results have been achieved for the rotating formation or surrounding/encirclement control problem [1–7].

In practice, many physical systems, including manipulators, vessel, unmanned aerial vehicles (UAVs), can be written in the strict-feedback form [8]. Some important researches of the strict-feedback single/multi-agent system have been presented [8–11]. However, there is no research to date on the rotating encirclement control of high-order multi-agent system.

Motivated by the above discussion, for the first time, we consider the multi-target rotating encirclement formation problem of strict-feedback multi-agent systems, and only bearing measurements of targets can be obtained. To this end, we divide the problem into three sub-problems: target localization, trajectory planning and trajectory tracking. Four estimators are designed to construct a reference trajectory for each agent, and an adaptive neural dynamic surface control scheme is presented to drive all agents to move along their desired trajectories.

## 2. PRELIMINARIES AND PROBLEM STATEMENT

### 2.1. Graph Theory

Let  $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A}, \mathcal{B})$  be a weighted undirected graph corresponding to  $n$  agents and  $m$  targets, where  $\mathcal{V} = \{v_1, v_2, \dots, v_n, s_1, \dots, s_m\}$

denotes the set of vertexes,  $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$  denotes the set of edges,  $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$  denotes the weighted adjacency matrix of targets,  $\mathcal{B} = [b_{ik}] \in \mathbb{R}^{n \times m}$  denotes the weighted adjacency matrix from targets to agents. Let  $d(v_i, v_j)$  denote the shortest distance from the vertex  $v_i$  to  $v_j$ , for instance,  $d(v_i, v_j) = 1$  if  $(v_i, v_j) \in \mathcal{E}$ . The neighbor agents set of the agent  $v_i$  is denoted by  $\mathcal{N}_i = \{v_j \in \mathcal{V} \mid (v_i, v_j) \in \mathcal{E}\}$  and the neighbor targets set of the agent  $v_i$  is denoted by  $\mathcal{N}_i^T = \{s_k \in \mathcal{V} \mid (v_i, s_k) \in \mathcal{E}\}$ . The neighbor agents set of the target  $s_k$  is denoted by  $\mathcal{N}_k^T = \{v_i \in \mathcal{V} \mid (v_i, s_k) \in \mathcal{E}\}$ .

### 2.2. Problem Statement

Consider a multi-target multi-agent system consisting of  $n$  agents (Index set  $\mathcal{I} = \{1, 2, \dots, n\}$ ) and  $m$  stationary targets (Index set  $\mathcal{T} = \{1, 2, \dots, m\}$ ) with bearing-only measurements, where the dynamic of agent  $v_i$  is written in the following  $q_i$ -order strict-feedback form.

$$\begin{cases} \dot{x}_{ij} = f_{ij}(\bar{x}_{ij}) + x_{ij+1} \\ \dot{x}_{iq_i} = f_{iq_i}(\bar{x}_{iq_i}) + u_i \\ y_i = x_{i1} \end{cases} \quad (1)$$

where  $\bar{x}_{ij} = [x_{i1}^T, \dots, x_{ij}^T]^T$ , and  $\bar{x}_{iq_i}, y_i, u_i \in \mathbb{R}^2$  represent the states, output and control input of agent  $v_i$ , respectively.  $f_{ij}(\bar{x}_{ij})$  is an unknown continuous nonlinear function.

The objective of this note is to design the distributed control scheme using bearing-only measurements of targets and the neighbor position information of agents, such that strict-feedback agents are capable of achieving the multi-target rotating encirclement formation, which is properly formulated by Definition 1 using the polar coordinate transformation  $y_i = \bar{r} + [l_i \cos(\theta_i), l_i \sin(\theta_i)]^T$ .

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**Definition 1<sup>4</sup>.** The multi-agent system is said to achieve the *multi-target rotating encirclement formation* if

$$\lim_{t \rightarrow \infty} \left[ l_i - \lambda \max_{k \in \mathcal{T}} \|r_k - \bar{r}\| \right] = 0 \quad (2)$$

$$\lim_{t \rightarrow \infty} \left[ \theta_i - \theta_j - \frac{2\pi(i-j)}{n} \right] = 0 \quad (3)$$

$$\lim_{t \rightarrow \infty} [\dot{\theta}_i - \omega] = 0 \quad (4)$$

where  $i, j \in \mathcal{I}$ ,  $r_k$  and  $\bar{r}(t) = 1/m \sum_{k \in \mathcal{T}} r_k$  denote the position of the  $k$ -th target and the geometric center of all targets respectively. The design parameter  $\lambda > 1$  determines the radius of the desired rotation formation and  $\omega$  represents the desired angular velocity.

To facilitate the latter control design and analysis, we make some reasonable assumptions.

**Assumption 1.** All agents are connected in some undirected communication topologies and each target connected to at least one agent via the directed edge.

**Assumption 2.** The radius of the desired rotation formation is bounded, i.e., there exists a positive constant  $d^*$  satisfying  $\max_{k \in \mathcal{T}} \|r_k - \bar{r}\| \leq d^*$ .

**Assumption 3.** The desired angular velocity  $\omega$  and angular acceleration  $\dot{\omega}$  are continuous and bounded, i.e., there exists positive constants  $\omega^*$  and  $\dot{\omega}^*$  such that  $\|\omega\| \leq \omega^*$ ,  $\|\dot{\omega}\| \leq \dot{\omega}^*$ .

## 3. CONTROL DESIGN

### 3.1. Target Localization

With bearing-only measurements, the following estimator is proposed to obtain the neighbor target's position of agent  $v_i$  according to Shao and Tian [7].

$$\dot{\hat{r}}_{ik} = \alpha_{ik} (I - \varphi_{ik} \varphi_{ik}^T) (x_{i1} - \hat{r}_{ik}) \quad (5)$$

where  $k \in \mathcal{N}_i^T$ ,  $\alpha_{ik}$  is a positive design parameter, and  $\varphi_{ik}$  is the unit vector from  $x_i$  to  $r_k$ .

### 3.2. Trajectory Planning

We design the following distributed estimators to obtain the estimations  $p_i$ ,  $\hat{l}_i$  and  $\hat{\theta}_i$  of the desired geometric center  $\bar{r}$ , polar radius  $l_i$  and polar angle  $\theta_i$ , respectively.

$$\begin{cases} \dot{p}_{ik} = \beta_i \sum_{j \in \mathcal{N}_i} a_{ij} [p_{jk} - p_{ik}] + \beta_i b_{ik} [\hat{r}_{ik} - p_{ik}] \\ p_i = \frac{1}{m} \sum_{k=1}^m p_{ik} \end{cases} \quad (6)$$

$$\begin{cases} \dot{\rho}_{i1} = \gamma_{i1} \max_{k \in \mathcal{N}_i^T} (\|\hat{r}_{ik} - p_i\|) - \rho_{i1} \\ \dot{\rho}_{i2} = \gamma_{i2} \max_{j \in \mathcal{N}_i \cup \{i\}} (\|\rho_{j1}\|) - \rho_{i2} \\ \vdots \\ \dot{\rho}_{iM} = \gamma_{iM} \max_{j \in \mathcal{N}_i \cup \{i\}} (\|\rho_{jM-1}\|) - \rho_{iM} \\ \hat{l}_i = \lambda \rho_{iM} \end{cases} \quad (7)$$

$$\dot{\hat{\theta}}_i = \delta_i \sum_{j \in \mathcal{N}_i} a_{ij} \left[ \hat{\theta}_j - \hat{\theta}_i - \frac{2\pi(j-i)}{n} \right] + \omega \quad (8)$$

where  $\beta_i$ ,  $\gamma_{i1}$ , ...,  $\gamma_{iM}$ ,  $\delta_i$  are positive design parameters, and  $M = \max_{i, j \in \mathcal{I}} \{d(i, j)\}$ , which can be chosen as  $M = n - 1$  if it is not prior information.

Then, with the polar coordinate transformation, the reference trajectory of agent  $v_i$  is provided as follows.

$$\hat{y}_i = p_i + \left[ \hat{l}_i \cos(\hat{\theta}_i), \hat{l}_i \sin(\hat{\theta}_i) \right]^T \quad (9)$$

### 3.3. Trajectory Tracking

Define dynamic surface errors as follows.

$$\begin{cases} z_{i1} = x_{i1} - \hat{y}_i \\ z_{ij} = x_{ij} - \hat{\eta}_{ij} \end{cases} \quad (10)$$

where  $\hat{\eta}_{ij}(t)$  is the first-order filter estimation of the virtual controller  $\eta_{ij}(t)$  with the time constant  $\tau_{ij} > 0$  and the filter error is denoted by  $\tilde{\eta}_{ij} = \hat{\eta}_{ij} - \eta_{ij}$ .

$$\tau_{ij} \dot{\hat{\eta}}_{ij} + \hat{\eta}_{ij} = \eta_{ij}, \hat{\eta}_{ij}(0) = \eta_{ij}(0) \quad (11)$$

Then, we present the following adaptive neural dynamic surface control scheme.

$$\begin{cases} \dot{\eta}_{ij} = -\kappa_{ij} z_{ij} - \hat{W}_{ij}^T S_{ij}(\zeta_{ij}) \\ \dot{u}_i = -\kappa_{iq_i} z_{iq_i} - \hat{W}_{iq_i}^T S_{iq_i}(\zeta_{iq_i}) \end{cases} \quad (12)$$

$$\dot{\hat{W}}_{ij} = -\Gamma_{ij}^{-1} [\sigma_{ij} \hat{W}_{ij} - S_{ij}(\zeta_{ij}) z_{ij}^T] \quad (13)$$

where  $\Gamma_{ij} = \Gamma_{ij}^T > 0$  is an adaptive gain matrix,  $\hat{W}_{ij}$  and  $S_{ij}(\zeta_{ij})$  represent the estimation of the optimal weight matrix  $W_{ij}$  and the basis function vector respectively.  $\kappa_{ij}$ ,  $\sigma_{ij}$  are positive design parameters.

## 4. MAIN RESULTS

With the proposed control scheme in Section 3, we can easily obtain the following reasonable results.

**Lemma 1.** Consider the estimator (5) under Assumptions 1–2. Then for any  $i \in \mathcal{I}, k \in \mathcal{T}$ , the estimation position  $\hat{r}_{ik}$  will asymptotically converge to the actual position  $r_k$  of the  $k$ -th target.

**Proof.** The proof is similar to Theorem 3.1 in Shao and Tian [7].

Then, we define the estimation of the targets' geometric center as follows.

$$\tilde{r} = \frac{1}{m} \sum_{i=1}^n \sum_{k \in \mathcal{N}_i^T} \frac{1}{|\mathcal{N}_k^T|} \hat{r}_{ik} \quad (14)$$

Apparently,  $\tilde{r}$  will asymptotically converge to the actual geometric center  $\bar{r}$ .

**Lemma 2.** Consider the estimator (6) under Assumptions 1 and 2. For any  $i \in \mathcal{I}$ , the estimation position  $p_i$  will asymptotically converge to  $\tilde{r}$ .

**Proof.** The proof is similar to Lemma 4 in Zhang et al. [4].

Then, combining Lemma 1 with Lemma 2, we can conclude that the estimation position  $p_i$  of the  $i$ -th agent will asymptotically converge to the actual geometric center  $\bar{r}$ .

**Lemma 3.** Consider the estimator (7) under Assumptions 1 and 2. For any  $i \in \mathcal{I}$ , the following equation holds.

$$\lim_{t \rightarrow \infty} \left[ \hat{l}_i - \lambda \max_{i \in \mathcal{I}, k \in \mathcal{N}_i^T} \left( \left\| \hat{r}_{ik} - p_i \right\| \right) \right] = 0 \quad (15)$$

In other words, the estimation  $\hat{l}_i$  of the polar radius will asymptotically converge to the above value.

**Proof.** The proof is similar to Lemma 5 in Zhang et al. [4].

Furthermore, with Lemma 1 and 2, it is easy to see that  $\lim_{t \rightarrow \infty} \left[ \hat{l}_i - \lambda \max_{k \in \mathcal{T}} \left( \left\| r_k - \bar{r} \right\| \right) \right] = 0$ , which implies that  $\hat{l}_i$  satisfies the condition (2).

**Lemma 4.** Consider the estimator (8) under Assumptions 1–3. For any  $i, j \in \mathcal{I}$ , the following equations hold.

$$\lim_{t \rightarrow \infty} \left[ \hat{\theta}_i - \hat{\theta}_j - \frac{2\pi(i-j)}{n} \right] = 0 \quad (16)$$

$$\lim_{t \rightarrow \infty} \left[ \dot{\hat{\theta}}_i - \omega \right] = 0 \quad (17)$$

In other words, the estimation  $\hat{\theta}_i$  of the polar angle satisfies conditions (3) and (4).

**Proof.** The proof is similar to Lemma 6 in Zhang et al. [4].

Thus, from the polar coordinate transformation of (9), we can easily conclude that the reference trajectory  $y_i$  satisfies conditions of the multi-target rotating encirclement formation in Definition 1. Then, with the same analysis process as given in our article at ICAROB2020, we present the following theorem.

**Theorem 1.** Consider the multi-agent system (1) in the strict-feedback form with stationary multi-targets. Suppose that Assumptions 1–3 hold. For any bounded initial condition, if we choose design parameters satisfy  $c_{i0} > 0$ , then all agents will achieve the multi-target rotating encirclement formation with the proposed control scheme in Control Design.

**Proof.** The proof is similar to our article at ICAROB2020.

## 5. NUMERICAL SIMULATION

The strict-feedback multi-agent system, design parameters are chosen as follows and communication topologies are shown in Figure 1.

$$\begin{cases} \dot{x}_{11} = x_{11} + \cos(x_{11}) + x_{12} \\ \dot{x}_{12} = x_{11}x_{12} + u_1 \\ \dot{x}_{21} = \sin(x_{21}) + u_2 \end{cases} \quad \begin{cases} \dot{x}_{41} = x_{41} \sin(x_{41}) + x_{42} \\ \dot{x}_{42} = x_{41} + x_{42} + u_4 \\ \dot{x}_{51} = 3 + \cos(x_{51}) + u_5 \end{cases}$$

$$\begin{cases} \dot{x}_{31} = 2x_{31} + x_{32} \\ \dot{x}_{32} = x_{31} + 2x_{32} + x_{33} \\ \dot{x}_{33} = x_{31} \cos(x_{32}) + u_3 \end{cases} \quad \begin{cases} \dot{x}_{61} = 2x_{61} + x_{62} \\ \dot{x}_{62} = x_{61}x_{62} + x_{63} \\ \dot{x}_{63} = x_{61} + \sin(x_{62}) + u_6 \end{cases}$$

$$\alpha_{ik} = 2, \beta_i = 4, \gamma_{ik} = 4, \delta_i = 2, \kappa_{ij} = 50, \tau_{ij} = 0.01, \sigma_{ij} = 10$$

Clearly, Assumptions 1–3 are satisfied. Figure 2 shows that all agents can precisely locate their neighbor targets. Figure 3 illustrates

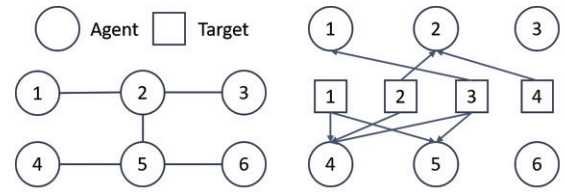


Figure 1 | Communication topologies.

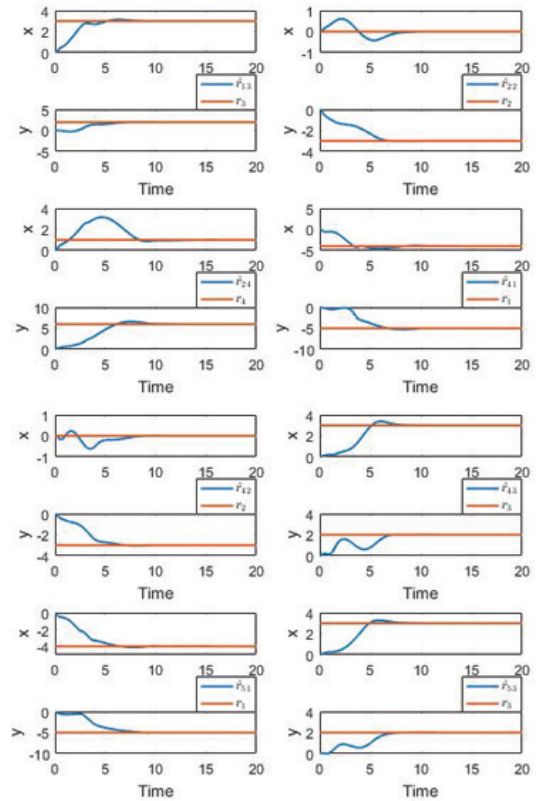
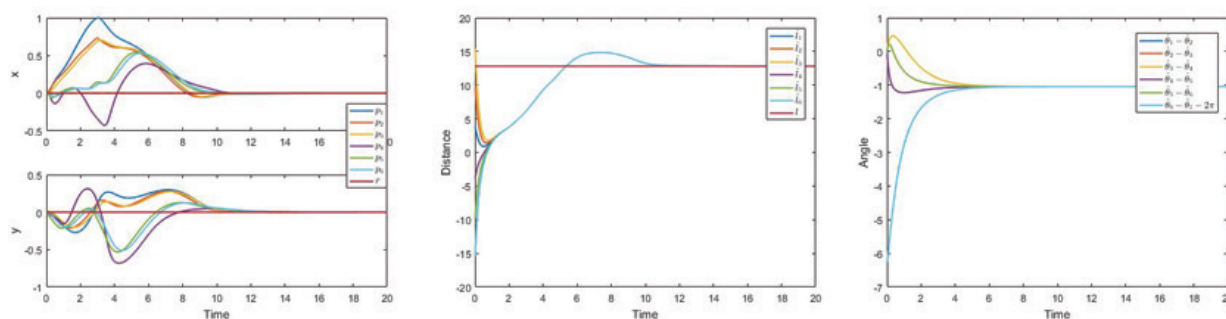
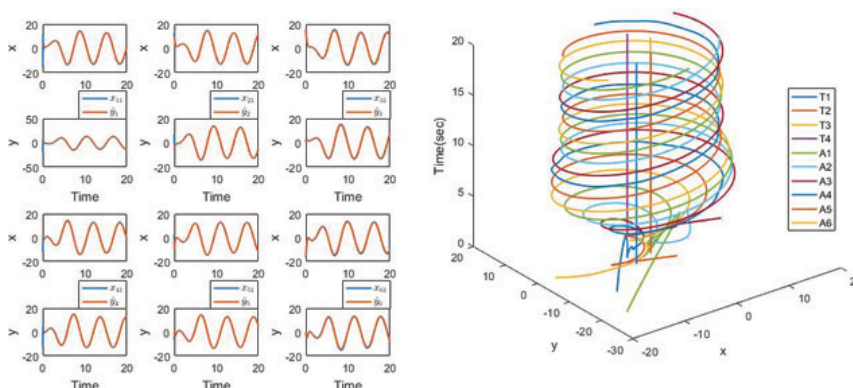


Figure 2 | Target localization.



**Figure 3** | Estimations of the targets' geometric center, the desired polar radius and polar angle.



**Figure 4** | Trajectory tracking and multi-target rotating encirclement formation in 3D.

that the reference trajectory satisfies conditions in Definition 1. Finally, Figure 4 indicates that the proposed control scheme can force all agents to track their reference trajectories and shows each agent's trajectory in 3D.

## 6. CONCLUSION

The collective multi-target rotating encirclement formation problem of strict-feedback multi-agent systems is investigated by dividing into three sub-problems. Our proposed control scheme can solve this problem well.

## CONFLICTS OF INTEREST

The authors declare they have no conflicts of interest.

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